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ABSTRACT

This conference proceedings contains three plenary session reports, 12 working group and 79 research reports, 35 short oral reports, 60 poster session reports, and two discussion group reports. Major papers (excluding "short orals" and "posters") include: (1) "Semantical Obstacles in Mathematics Understanding" (Carlos Arteaga and Manuel Santos); (2) "Mathematical Beliefs and Conceptions of College Algebra Students" (Victor V. Cifarelli and Tracy Goodson-Espy); (3) "Mathematical Intimacy: Local Affect in Powerful Problem Solvers" (Valerie A. DeBellis); (4) "A Modeling Approach to Non-Routine Problem Situations" (Helen M. Doerr); (5) "Posing Questions or Reformulation of Problems as an Activity to Perceive the Structure of Mathematical Problems" (Juan Estrada); (6) "An Analysis of Mathematics Problems in Middle School Textbooks" (Yeping Li); (7) "Recognizing Isomorphism and Building Proof: Revisiting Earlier Ideas" (Ethel M. Muter and Carolyn A. Maher); (8) "Students' Strategies for Searching the Instantaneous Flow in a Graphical Representation of Time versus Volume" (Rodolfo Oliveros); (9) "An Investigation of the Ability to Process and Comprehend Text on Student Success in Solving Algebraic Word Problems" (David K. Pugalee); (10) "Variables Influencing the Effectiveness of Small-Group Work: A Year Long Study of Collaborative Math Problem Solving in Two Fourth-Grade Classrooms" (Rose Sinicrope, Marion A. Eppler, Marsha Ironsmith); (11) "Creating the Conditions for Conceptual Change" (Judy Clark and Joan D. Lukas); (12) "Assessing Students' Attitudes toward Mathematics from a Multilevel Perspective" (George Frempong); (13) "Studying the Impact of Reformed Mathematics Curricula" (Mary C. Shafer and Norman Web); (14) "Connecting Students' Everyday Mathematics and School Mathematics" (Marta Civil, Melanie Ayers, Jose David Fonseca, and Leslie Kahn); (15) "A Vygotskian Action-Research Model for Developing and Assessing Conceptual Models and Instructional Materials Inter-Actively" (Karen C. Fuson, Bruce Sherin, and Steven T. Smith); (16) "Relating Equity to Classrooms Which Promote

Understanding: Identifying Relevant Issues" (Lynn Liao Hodge); (17) "The Power of One in a Mathematics Classroom: Lisa and Divisibility by Four" (Kathy M.C. Ivey); (18) "Students' Use of Function Ideas in Everyday Activities" (Joanna O. Masingila and Helen M. Doer); (19) "Mathematics Power: Developing the Potential to Do Work" (Rodney E. McNair); (20) "Using the Metaphor of Voice to Investigate the Mathematical Experiences of African American Students" (Vivian R. Moody and Patricia S. Moyer); (21) "The Impact of a Secondary Preservice Teacher's Beliefs about Mathematics on Her Teaching Practice" (Babette M. Benken and Melvin (Skip) Wilson); (22) "Implementing Mathematics Reform: A Look at Four Veteran Mathematics Teachers" (M. Lynn Breyfogle and Laura R. Van Zoest); (23) "Reflections about Listening and Dialogue as Evidence of Teacher Change" (George W. Bright, Anita H. Bowman, and Nancy N. Vacc); (24) "Shifting Beliefs: Preservice Teachers' Reflections on Assessing Students' Mathematical Ideas" (Patricia S. Moyer and Vivian R. Moody); (25) "Understanding How Prospective Secondary Teachers Avoid Accommodating Their Existing Belief Systems" (Daniel Siebert, Joanne Lobato, and Stacy Brown); (26) "Self-Estimating Teachers' Views on Mathematics Teaching--Modifying Dionne's Approach" (Gunter Torner); (27) "Learning to Teach Mathematics Using Manipulatives: A Study of Preservice Elementary School Teachers" (Sunday A. Ajose); (28) "Prospective Elementary Teachers' Constructions of Understanding about Multiplication with Rational Numbers" (Diane S. Azim); (29) "Narrative as a Tool for Facilitating Preservice Mathematics Teacher Development" (Olive Chapman); (30) "Teacher Change during an Urban Systemic Initiative" (Thomas G. Edwards and Sally K. Roberts); (31) "Cases as Contexts for Teacher Learning" (Megan L. Franke, Elham Kazemi, Jeffrey Shih, and Stephanie Biagetti); (32) "Preservice Elementary Teachers Learning to Integrate Mathematics with Other Subjects through Interdisciplinary Instruction" (Susan L. Hillman); (33) "Using Conceptual Design Tools to Foster Learning in the Game Design Environment" (Yasmin B. Kafai, Megan L. Franke, Jeffrey C. Shih, and Cynthia C. Ching); (34) "Redefining 'The Object' of Assessment in Clinical Interviewing" (Rochelle G. Kaplan and Peter M. Appelbaum); (35) "Professional Development and Systemic Reform: Some Important Connections" (Andrea Lachance and Jere Confrey); (36) "An Analysis of Two Novice K-8 Teachers Using a Model of Teaching-In-Context" (Cheryl A. Lubinski, Albert D. Otto, Beverly A. Rich, and Patricia A. Jaberg); (37) "Preservice Teachers' Ideas about Teaching Mathematics: Contributions to Frameworks for Teacher Education Courses" (E.E. Oldham, A.E. van der Valk, H.G.B. Broekman, and S.B. Berenson); (38) "Classroom Teachers Becoming Teacher Educators: 'Just' Facilitators or Active Agents?" (Deborah Schifter and Susan Jo Russell); (39) "Using Thought-Revealing Activities to Stimulate New Instructional Models for Teachers" (Roberta Y. Schorr and Richard Lesh); (40) "The Role of Formal and Informal Support Networks as Teachers Attempt to Change Their Practice from Traditional to Innovative Reform-Oriented Teaching of Mathematics" (David Feikes); (41) "Enhancing the Pedagogy of University Faculty through Mentoring and Reflection: Preliminary Observations" (Lynn C. Hart); (42) "Developing Teachers' Ability to Identify Student Conceptions during Instruction" (Miriam G. Sherin); (43) "Characterizing a Perspective on Mathematics Learning of Teachers in Transition" (Martin A. Simon, Ron Tzur, Karen Heinz, Margaret Kinzel, and Margaret Schwan Smith); (44) "Investigating Teachers' Insights into the Mathematics of Change" (Janet S. Bowers and Helen M. Doerr); (45) "Technology, Students' Understanding of Graphs, and Classroom Interactions" (Maria L. Fernandez); (46) "Learning the Standard Addition Algorithm in a Child-Centered Classroom" (Betsy McNeal and Betty Tilley); (47) "Megan: 'Seventeen Take Away Sixteen? That's Hard!'" (Catherine A. Pearn); (48) "Thinking in Units: An Alternative to Counting as a Basis for Constructing Number" (Grayson H. Wheatley and Anne M. Reynolds). (ASK)

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North American Chapter of the International Group for the

Psychology of Mathematics Education

VOLUME 2

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Proceedings of the
Twentieth Annual Meeting

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of the International Group
for the

Psychology of Mathematics Education

Volume 2

PME-NA XX

October 31-November 3, 1998
North Carolina State University
Raleigh, North Carolina U.S.A.

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- A vita and a writing sample.

History and Aims of the PME Group

PME came into existence at the Third International Congress on Mathematical Education (ICME 3) held in Karlsruhe, Germany, in 1976. It is affiliated with the International Commission for Mathematical Instruction.

The major goals of the International Group and of the North American Chapter (PME-NA) are:

1. To promote international contacts and the exchange of scientific information in the psychology of mathematics education;

2. To promote and stimulate interdisciplinary research in the aforesaid area with the cooperation of psychologists, mathematicians and mathematics teachers;

3. To further a deeper and better understanding of the psychological aspects of teaching and learning mathematics and the implications thereof.

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1998 PME-NA Proceedings

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1995	PME 19	Brazil	SE 057 135
1996	PME 20	Spain	SE 059 578
1997	PME 21	Finland	SE 061 119-121
1998	ONE 22	South Africa	Not yet avail.

Preface

The program for PME-NA XX was discussed and changes proposed by PME-NA members attending the October, 1997, annual meeting at Illinois State University. The theme of this conference is the richness and power of students' mathematical ideas where students are broadly defined to include children, adolescents, and adult learners. This theme is the focus of three plenary papers. Pat Thompson and Paul Cobb debate the merits of psychological frameworks versus sociocultural frameworks in their paper. Alan Schoenfeld discusses the theoretical implications of the Berkeley model for Teaching-in-Context along with standards to judge such models. Jere Confrey explicates a splitting-based analysis of multiplicative structures with examples of students who successfully build mathematics structure.

At the 1997 annual meeting, the Steering Committee proposed that preliminary proposals could be submitted in either English or Spanish, and this change in procedure was approved by the general membership. Another change proposed was to create 10 Working Groups with an appointed organizer and panel members as a means of increasing PME-NA attendance by senior researchers. The purpose of these working groups was to establish a community of researchers with common areas of expertise. Organizers of each working group have established goals and strategies to increase the scholarly activities within each of these 10 communities. It was expected that many of the working groups will continue to collaboratively pursue common research interests over the course of this year. The following working groups and organizers were established for pilot in 1998:

- Advanced Mathematical Thinking - Kathleen Heid
- Algebra - David Kirschner & Carolyn Kieran
- Collegiate Mathematics - Ed Dubinsky
- Gender and Mathematics - Suzanne Damarin & Diana Erchick
- Geometry and Technology - Douglas McDougall
- Probability and Statistics - Carolyn Maher
- Rational Number, Ratio, and Proportionality - Tom Post
- Representations and Visualization - Fernando Hitt
- Socio-Cultural Theories - Judit Moschkovitz & Karen Fuson
- Teacher Education - Martin Simon

Beyond the papers of the 4 plenary speakers and 10 working groups, there are papers from 2 discussion groups, 79 research reports, 40 short oral reports and 50 poster sessions. There were 232 proposals submitted for review. The acceptance rate for research reports was 56%. The research reports, discussion groups, short orals, and poster presentations are organized by topic following the pattern begun with the Proceedings of the 1994 PME-NA meeting. Proposals for all categories were blind reviewed by three reviewers with expertise in the topic of submission. Cases of disagreement among reviewers were refereed by a subcommittee of the Program Committee at North Carolina State University.

Submissions for the Proceedings were made on disk; read, edited, and formatted by the editors. The format of the papers was adjusted to make them uniform and to conform to the page limit specified in the documentation for manuscript submission.

The editors wish to express thanks to all those who submitted proposals, the reviewers of proposals, the PME-NA XX Steering Committee, and the PME-NA XX Program Committee. The Program Chair would like to extend special thanks to the mathematics and science education faculty at North Carolina State University for their support and generous contributions to make this a successful professional experience for the community of mathematics education researchers.

The Editors

Sarah B. Berenson, Chair of PME-NA
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Wendy N. Coulombe
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PROBLEM-SOLVING RESEARCH REPORTS

SEMANTICAL OBSTACLES IN MATHEMATICS UNDERSTANDING

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What do students see and understand when they are asked to examine a mathematical statement? This is an important question to evaluate the students' potential to work on mathematical problem-solving activities. This study documents the work shown by high school students who were asked to understand and use a mathematical definition. Results showed that students, in general, lack methods and strategies that help them identify the essentials of the definition and frequently they give different interpretations of it and use them inconsistently. Here, it is important to attend the type of knowledge that students actually bring into the classroom and take it into account to implement problem solving activities.

Introduction

Some high school teachers who try to implement problem-solving activities in their classroom often complain about the type of mathematical resources that their students bring into the classroom arena. It is common that when students are asked to solve nonroutine problems they experience difficulties not only in the process of designing a plan of solution but also during the initial stage of understanding the statement of the problem. It may be that the way students often learn basic mathematical ideas influence directly the way they access and use those resources in problem-solving activities. What does it mean that students have understood a definition or mathematical procedure or basic fact? is a question that needs to be addressed if one is interested in documenting the students approaches to mathematical problems. The present study documents what students show when they are asked to reflect on the use of simple definition or to access basic knowledge to solve problems in which the efficient use of resources is required.

Background to the Study

It is recognized that the type of knowledge that students bring into the classroom plays an important role in their understanding of mathematical ideas (Schoenfeld, 1985). Indeed, some instructors spend a lot of time trying to provide mathematical resources that students need or will require to solve problems. Here, there is an explicit distinction between learning basic resources and learning to solve problems. Perhaps, if the students are asked to work on their study of basic facts or resources in the same way as they learn to solve nonroutine problems, then they could show more efficient way to access and apply those resources when they are required in problem situations. This idea is completely consistent with what Hiebert et al. (1996) called problematizing the subject. That is, in the process of understanding a definition, algorithm, concept, or mathematical problem students have to think of a set of questions that need to be discussed as a necessary condition in their learning.

Subject, Procedures, and Conceptual Framework

Fifty-two tenth-grade students participated in the study. The students had taken a first algebra course which included the study of basic properties of number systems (integers, rational, irrational, real), algebraic expressions (reduction of similar terms), linear equations, system of linear equations, quadratic equations, and functions. The course that they were going to take (the data were collected during the development of this course) included similar content but a main difference was that there were a couple of new topics (complex numbers, inequalities, and the study of equations with degree greater than 2). The teacher planned a problem-solving oriented course. He basically wanted to introduce ideas related to "searching for patterns", "use of tables", "reformulation of problems," and explicit discussion among different ways or methods of solutions to the problems. Given that the problems initially selected by the teacher, for the students discussion and examples, required the use of resources studied previously, it was important to document to what extent the students had actually understood them. Here, the teacher asked the students to work on a questionnaire that included questions in which they needed to work on the meaning and application of definition and basic procedure. The analysis of the information gathered from the students' responses was first based on the identification of key elements of the definition and then to what extent, they were present in the students' work. That is, it was important to document what Greeno et al. (1992) identify as affordances or essentials of the situation in study. The results shown by the students were crucial to redesign the initial goal of the problem-solving course and to focus on aspects that could help students to understand the statement of a problem. In addition, it was evident that in order to understand basic resources, the students needed to examine and answer a set of questions in which the strength and limitations of such resource are shown.

The understanding of a definition. In this work, there is interest in the students' responses given to a set of questions in which they had to show their understanding of a given definition. That is, it was important to document how they understood and applied the meaning of that definition. The definition, given in written form to the students, was: Two positive integers are called COMPATIBLES if they only share two common divisors.

In each of the next six statements respond YES or NO and give an argument to support your response:

(a) ( ) Are 6 and 15 compatibles numbers?
(b) ( ) Are 9 and 16 compatibles numbers?
(c) ( ) Are 6 and 18 compatibles numbers?
(d) ( ) Are 1.5 and 3 compatibles numbers?
(e) ( ) Is 5 a compatible number?
(f) ( ) Is 12 a compatible number?

Note that the first three questions included three pairs of positive integers and only one pair fits the definition. The idea here was to explore whether the students were able to identify ways to get the factors of the given numbers and locate the common ones. The fourth question included a number which was not an integer (1.5), therefore, the definition could not be applied. In the last two questions, the statement only included one number (not a pair) and the definition, here, could not be used either.

The criteria used to evaluate the students' responses focused on to what extent their work involved essential aspects of the definition. Particularly, there was interest in documenting the extent to which students utilized aspects of the definition that included:

(a) the definition should be applied to only pair of numbers,
(b) the pair of numbers must be positive integers,
(c) the existence of common divisors to both numbers,
(d) the set of common divisors (way of counting them) and mentioning when there are only two, less or more than two, and
(e) the list of common divisors.

Results

Fifty-two students were asked to work on the definition and answer the questions. Students who only responded YES or NO without supporting their responses or did not answer the question at all were not considered for the analysis. The presentation of the results begins with the identification of the number of students who showed in their responses some of the criteria listed above. For example, 46 students responded to the questions (a), (b), and (c). Only 19 students, somehow, used the information in which it was important to attend to the common divisors. Seventeen students mentioned the number of common divisors but only 7 students provided a list of them.

Question (d) was responded to by 48 students; only 6 realized that 1.5 is not an integer and therefore, the definition could not be applied to that pair. It is interesting to observe that 42 students did apply the definition directly.

Forty-seven students responded to questions (e) and (f). Only 7 argued that there was one number only and the definition required a pair of numbers to be used; the other 40 students did not see that the definition involved a pair of numbers and responded to these questions by applying it to one number.

It is clear that the majority of the students experienced difficulties in understanding the key aspects involved in the given definition. A closer analysis of the students' responses showed that they often changed the original definition or introduced other information when applying it to some particular cases.

Two students out of the 52 showed that they understood the definition and responded consistently to all of the questions. Their responses are reported in Table 1.

Table 1

Student	Are 6 & 15 compatibles?	Are 9 & 16 compatibles?	Are 6 &18 compatibles?
A	No, because they only share one divisor	No, because they do not share any divisor	Yes, because they share two divisors: 2 and 3
B	Yes, because 3 and 1 are only their divisors	No, because they do not have common divisors	No, because their divisors are 2, 6 , 3 & 1

Here, it is important to observe that student A does not include the unit or the number itself as possible divisors. However, his reasoning is consistent throughout all of his responses (see Table 2).

Other group of students seemed to interpret the definition as "Two numbers are COMPATIBLES if one of them divides the other". Examples of these type of students' responses are shown in Table 3.

It is important to mention that these 5 students abandoned their definition to answer the questions in which there was only one number. Their

Table 2

Student	Are 1.5 &3 compatible?	Is 5 compatible?	Is 12 compatible?
A	No, because 1.5 is not an integer	No, because we need two integers	No, this is only one integer number
B	No, because 1.5 is a decimal fraction	No, because there should be another integer	No, because there should be another integer

Table 3

Student	Are 6 & 15 compatibles?	Are 9 & 16 compatibles?	Are 6 &18 compatibles?
C	No, because 6 can't divide 15	No, because 9 can't divide 16	Yes, because 18 can be divided by 6
D	No, because they can't be divided by one of them	No, because 16 can't be divided by 9	Yes, because 6 is a divisor of 18
E	No, because 6 is not its divisor	No, because 9 is not a divisor of 16	Yes, because they are exact divisors
F	No, because they are not divisible	No, because the result (division) are not integer numbers	Yes, because they both are divisible.
G	No, because 6 is not a divisor of 15	No, because they can't be divided	Yes, because 6 fits in 18

responses included "no because it is an odd number; yes, because it has various divisors; no because it does not have two divisor, etc." Here, it was clear that students did not realize that the original definition involved a pair of numbers.

Yet another group of students interpreted the definition in the following manner: Two numbers are COMPATIBLE if each one has two divisors. (See Table 4.)

These students did not maintain their definition to examine the other questions. They applied the definition even when they have only one number. The word integer that appears in the original definition was totally ignored. (See Table 5).

Finally, there were other interpretations of the original definition given by no more than one student. One student, for example, only checked the condition of finding more than one divisor as the only condition to apply the definition. In other cases, the students were not consistent in their responses. That is, they used a criterion to answer a question and changed to other interpretation in the other questions. (See Table 6.)

Discussion of Results

It is clear that the students experienced difficulties in, first, identifying the key components of the definition, and later in applying them to exam-

Table 4

Student	Are 6 & 15 compatibles?	Are 9 & 16 compatibles?	Are 6 &18 compatibles?
H	No, because 6 has more than 2 divisors	No, because 16 has more than 2 divisors	No, because both have more than 2 divisors
I	No, because 6 has 3 divisors and 15 also has 3 divisors	Yes, because both have 2 divisors	No, because 6 has 3 divisors and 18 has four
J	Yes, because 6 has two common divisors: 3 & 2 and 15 has common divisors 5 & 3	No, because 9 only has one divisor 3 and 16 has common divisors 2, 4, 8.	Yes, because 6 has 3 & 2 as common divisors while 18 has 9 and 2

Table 5

Student	Are 1.5 &3 compatibles?	Is 5 compatible?	Is 12 compatible?
H	Yes, because they are divisible by themselves and one	Yes, because it is divisible by itself and the unit	No, because it has more than one divisor
I	Yes, because both have two divisors	No, because it has only one divisor	No, it has more than 2 divisors
J	No, because 1.5 is a decimal fraction	No, because there should be another integer	No, because there should be another integer

Table 6

Student	Are 6 & 15 compatibles?	Are 9 & 16 compatibles?	Are 6 &18 compatibles?
K	Yes, because they are divisible by 1 and 3	No, because they are not compatible	Yes, because they are divisible by 1, 2, 3, and 6
Student	Are 1.5 &3 compatibles?	Is 5 compatible?	Is 12 compatible?
K	Yes, because they are divisible by 1 and 1.5	Yes, because it is divisible by 1 and 5	Yes, because, it is divisible by 1,2,3,4,6, 12

ine particular cases. Teachers rarely spend time exploring what students see or understand when they interact with basic mathematical ideas. This study shows that the process of reading and understanding mathematical statements involves the use of strategies, on the part of the students, that allow them to examine the pertinence and importance of the information. Is it important to have two integers numbers? What does it mean to have common divisors? What does it mean that two integers only share two common divisors? These are fundamental questions that students need to reflect on while trying to understand the definition. Similarly, it seems that students lacked a sense of contrasting their responses, since it was notable that they often used an interpretation to respond the first three questions and showed other interpretation to answer the rest of them. An explicit reflection on whether or not the argument used to explain their response was consistent throughout their work seems to be absent in the students' answers. Perhaps, the students are used to hearing from their teachers what aspects of the definition (in this case) are important to pay attention to and then apply a procedure to search for common factors to find compatibles numbers.

This study shows that when students have to explore the essentials of a mathematical statement on their own and analyze them in an ample perspective, including here the use of a procedure or an algorithm, they lack strategies that help them make sense of the statements and they frequently reduce their approaches to trying to apply rules or procedures. Another feature that appeared consistently in the students' work was the difficulty they experienced in writing up the support of their responses. They sometimes repeated part of the question "no, because they are not compatibles" or were not explicit in their explanations. It is clear, from this study, that students need to pay attention to the development of their writing skills.

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MATHEMATICAL BELIEFS AND CONCEPTIONS OF COLLEGE ALGEBRA STUDENTS

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Among four-year universities, the number of students required to enroll in College Algebra classes forms a critical mass that presents special challenges for the mathematics faculty whose mission it is to provide quality instruction. The large numbers of College Algebra students enrolled in these classes can be traced back to the 1970s, when remedial enrollments increased 72% (Leitzel, 1987). The National Center for Education Statistics reports 34% of entering freshmen at two-year public colleges were required to take remedial coursework in mathematics as were 18% of the freshmen enrolling at public four-year institutions (NCES, 1995). Other sources have placed similar figures much higher (Watkins, 1993). Because College Algebra serves as the core mathematics requirement for many majors, both universities and community colleges are looking for innovative ways to address the needs of College Algebra students.

Purpose and Theory

The purpose of the study was to examine the beliefs and conceptions of College Algebra students, with the view that their mathematical conceptions and beliefs interact to influence their cognitive actions in mathematical learning situations. The role of mathematical beliefs in the evolution of mathematical concepts needs to be documented and explored. According to Schoenfeld, the mathematical beliefs of students help constitute their "mathematical world view" (Schoenfeld, 1985, p. 157), and hence play a crucial role in the ways they "see" the mathematical problems they face. Furthermore, these mathematical perspectives are abstracted by students "from their mathematical experiences with objects in the real world and from their classroom experiences with mathematics" (Schoenfeld, 1985, p. 157). Vergnaud hypothesized a formal connection between the mathematical beliefs and conceptual actions of learners, asserting that problem solvers often demonstrate their "mathematical beliefs-in-action" as they solve problems, and that these beliefs serve them as conceptual models upon which they can develop successful solution strategies (1984, p.7). Given the theoretical underpinnings discussed above, one of the research questions the study addressed was- What is the role played by the students' mathematical beliefs in the evolution of their mathematical knowledge?

Methods

Subjects came from College Algebra classes at two universities in the southern United States. A total of 115 subjects completed a mathematics beliefs and attitudes survey developed by Yackel (1984). In addition, 25 of the students participated in a series of individual teaching interviews, which occurred bi-weekly and lasted about 40 minutes each. Each interview included approximately 20 minutes where the students solved algebra tasks given by the researchers; during the remaining time, the students introduced their own problems and questions.

Analysis

The analysis proceeded as follows. First, the survey of beliefs and attitudes was compared with students' activities in the interviews. Next, the interview data were examined through protocol analysis. The video-taped recordings were examined to identify instances where significant conceptual structuring activity appeared to occur. This enabled the researchers to focus on episodes of novel activity, and make inferences about the constructive role played by the subject's mathematical beliefs in the evolution of their conceptual knowledge. In addition to the video protocols, transcripts of the videos, paper-and-pencil records, the researchers' field notes, and the subjects' written tests were examined and used to develop case studies.

Our survey results are consistent with what other researchers have found regarding the nature of mathematical beliefs and its impact on performance (Frank, 1986; Sackur and Drouhard, 1997; Schoenfeld, 1985). For example, 90% of the students demonstrating high-level achievement in the classes demonstrated flexible mathematical beliefs, viewing mathematics as a tool of their reasoning that is supposed to make sense to them, and that the teacher's way of solving math problems represents only one of many possible solutions. In contrast, 88% of the students demonstrating low-level achievement in the classes appeared to have more rigid beliefs about mathematics, viewing mathematics as a collection of rules and tricks, where the teacher determines what is correct and the student's goal is to imitate the actions of the teacher.

In this paper we focus on the mathematical activity of two subjects by examining episodes that illustrate the significant interplay between the students' beliefs, their conceptions, and their demonstration of mathematical structure through their problem-solving activities. We will approach these episodes by examining: (a) how the students conceived of and interpreted their problems initially; (b) the complexities of their mathematical ideas; and (c) how they worked through the dilemmas and difficulties they faced as they solved their problems.

An interview with Brad: Brad was a first-year business major who had taken College Algebra the previous semester and earned a grade of D.

He was repeating the class, a practice common among College Algebra students, because he needed a grade of C to satisfy his academic major.

During the first interview, Brad worked a series of tasks that involved simplifying radicals and applying the laws of exponents. After completing the tasks, Brad asked a question from the current homework on radicals. Brad had tried to simplify the expression $2\sqrt{50} + 12\sqrt{8}$ using the laws of radicals, and he was concerned that his answer, 34, did not agree with the answer given in the book, $34\sqrt{2}$. The interviewer asked Brad to re-work the problem at the blackboard.

> Brad: *I've worked it out twice but I didn't get the answer that's in the back of the book. First thing I do is look at radicals and see if I can simplify anything, just to drop one of the radicals. And um ...., you can't break these down into terms that can't so ... 50 will break down into 2 and 25, which are both perfect squares (sic), so that's what I went ahead and did. (Write $2\sqrt{25 \Diamond 2}$) Some people like to break them up, I'll keep them together. And 8 is not a perfect square either, but I know that 2 and 4 are (sic), which are factors of 8, so I went ahead and wrote that down (Writes $12\sqrt{2 \Diamond 4}$) (Re-writes entire expression)*

Brad's retrospective reporting of how he tried to solve the problem demonstrated an overall understanding of the task—that he could both re-capitulate and monitor his prior activity in an objective manner. In addition, while Brad invoked an appropriate strategy, his problem involved making sense of a discrepancy between his answer and that given in the back of the book.

> Brad: *And from here I just go ahead and take the square root of this, 25, which would bring the 5 out front, which would leave me ... 2◊5 and go ahead and bring the 2 out which would be 1, right ? ... or now it'd just be a 2, right ?...(stares in space, rolls eyes) ... and then plus 12 then ... that that'll just be 1 and bring out a 4, which will be 2, and we multiply by 2, ... 2 x 5 will be 10, and 12 x 2 will be 24, which will leave you with 34, but that's not what the book got.*

Brad's hesitation in asserting that he could simplify $\sqrt{2} = 1$ ("go ahead and bring the 2 out which would be 1, right ?") indicated that he was becoming aware of the probable source of his problem, that $\sqrt{2}$ may not simplify to 1 as he had previously thought.

> Brad: *The book got $34\sqrt{2}$. I can't figure where $\sqrt{2}$ is ?*
>
> Interviewer: *It looks awfully close, only thing is the $\sqrt{2}$ there in the answer. Why don't you look back at an earlier step and see if there's some place where there could've been a $\sqrt{2}$ and maybe it got lost in the shuffle when you reduced things. Where do you think the $\sqrt{2}$ might be ?*

Brad: *(reflects) ... There and there. (points to* $\sqrt{25} \lozenge 2$ *and* $12\sqrt{2} \lozenge 4$ *)*

Interviewer: *So, what did you do at that point in the process ?*

Brad: *I just took the square root of 25, which was 5 and the square root of 2, ... (long reflection here) ... that's not perfect! ..., yes, it's perfect, ... yeah for some reason, I cannot ... (realizes he has a problem here, but tries to work it out)*

Interviewer: *So the question is, is* $\sqrt{2}$ *perfect ?*

Brad: *... no, it's not, is it. You know what I was getting confused with ? ... is because that (writes* $^{2}\sqrt{2}$ *) and for some reason I thought (cancels 2s in expression* $^{2}\sqrt{2}$*, e.g.,* $^{\cancel{2}}\sqrt{\cancel{2}}$ *). Maybe that's where I got lost, that has to be it, because there's no other place.*

Interviewer: *So why don't you fix it from this point on ?*

Brad: *OK, so just go from this line ? (goes back to his board work and starts with* $2\sqrt{25} \lozenge 2 + 12\sqrt{2} \lozenge 4$*) Um.* $2 \lozenge 5 \sqrt{2}$ *Still gonna keep the* $\sqrt{2}$ *and it's gonna be plus 12, um* $\sqrt{4}$ *is 2, still have* $\sqrt{2}$ *there. Then we go ahead and multiply that to be* $\sqrt{2}$*,* $5 \lozenge 2\sqrt{2}$ *plus* $12 \lozenge 2 \sqrt{2}$*, that stays there. (writes* $10\sqrt{2} + 24\sqrt{2}$*).*

Brad: *(several seconds reflection) I guess this is just like 10X + 24X, the X stays the same and you just go ahead and bring down the radical. Then 24 and 10 is 34 (writes* $34\sqrt{2}$ *) O.K. That's what I was doing. That's the kind of mental lapse I'll have, that right there. ... that's crazy, for some reason it didn't register with me on the homework And that's the kind of crazy thing I do ... crazy little careless mistakes like that. It kills me on the test. I usually catch it on the homework, I checked it twice.*

We believe that Brad's episode is noteworthy for the following reasons. First, Brad was able to distance himself from his prior activity and objectively review, monitor, and then report results to the interviewer. College Algebra students are seldom able to engage in such retrospective analysis of their actions. That Brad was able to demonstrate such a grasp over his actions indicates both the robust nature of his conceptions (his knowledge of what he needed to do) and the strength of his convictions about how these types of problems are to be solved. He systematically set about to simplify the radicals (#2-3) and never wavered from his belief that his overall reasoning was sound—he knew what he needed to do to solve the problem with the radicals and could carry out and evaluate the efficacy of his actions. Second, Brad's inability to self-diagnose and correct his erroneous idea about cancellation of radicals ($^{\cancel{2}}\sqrt{\cancel{2}} = 1$), suggests that his misunderstandings were deep-rooted within his flow of continuous action.

While Brad could "see" an overall structure of appropriate solution activity to carry out, he had great difficulty isolating the source of error even after repeated attempts. It was only with the intervention of the interviewer's questions that Brad became aware of the error and set about to correct his solution accordingly.

An Interview With Carrie: Carrie was a second-year student whose performance in the class was consistently in the B-to-upper-C range. Carrie's responses on the beliefs survey indicated that she believed mathematics to be difficult because in order for one to be successful solving problems, one must remember many rules and procedures. She indicated that she thought mathematics was important for many careers but that she personally took mathematics courses only because they were required. She also indicated that she thought some people were naturally better at mathematics than others but she strongly disagreed when asked if mathematical ability was determined by gender. During the latter part of the first interview, Carrie introduced a rather difficult complex fraction problem from homework that had puzzled her.

Summary of activity. Carrie began her work on this problem by factoring m^2-4 (#2). Immediately thereafter, Carrie came to her first major decision–was this a division problem? Initially, Carrie stated that she did not know what to do with the denominator, 1/(m+2). In describing her source of indecision, it appeared that the numerator posed the more immediate problem for her. Carrie stated that she wanted to work on the numerator using the least common denominator which she identified very quickly as (m+2)(m-2). While Carrie had some difficulty describing what she wanted to do, her actions indicated that she understood the process for combining rational expressions. She correctly combined the terms in the numerator (#3); however, she also altered the denominator from 1/(m+2) to m+2. The interviewer intervened with a question and the subsequent episode served as a second major decision point for Carrie as she solved her problem.

I: How did you get this in the denominator (points to m+2)?

Carrie: Do I apply the same LCD to this part or do I do it separately? Basically I get the LCD which is that [points to (m-2)(m+2)] and so all it is going to be is 1. (m+2), it's already there, so it's like..one.

I: You keep saying one, I'm not sure what you mean [The interviewer inferred that Carrie was mentally dividing out (m+2)].

Carrie: (pause) O.K. see my LCD for this part, 1/(m+2)?

I: Yes, what are you going to do with it? Carrie: (pause) Here it is simplified?

I: Yes

Carrie: Should I leave it as it is?

I: Yes. What should you do now? Carrie: Well, you don't want to mark it out (Indicates cancellation in the numerator). So I want to multiply it out.

The interviewer's interpretation of Carrie's activity was that she had confused the two common methods for simplifying complex fractions and was trying to apply both methods simultaneously. In her first attempt (#3), Carrie appeared to be trying to mentally multiply both the numerator and denominator with the LCD, (m+2)(m-2). In the subsequent attempt, she considered the denominator separately and determined that her simplification of the denominator was incorrect. Carrie then returned to the numerator and addressed the issue of how to multiply m(m-2)(m+2). She noted that she understood how to apply the "FOIL method"[1] but she wasn't sure if she should multiply by the m first. After the interviewer suggested that she could multiply in any order that she wished, Carrie wrote: (m^3 - 4m - 1)/(m-2)(m+2) (#4) and then mused as to how this answer should be written in relation to the rest of the problem. At this time, Carrie reached the third major decision point of her problem solving process, when she again reflected as to the kind of problem she was faced with. She immediately declared that it was a division problem and began to work on it using the invert and multiply method. Carrie hesitated for a moment as she considered whether or not she should try to factor m^3 - 4m - 1. After deciding against factoring, she divided out the common factor of m+ 2 and wrote her final answer, m^3- 4m - 1/m-2.

Carrie's activity indicated that she had possession of some basic mathematical tools that many students at this level have not yet mastered. Carrie could factor polynomials, combine rational expressions, and simplify algebraic expressions. However, on the basis of her survey responses and interview data, we claim that her "mathematical world view" (Schoenfeld, 1985, p. 157) is procedurally based. For example, in utilizing her rules to simplify the complex fraction, she demonstrated solution activity that ultimately led to results that did not make sense to her. In order for her to resolve the confusion regarding what she perceived as similar solution paths, she was unable to mentally coordinate the two methods, one against the other, and determine which one to apply. Rather, she needed to choose one of the paths and physically carry out the process. She appeared unable to mentally carry out a process and evaluate the results it would yield. Finally, we noted that while Carrie immersed herself within the problem, she sometimes became lost inside the details of specific sub-tasks. We contend that one reason for this is that Carrie does not see mathematics as a world of connected mathematical ideas. Survey data revealed that she believes that "mathematics consists of many unrelated topics" (Yackel, 1984).

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[1] The **FOIL** method refers to a memory device used by algebra students in the U.S. to remind them how to mutliply together a pair of binomials—**First**, **Outer**, **Inner Last**.

Conclusions

We posit that the experiences of Brad and Carrie are somewhat typical of College Algebra students. Such students enter college with a collection of mathematical rules, procedures and rigid expectations concerning what it means to do mathematics. As a result, the mental structures they invoke to help them organize and direct their mathematical actions are often fragmented. For example, memorizing the definition of a linear equation may help students recognize when they have a linear equation; however, it does not ensure that they will be able to mentally reflect upon, critically examine, and choose from among potential solution strategies. While memorizing definitions and rules is an important part of learning mathematics, it is not sufficient for the development of such reflective activity. For students such as Brad and Carrie, instructional practices that merely review and reinforce procedural tasks are not likely to benefit their mathematical development. Rather, these students need to face mathematical tasks that present dilemmas for them, the resolution of which contributes to their evolving awareness of algebraic concepts and, hence, to their evolving mathematical knowledge. Our continued work in this area is directed at developing instructional activities of this nature.

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MATHEMATICAL INTIMACY: LOCAL AFFECT IN POWERFUL PROBLEM SOLVERS

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This paper includes an introduction to and description of a new phenomenon of local affect demonstrated among powerful problem solvers. Aspects of mathematical intimacy include intimate interactions and intimate relationships. Intimate interactions are comprised of intimate behaviors and experiences. In the context of mathematical problem-solving, these emotive aspects have powerful potential to encode meaning and to influence problem solving performance.

Research into students' mathematical thinking has for some time focused on the cognitive aspects of learning. Recently, mathematics education research on affect (Goldin 1988; McLeod 1989, 1992; Lester, Garofalo, & Kroll 1989; DeBellis & Goldin 1991, 1997; DeBellis 1996) has provided educators with a different perspective of student thinking and of how such thinking may influence performance. In fact, other scientific fields (e.g., neuropsychology and computer science) are also finding that emotions play an essential role in decision making, perception, learning, and more — that is, they influence the very mechanisms of rational thinking (Picard, 1997). Picard proposes that in order for computers to be genuinely intelligent, computer scientists must give them the ability not only to recognize and understand, but also to build knowledge through emotional connections. At the same time, progress in mathematics education research has formed a similar conclusion. DeBellis and Goldin (1997) view affect, and its detailed interplay with cognition, to be the most fundamental and essential system of representation in powerful mathematical learning and problem solving. Hence, this idea — that emotive aspects of knowing can influence one's acquisition of knowledge — has opened new opportunities for research in learning.

McLeod (1989) described the affective domain as including three components: emotions, beliefs, and attitudes. DeBellis and Goldin (1997) presented a research-based theoretical framework for describing the affective domain in mathematical problem solving which, in addition to McLeod's components, includes components of values/morals/ethics as it pertains to problem-solving affect, the notion of mathematical self-acknowledgment, and the idea of meta-affect. An essential idea in this framework is that affect, as a *representational* system, interacts with other modes of representation (as defined by Goldin, 1988, 1997) and encodes important information (meaning) and influences problem-solving performance. Every individual constructs complex networks of affective pathways, contributing to or detracting from powerful mathematical problem-solving ability (DeBellis and Goldin, 1997). What kinds of emotional experiences happen during mathematical problem solving that are characteristic of powerful problem solvers? One type of emotional occurrence is described below. I call this notion of affect, "mathematical intimacy."

Mathematical intimacy is a changing state of feeling during mathematical problem solving, and is classified as local affect. It is important to note the distinction between local affect and global affect. The former refers to changing states of feeling during mathematical problem solving (Goldin, 1988). For example, a child might temporarily experience the feeling of frustration while solving a problem, yet this emotion might change quickly to excitement as this child *perceives* to have discovered a solution. Conversely, global affect refers to more stable, longer-term affective constructs such as beliefs and attitudes. For example, the above-mentioned child's attitude toward solving mathematical problems might be "dread" because "he usually gets it wrong."

Research on the influence of affect on strategic decision-making during mathematical problem solving has suggested the following: Powerful affect is positive; the problem solver discerns and regulates negative local affect to useful purpose; local affect frequently changes; local affect is under the control of the solver; it effects the construction of mathematical knowledge; there are important interactions between global and local affect; and the problem solver's affect interacts with a variety of external factors (DeBellis, 1996). Mathematical intimacy is a powerful local affect whose description is based on the psychology of intimacy.

Psychological Aspects of Intimacy

Intimacy is a complex and elusive concept that has generated a variety of definitions, theories, and philosophies over the years. In fact, psychology literature provides disagreement about the essential meaning of the term. Intimacy is typically viewed as an interpersonal relationship which includes qualities of closeness and depth in the experience of human attachment. The word *intimacy* is derived from the Latin *intimus,* meaning inner or inmost. To be intimate with another is to have access to, and to comprehend his/her inmost character; it is a privileged knowledge of what is disclosed in the privacy of an interpersonal relation, while ordinarily concealed from the public view (Sexton and Staudt Sexton, 1982). There are many types of intimacies, ranging from fleeting to long-term, some of which do not remain static. Thus, "intimacy" has been characterized in the literature as ranging from casual contact to deep, close, time-tested association.

Although most psychological research on intimacy refers to the experience as interpersonal, some have claimed the experience is primarily intrapsychic (Maslow, 1970) or is a combination of dynamic personal or apersonal (collective unconscious) factors (Jung, 1928). Prager (1995) proposes a multi-tiered concept of intimacy, and describes two components thereof: (1) *intimate interactions* which are dyadic communicative exchanges, and (2) *intimate relationships* in which people have a history and anticipate a future of intimate contact over time. She claims intimate relationships, both conceptually and in fact, are built on multiple intimate interactions. In contrast, intimate interactions can exist without intimate relationships. The presence of an intimate relationship influences the probability and frequency of such interactions. She further proposes intimate interactions themselves have two components: *intimate behaviors* and *intimate experiences*. Intimate behaviors refer to the actual observable verbal or nonverbal behaviors people engage in when interacting intimately. Intimate experiences are the positive feelings and perceptions of understanding people have during, and as a result of, their intimate interactions (e.g., warmth, pleasure, affection). Thus, intimate experiences are psychological constructs which are inferred by others. These feelings are intrinsically rewarding to the individual, may serve to foster other positive interactions, and can enhance individual well-being. Intimate interactions, then, are composed of behaviors and experiences, while intimate relationships are composed of multiple intimate interactions and their experiential byproducts.

A Proposed Description of Mathematical Intimacy

For research on affect in mathematics education, it is helpful to admit a definition of intimacy which is not restricted to interpersonal experiences. Mathematical intimacy is rooted within an individual and is derived from intrapsychic exploration with the mathematical content. To be intimate with mathematics is to have access to, and comprehension of its inmost structure. This form of intimacy occurs between an individual problem solver and the mathematical content, and hence is dyadic in nature.

Mathematical intimacy is comprised of two components consistent with Prager's intimacy definition: intimate relationship and intimate interaction. Intimate mathematical interactions are characterized by both intimate mathematical behaviors and intimate mathematical experiences. Intimate mathematical behaviors might include the distance a problem solver places between himself and his work, cradling his work, temporary loss of hearing external noises because he is so focused and consumed by the interaction, and hesitation in sharing mathematical solutions.

Intimate mathematical experiences are the positive feelings and perceptions of understanding which a problem solver incurs while solving a problem or thinking about a mathematical concept. Examples of mathematically intimate experiences include elation, excitement, warmth, amusement, affection, time warp, or perceived beauty in mathematics. These experiences are more than enjoyable or otherwise positive; they are intimate and thereby build a bond between problem solver and mathematical content. The mathematically intimate learner appears to make mathematics his own as he acquires intimate knowledge and through repeated intimate interactions builds a mathematically intimate relationship. Mathematical intimacy assists powerful problem solvers in constructing deeper mathematical knowledge since this emotion interacts with other modes of representation and encodes important information (meaning) about the mathematical content.

Mathematical intimacy may foster other positive byproducts which include: The willingness to take mathematical risks, since intimacy provides some sense of safety; the notion of perseverance, since intimate experiences include feelings of loyalty, devotion, and passion; and confidence, since intimacy enhances a problem solver's well-being. However, possible negative byproducts might include frustration or anger, since a problem solver might feel disappointed or betrayed by an unexpected outcome.

Access to mathematical intimacy may depend upon a person's willingness to engage the content at a vulnerable level of interaction. Mathematical learning then becomes "the privileged knowledge" gained through intimate mathematical interactions. It may be the degree in which mathematics is understood directly relates to one's level of mathematical intimacy.

An Example: Jerome

In qualitative, exploratory investigations, DeBellis (1996) studied elementary school children over a two year period through a series of three carefully structured task-based interviews, designed to maximize nondirective mathematical problem solving and construction of external representation. Videotapes of four subjects were analyzed for interactions between affect and cognition, using fine-grained protocol analysis, inferences made by observers, and a validated facial movement coding system. Empirical evidence for mathematically intimate behaviors was collected through voice inflection, timed pauses in speech, facial expression, and posture. Mathematically intimate experiences were inferred through the analysis of a facial coding scheme, posture, and verbal utterances. Each interview consisted of several mathematical exercises, culminating in a difficult non-routine problem and lasting approximately forty-five minutes.

Jerome impressed this researcher to be a straightforward, expressive young man, both in words and in actions. He is a New Jersey Caucasian male, who, from nine years old to twelve years old, participated in task-based interviews. He attended elementary school in a working class community. Predominant global affect characteristics included a self-professed stubborn demeanor, a liking of mathematics, and the frequent admission of, "I don't know" while solving non-routine problems.

Jerome's body gestures included cradling his work as he solves the problem so as to separate the activity between the content and himself from the clinician. At times, he can be seen at close physical proximity to his paper. Yet, during the same mathematical episode, Jerome physically pushes himself away from his work when his experience contradicts his expectation. His facial expressions included raised eyebrows, furrowed brow, pressed lips, wide-eyes, and smiles.

Results and/or Conclusions

Powerful problem solvers seem to have a fundamental willingness to become intimate with most any mathematical problem. These solvers construct a higher level of meaning through this intimacy level than others who are less intimate with the content. They build personal meaning with the content, not as an observer, but as an active participant. Emotions that manifest themselves between a mathematically intimate child and his work may include on the one hand feelings such as devotion, loyalty, bonding, privacy, trust, affection, and pride, but may also include feelings of deception, betrayal, disappointment, despair, frustration, and anger. As a result, when mathematically intimate children present their work to an external agent (i.e., teacher, parent, peer, or clinician) there exists a high emotional risk since they have identified so closely with their mathematics.

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A MODELING APPROACH TO NON-ROUTINE PROBLEM SITUATIONS

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A modeling approach to non-routine problem situations explicitly focuses on the activities of creating symbolic, graphical, and numeric representations and descriptions of situations that are meaningful to the learner. The approach elaborated in this paper is based upon a two-stage sequence of activities that provides learners with an opportunity to first create and then extend their ideas about selecting, ranking, and aggregating data in such a way that decisions can be made. The characteristics of each stage are illustrated by student work at the small-group and whole-class levels. Student reasoning about the relationships between and among quantities and the quantification of qualitative data are discussed. The results of this classroom-based case study suggest that students were able to create generalizable and re-usable systems (or models) for selecting and ranking. As their resulting models were shared within the classroom, the students did not appear inclined to identify some models as being ìbetterî than others.

Introduction

Current K-8 mathematics curricula largely focus on quantities such as counts, shapes, measures, and an assortment of notions about ratio, rate, and proportion. While this set of quantities is a rich and important core of mathematical ideas, often the kinds of quantities needed to mathematize realistic, meaningful situations include a much wider array of quantities, such as accumulations, probabilities, frequencies, ranks, signed quantities, and vectors. The operations on these quantities likewise need to be extended beyond the four basic operations of arithmetic to include sorting, weighting, selecting, and transforming entire data sets rather than single, isolated data points. In this paper, I discuss the results of an analysis of a two-stage sequence of modeling activities in which middle school students investigated non-routine problem situations where the core mathematical ideas focused on the creation of ranked quantities, operations, and transformations on those ranks, and, finally, the generation of relationships between and among quantities to define explanatory and predictive relationships.

Theoretical Framework

Many textbook problems that are posed to middle school students do not involve the students in creating, modifying, or extending representations of meaningful problem situations. In solving typical school *word problems*, students generally try to make meaning out of questions that are already carefully quantified situations. The solution process is generally an exercise in mapping the problem information onto an invariant model using some level of symbolic notation (such as numbers and the four basic operations of arithmetic) so that an answer can be produced. The mapping of the problem information onto arithmetic operations is typically a one-step map. Such activity rarely involves students in an explicit creation or examination of the underlying mathematical model. Seldom is the model itself transformed, modified, extended or refined as part of an iterative development.

A modeling approach to problem situations explicitly focuses on the activities of creating symbolic, graphical, and numeric representations and descriptions of situations that are meaningful to the learner. Engaging in this kind of model building is not seen as finding a solution to a given problem but rather as developing a tool that a learner can use and re-use to find solutions (Bransford, Zech, Schwartz, & The Cognition and Technology Group at Vanderbilt University, 1996; Doerr, 1997). Each stage of the modeling process includes multiple cycles of interpretations, descriptions, conjectures, explanations, and justifications that are iteratively refined and re-constructed by the learner, ordinarily interacting with other learners. Once created, models can be extended, explored, and refined in other contexts. Generalizing and re-using models are central activities in a modeling approach to learning mathematics. Thus, a modeling perspective leads to the instructional design of a sequence of activities that begins by engaging students with non-routine problem situations that elicit the development of significant mathematical constructs and then extending, exploring, and refining those constructs to other problem situations.

Description of the Study

The initial model eliciting activity was designed to motivate and guide the students' inquiry and to confront them with the need to develop a model to describe and explain a problem situation. This activity (described in more detail below) was intended to be a problem situation that students can readily understand through their own first-hand experience and to provide a significantly rich mathematical site that will lend itself to refinements, extensions, and powerful generalizations (Lesh, Hoover & Kelly, 1993). The sequence of problem situations used in this research study were designed to be completed by a small group of students, thus providing a social setting for the negotiation of conjectures and justifications, and for the clarification of explanations. In addition to the sharing which would take place within the small groups, discussion of specific systems created by students provided a forum that could potentially lead to the sharing of either common or multiple systems within the whole class. The overall curricular unit was designed to provide activities that support (a) the generation of a model in the first place, and (b) the extension, exploration, and refinement of the significant mathematical relationships, representations, and assumptions.

This study took place in an urban middle school setting, with low-to-average-achieving eighth grade students. In this particular school system, higher-achieving students are selected to take a first year algebra course. The study took place in four classes, all taught by the same experienced middle school teacher. The activities had been piloted by the teacher a year before this study took place. This pilot phase enabled the teacher and the researcher to modify and extend the problem situations which the students investigated. The teacher had a classroom set of four-function and fraction calculators available for the students. Small-group work and pair work was occasionally done in the classroom, but there was not a well-established classroom practice of effective student learning in group settings. Two of the four classes were observed by the researcher. The teacher and researcher met regularly for the planning of the unit, for revisions and modifications made during the unit, and for reflection on the students' learning.

Data Sources and Analysis

Each class session of the overall unit was video-taped, and during small-group work, the groups were observed by the researcher. All of the written work, including student journals and class work, done by the students was made available to the researcher. Extensive field notes were taken during the class sessions. The video-tapes of class sessions were reviewed and selected portions were transcribed for more detailed analysis.

Description of the Modeling Sequence

The overall sequence of the unit consisted of four problem situations (two of which are discussed in this paper) centered on the core mathematical idea of ranking, weighting ranks, and selecting ranked quantities. Since the problem situation is focused on ranking, of necessity the students analyze and transform entire data sets or meaningful portions thereof, rather than single data points. The students had no specific formal exposure or instruction on these ideas prior to the unit. Rather, the unit was designed so that the students could readily engage in meaningful ways with the problem situation and could create, use and modify quantities (e.g., ranks) in ways that would be meaningful to them and in ways that could be shared, generalized, and re-used in new situations.

The first problem in the sequence, *The Sneaker Problem*,was designed to elicit the notion of ranking in the first place and the multiplicity of factors that would lead to the need for selecting based on ranks. This problem was then followed by *The Summer Camp Problem* (modified from Lesh et al. (1989)) that was designed to extend, explore, and refine their notions of ranking and selecting. Finally, the unit concluded with two problem situations in which the student could apply and further extend their ranking systems.

The unit began by posing *The Sneaker Problem* to the students. The students encounter the notion of multiple factors that could be used in developing a rating system for purchasing sneakers and the notion that not all factors are equally important to all people. They were asked "What factors are important to you in buying a pair of sneakers?" This generated a list of factors and not all of these factors were equally important to the students; the students then worked in small groups to determine how to use these factors in deciding which pair of sneakers to purchase. This resulted in different group rankings of the factors. The teacher then posed the problem of how to create a single set of factors that represents the view of the whole class; in other words, the group ranks needed to be aggregated into a single class ranking.

The second problem was designed to extend, explore, and refine the idea of ranking, sorting, selecting, and using non-quantitative data. The students were given *The Summer Camp Problem* with a complete data set for 12 girls:

You are a counselor at the City Summer Recreational Program. Your program specializes in track and field events. In the preliminary events, held on the first day, the 12-year-old girls performed as shown in Table 1.

Table 1. Swim levels, from highest (Shark) to lowest (Sunfish).

Name	50-yd Run	100-yd Run	High Jump	Long Jump	Swim Level
Andrea	7.9	14.6	4'2"	10'9"	Bass
Carrin	9.4	16.5	3'4"	10'0"	Sunfish
Deanna	7.0	13.5	4'4"	11'6"	Shark

Based on this information, and the comments from the school coaches, you are to assign these 12 girls to three groups. These three groups will be competing against each other in the summer program and you want the competition to be fair.

The studentsí solutions were to include a specific recommendation for teams, and, most importantly, an explanation of their system for deciding how the teams were created. In this sequence, the students were explicitly asked to develop systems which can in turn be shared with other students and re-used in other situations.

Results and Discussion

These results come from the analysis of the observations of classroom discussions taken as a whole and the examination in detail of the small-group interactions. In the whole-class discussion, the students easily identified ten or more factors that they would consider in buying a pair of sneakers, including color, size, style, brand name, and even cost. Within their small groups, they usually listed these items from most important to least important through a process of negotiation, discussion and, in some cases, voting. In this early stage, most groups did not explicitly assign a ranking value to the elements of the list. Rather, most lists were simply in an implied order from most to least important. Where the elements were numbered, this number seemed to serve more as a label than as a numerical quantity which could in turn be operated on. In other words, the list could have just as easily been labeled A, B, C, D, and so on. As the lists were shared, it was immediately evident to the class that while no two lists were exactly the same, there were some items that appears in common at or near the top of most lists. It was at this juncture that the teacher posed the problem of how to find a single list that she could use in deciding how to choose a pair of sneakers.

In aggregating the ranked sneaker data for the whole class, students developed three systems. The first system was to arithmetically average the ranks and then to explicitly re-rank those averages. Significantly, this system indicates a shift from seeing the ranks as labels to seeing them as quantities that can be operated on by averaging. The factors now have explicit numerical values, rather than implicit places in a list. Moreover these newly constructed quantities were assigned explicit numerical ranks.

A second system used to determine an aggregated class rank was a frequency-based strategy. The factor with the greatest number of one's became the first ranked and so on for the second and third factors. The students were easily able to identify the lowest ranking factors by this system as well. The factors in the middle were assigned ranks by either an estimation strategy that yielded a *close enough* rank or by successive pairwise comparisons. In one instance, the occurrence of a tie lead to a refinement of the system. The tied factors were assigned a common rank, and the next rank was skipped. A third system used to determine an aggregated class rank simply ignored the group data and resorted to a *voting* strategy, whereby individual members of the class were polled. Some students argued that this strategy was *fairer* than using the group data since it took into account the opinion of everyone in the class.

The Summer Camp problem was designed to provide an opportunity for students to extend, explore, and refine the ranking constructs elicited by the Sneaker problem. The camp problem evoked several early and unstable interpretations that were generally, but not always, revised and refined within the small groups of students. These early interpretations included averaging the scores in all four events without regard to the units associated with the quantities, averaging the two running scores to obtain a single score, operating on feet and inches as if it were a single decimal quantity, and ranking without regard to whether or not a high score was ìbetterî than a low score. Some groups of students ignored the swimming scores, but others assigned them a numerical rank from one to four and used this along with the other ranked data.

Most groups ranked each player on all four events, but no group gave a single rank for each player. Rather, having four ranks for each player, the three best players were selected by frequency of high ranks and then the poorest players were identified. The students devised several systems for assigning the teams. Two of the most noteworthy systems were a high/low matching system that put the best and worst players on the same team and then filled in the middle players. A second system was a pairwise comparison that successively filled slots on teams by comparing the four ranks of pairs of players. In some cases, the students refined their teams by using the coaches comments as a basis for adjusting team rosters. As the resulting models were shared within the whole-class setting, students did not find conflict in different results nor did they appear inclined to identify some models as being "better" than others. This result, while somewhat surprising, may be in part accounted for by the relatively high agreement among students as to the identification of the "best" and "worst" players and relatively little need for fine judgments about more average players.

Conclusions

In the first stage of the modeling sequence, the studentsí work lead to the creation of ranks which were at first implicit, but later became explicit quantities to be operated on by averaging or totaling. In the second stage, the students developed several systems for ranking data, aggregating the ranks, and then assigning groups based on ranks. The grouping based on ranks was an extension and refinement of the studentsí first stage models which aggregated group ranks into a single rank. The sharing of systems within the classroom suggested that students created multiple systems (or models) that were characterized by the central idea of a rank, but that they showed little tendency to make comparative judgments among models.

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POSING QUESTIONS OR REFORMULATION OF PROBLEMS AS AN ACTIVITY TO PERCEIVE THE STRUCTURE OF MATHEMATICAL PROBLEMS

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This investigation reports the results on students' behavior when exposed to a mathematical description of two tasks (geometrical figures) and when required to pose mathematical problems. The tasks don't explicitly require the use of the description when formulating the problem. We wished to observe whether or not the student could establish connections between both processes; we wished to find out the relation between the level of the description and the problem posed. The questions which we are investigating are to see if problem posing tasks help the student to become aware of the relevant mathematical relations involved in a given situation. We consider that perception plays an important role in problem solving.

Introduction

Schoenfeld (1985) reported that students tend to perceive superficial information when they first approach a problem. However, it is posible to improve considerably the behavior of students if they carry out activities which stimulate their awarness of the deep structure involved in the problems. Although there exists information on how students behave when facing problematic situations, it is necessary to study what students do or perceive, not only when solving problems, but when they are required to explore and describe mathematical relations containing certain information (figures, tables, and graphs); and based on this, to pose problems, that is to say, this paper investigates the behavior of students when facing situations in which there is no a given problem and the task is precisely to formulate the problems. We hope that the students realizing these types of activities before facing problem solving will be in better conditions to approach mathematical problems more successfully than students who have not engaged in these activities (Santos, 1996) and particularly to identify the key components of the problems.

Conceptual framework, participants, organizations, and analysis of information.

One of the difficulties facing students when trying to understand a mathematical statement or a situation including drawings, is that they don't grasp the fact that all the information given in the mathematical statement is connected; unless the student is aware of these connections he will not be able to solve the problem (Krutetskii, 1969). From the point of view of

Greeno, Morre, and Smith (1993) in order to carry out any activity it is necessary to perceive the affordances which support such activity. This investigation asks students to formulate questions or problems. This implies that the students realize that the situations (geometrical figures) have properties and/or relationships. An important point here is that the type of the activity being carried out by an agent in a learning situation depends on the purpose of the task and on the affordances being perceived. The following questions provide a base to guide the study: (a) Which affordances were attended by the students in the geometrical figures? (b) To what extent are these affordances used to formulate questions or problems? (c) Did the formulation of questions or problems stimulate the students to think and reflect, and especially to become aware of the relevant mathematical relationships.

The investigation was carried out with 21 high school students with an average age of 17. They had all taken courses in algebra, euclidean geometry, and analytic geometry. We must state that the students lacked experience as problem posers . This was the first time for them. There were two phases of observation in the study. During the first phase, 19 students, working individually, were asked to describe the properties of two figures and then report their work, they spent about 30 minutes in each task. The second phase involved a pair of students working on the same tasks. Here, they were required to solve the questions or problems they posed. We filmed students. The time taken for both tasks was aproximately 1 hr. 28 min. It should be noted that the tasks were analyzed and discussed by the investigators before working with the student. The idea was see the possibilities and potentials of each task and also to see what cognitive skills would be required from the student. It should be noted that we did not expect the students to necessarily see the same relations that we did.

The two tasks that were used and a brief discussion of potential questions are presented next:

1. On the parallelogram ABCD (figure 1) below the bisector of two consecutive angles (DAB, ABC) are intersected at the opposite side P. State everything you can about the sides, angles and triangles. Once this is done, pose questions or problems.
2. On the rectangle below (figure 2) a point P is an arbitrary point on the diagonal CB. From P two perpendiculars are drawn to AC and CD respectively. These perpendiculars intersect AB and CD in E and H respectively and AC and BD in G and F respectively. Formulate questions or problems.

Skills and abilities required to carry out the tasks.

The first figure offers the student the possibility to notice the following aspects: DC and AB are parallel segments, AP is transversal, therefore $\angle$ DPA = $\angle$ PAB, and as AP is a bisector, therefore $\angle$ DAP = $\angle$ DPA, then the

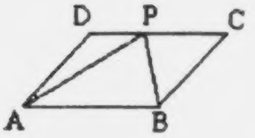

Figure 1

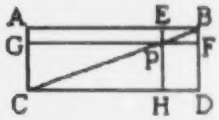

Figure 2

triangle ADP is isosceles. Similary, it is observed that Δ PCB is also isosceles. Based in this information, it can be proved that the ∠ APB is a right angle. Also, compare the areas of triangles APB and DCP with the area of Δ ADP etc. We would like to think that the students would formulate questions like: What properties does this figure have? What is a bisector? Given any parallelogram do the bisectors of two consecutive angles always intersect at the opposite side? Under which conditions will it happen? What happens with the figure if point P is moving and the side AB remains fixed? What path does point P trace? What happens to the value of angle APB when point P is moving? What other aspects remain to be considered? For the second task, some potential questions include: What happens with the area of the rectangles AEGP and PFDH if point P is moving on the side CB? What can you say about the area of AEGP and PFDH? To discuss what students did, we present the information by focusing on the students' work shown individually and contrasting it with the work done by two students who approached the tasks as a pair.

Students' performance and discussion results.

The information was organized and presented in accordance with the students' descriptions shown in their written report and questions they posed. The work of Martha is shown (See Table 1) to illustrate the type of information that was analyzed. The first column shows the literal description of tasks 1 and the second column the questions she formulated.

The work reported by the 21 students was organized as in the above table. Four different types of student responses emerged from the data:

1. Using direct information. Here the student simply asked information that was given in the statement of the task. For example, one student wrote: "Show that line BC is parallel to line AD," or "Show that ∠ DAP = ∠ PAB."
2. Finding or determining a specific measurement. Here the students showed two different levels in their responses. One in which they provided some information and asked to find a quantity: "Find the hight of the figure knowing that DC = 4 and BC is 5 cm." The second level included questions in which the students asked for certain calculation without providing any other information: "Find the area and the perimeter of the figure," or "Find the length of the angles which are within the figure."

Table 1

Parallelogram	Question or problems formulated
The figure has parallel sides, that is AB and CD; BC and AD. On BC, the bisectors met each other on the middle point P. The segment AB is the base of a triangle. There are three triangles, they are, ABP; APD; and DPC. On the point P there are three angles whose sum is 180 degrees. The bisectors form three triangles.	Find the measurement of the triangle formed by the bisector in the parallelogram measuring 4 and 8. Find the measurement of the hypothenuse of the triangle CPH, if AE measures 10 cm and BD 6 cm, AG 4 cm.
Rectangle	
Sides = are parallel AB and CD, and AC and BD are parallel. 4 rectangles and 4 triangles are formed. 2 of the 4 triangles are equal, and the other 2 are also equal.	

3. Proving a property. Here, only one student formulated a statement in which he explicitly used the term "prove", the question he posed was "Prove that triangles DAP and PCB are congruent."
4. Stating ambiguous statements. There were students who experienced difficulties in writing their reponses. They often described aspects of the figure by relying on the appearance of the diagram, without questioning their plausibility: "In parallelogram ABCD for bisectors AP, BP one can see at glance that they have the same distance equal to their angles and that they are the same triangles but looking at their area small differences and therefore it could not be possible to have a perfect answer."

It seems that the students perceive the geometrical figures (parallelogram, rectangle) as labels or names only. That is, they don´t realize that the figures involve properties and/or relationships. When using the term rectangle and parallelogram, the students might see the figure as a whole and therefore, cannot see the need for further description. Here, they might have considered the activity useless. For example, after working the first task, 8 students were asked if they knew what a bisector was. They replied affirmatively; however in their written answers they did not mention this. Generally speaking, no student saw the bisector as a transversal line between 2 parallels. This might explain why the students fixed their attention to superficial aspects, such as "3 triangles are formed", "there are 4 rectangles", "there are 9 angles".

It seems that student don´t know how to formulate questions or problems and some see no difference between posing questions and providing descriptions. This problem may be related to their lack of language, their lack of grammatical knowledge, or their lack of writing skills: "Say, if triangle ABC is equal to Δ BCD," or "To make the theorems of Pythagoras the perimeter of the rectangle it is only necessary to assign a measurement to the sides and the given triangles." The majority of the questions that students posed were like: "find or state a quantity," "state the area of triangle APD if AP is 3 cm and has a height of 4 cm."

As it was mentioned before, there was a pair of students who worked the same tasks. It is important to pinpoint some important differences when compared with the individual work of the students. The pair's analyses of the figures shows that they went beyond the superficial level, but their thinking was empirical, e.g., in the parallelogram, the pair showed awareness of the properties of the bisector and that those bisectors get intersected at the middle point on the opposite side. They also mentioned that the opposite angles$\angle$DAC and DCB were equal and that the opposite sides were also equal. In other words, they tried to find relationships among the components or parts of the figures. The pair perceived the figures differently than the individual workers. Their activity was directed towards "unpacking" the properties or relationships (e.g., "What else can we state that is not present?").

The differences noted between the pair and the individual approaches (e.g., the descriptive task) might be due to the social interaction between the pair–especially the girl who constanty pushed questions like "Why?", " How do you know?".

Conclusions

One of the ideas that permeated this investigation is that the formulation of questions or problems depends on the affordances that are perceived in the descriptive task, a more profound analyses of this task would lead to a better quality of the proposed problems. One way of achieving this is precisely the posing of question or problems. The students however, had no previous experience in this type of activity. From the results, we also noted that the students had no experience in expressing their ideas in a writting manner which affected the expectations of this study. Since this activity was new for the students, it is very difficult to draw conclusions about the potentialities of problem posing activities. We noted, however, that social interaction is crucial here and it should be taken in consideration in future investigations or in any teaching which emphasizes this type of activity. The next face of the investigation would be to record the effect on students exposed to a lenghtly experience in these type of activities.

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AN ANALYSIS OF MATHEMATICS PROBLEMS IN MIDDLE SCHOOL TEXTBOOKS

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This study analyzed problems presented in common content units in several middle school mathematics textbooks. A three-dimensional framework was developed for analyzing textbook problems in this investigation. The study found that the problems presented in five different US textbooks showed dramatic variations on the dimensions of mathematical density and problem requirements. The results echo the findings from a recent TIMSS curriculum study on textbook content topic coverage and emphasis, and reflect varied efforts that different textbooks have in responding current reform calls in school mathematics. The results indicate the importance of reaching a broad consensus on providing the best for student learning mathematics. Reform requirement of restructuring textbook problems that envision a nation's educational expectations for a new generation stands.

Background and Purpose

Contemporary school mathematics has emphasized the pursuit of high mathematics achievement, and the development of students' problem-solving ability. However, American students, especially after elementary school, often had poor performance in national and international mathematics assessments. Many researchers argued that math textbooks are a key factor in influencing students' math attainment (Flanders, 1994; McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers, & Cooney, 1987; Willoughby, 1984), specifically the problems presented in the textbooks (Nibbelink, Stockdale, Hoover, & Mangru, 1987; Nicely, 1985). In fact, a recent TIMSS curriculum study has indicated that the U.S. math textbooks have produced a splintered vision in school mathematics on what is important for students to learn (Schmidt, McKnight, Raizen et al., 1997). Although recent American textbooks have shown positive changes in content presentation and organization (Chandler & Brosnan, 1994; Li, Carter, & Ferrucci, 1997), how the problems presented in math textbooks help students attain mathematical achievement and obtain problem-solving skills has not been explored. It was the purpose of this study to analyze and compare the math problems presented in several middle school textbooks. Specifically, as a previous study on content analysis (Li et al., 1997), this study analyzed mathematical problems presented in the lessons on the addition and subtraction of integers in several American textbooks.

Theoretical Framework

Mathematical problems have been analyzed with different factors in different studies. Stigler, Fuson, Ham, & Kim (1986) studied addition and subtraction word problems in U.S. and Soviet elementary school textbooks using a classification scheme, which was based on problems' mathematical (i.e., the unknown's position) and contextual (i.e., problem's semantic structure) characteristics. In a cognitive analysis of algebraic problem difficulty, Tabachneck, Koedinger, & Nathan (1995) considered some mathematical and contextual factors that are embedded in algebraic problems, such as number of operators, kinds of operators, and situational factors. In both investigations the choice and emphasis afforded different problem factors were a function of the characteristics of the problems being analyzed and the purpose of the study. Nevertheless, both the dimensions of a problem's mathematical and contextual characteristics were considered in analyzing mathematical problems in these studies.

In addition to the mathematical and contextual aspects, some researchers have found that different requirements levied by mathematical problems could dramatically influence students' problem solving performance (e.g., Silver, Shapiro, & Deutsch, 1993; Zhang, 1992). Thus, the differences embedded in problems' requirements can show another dimension of factors that influence students' math performance. Consequently, a three-dimensional framework was developed in this study for analyzing the mathematical problems presented in different textbooks. Specifically, the three dimensions were: (a) Mathematical Depth, (b) Contextual Features, and (c) Performance Requirements. Several categories under each dimension were also identified and used for coding the problems in this investigation.

Methods

Materials. Mathematical problems in lessons on addition and subtraction of signed whole numbers in five middle school textbooks were selected for analysis. These texts were Mathematics Plus (MP, 1994), Addison Wesley Mathematics (AW, 1995), Exploring Mathematics (EM, 1996), Mathematical Connections (MC, 1997), and Heath Passport to Mathematics (HP, 1997). All textbooks were intended for use in Grade seven and were commonly being used across the country in various settings and with diverse populations.

Procedures. Mathematical problems selected from the textbooks were those exercises or questions that did not have accompanying solutions and/or answers. In all these textbooks, mathematical problems appeared under the headings: "check for understanding", "problems", "practice", "application", or "problem solving" within or immediately following sections of the text introducing integer addition and subtraction. Mathematical problems did not include exercises in review sections or questions for classroom or small group discussion. All problems could be solved using integer addition or subtraction.

Each problem was coded independently by a rater with the categories specified in the above framework. The problems in two randomly chosen textbooks were coded independently by another rater. An acceptably high degree of agreement in this coding between raters was obtained (91%). There were no unresolved disagreements between the raters. The following are examples of the problems and coding.

EXAMPLE 1: Evaluate the expression p+s-2 when p=8 and s=10.

Coding: Multiple math procedures (M), Letter as an unknown or a variable involved (L); Purely mathematical (PM); Answer only (A), Procedural practice (PP).

EXAMPLE 2: One day the temperature in Chicago was 5°C. The next it was 3°C. How many degrees had the temperature dropped?

Coding: Single math procedure (S), No letter as an unknown or a variable involved (NL); Illustrative context given in words (ICW); Answer only (A), Problem solving (PS).

Results and Discussion

The figures below show the percentages of mathematical problems classified in terms of Density; Level of abstraction; Contextual features; Response types; and Cognitive requirements for the five textbooks.

Table 1
Framework for Coding Mathematical Problems in Textbooks

1. Mathematical Depth	
(1) Density	• Single procedure (S) • Multiple procedures (M)
(2) Level of abstraction	•Letter/variable/unknown given (L) • No Letter/variable/unknown given (NL)
2. Contextual Features	•Purely mathematical context (PM) • Illustrative context given in words (ICM) • Illustrative context given with pictorial representation (ICP)
3. Problem Requirements	
(1) Response types	• Answer only (A) • Explanation only (E) • Mathematical expression (ME) • Mathematical solution (MS)
(2) Cognitive requirements	• Conceptual understanding (CU) • Procedural practice (PP) • Problem solving (PS) • Special requirements (S)

The figures show dramatic differences among the problems being coded across these textbooks. Comparatively, there are larger differences in terms of "density," "response types," and "cognitive requirements" than the differences in the categories of "level of abstraction" and "contextual features". In all five textbooks the mathematical problems in lessons on integer addition and subtraction were overwhelmingly found to contain purely mathematical contextual features and to have no letter as an unknown or variable. Specifically, in contextual features these textbook problems on integer addition and subtraction showed only up to 11% difference from each other with the vast majority of problems being purely mathematical. Likewise, there was up to 18% difference among these textbooks involved the use of letter(s) as variable or unknown. Such difference range appeared less dramatic in this measure of Mathematical Depth than density, which illuminated the complexity of math skills required for solution.

In the measures of Problem Requirements, the categories of "response types" and "cognitive requirements", there were dramatic variations across these five textbooks. Although the majority of the textbook problems on addition and subtraction of integers from all textbooks except MP were found to require an answer-only response, problems classified as explanation-only could have as much as 10% in one textbook but only 1% of those in another. Even larger percentage variations were found among textbook problems that required a mathematical expression (26% to 1%). Because such explanatory factors in the problem sets reflect the direction of the emphasis given to enhancing communication skills in American standards based reform documents, the dramatic variations noted may indicate that these texts had different efforts in responding current reform calls in school math. For cognitive requirements, all textbooks especially Exploring Mathematics (EM) were found to dominate problems that require procedural practice. However, there are also quite large amount of problems in all texts (varying from 19% to 33%) that require conceptual understanding for solution. This evidence of a large portion of attention on conceptual understandings in most of these texts is in line with the focus of current US mathematics education reform efforts.

These results provide some indication of the types of differences and similarities that may be gleaned from a detailed qualitative analysis of the mathematical problems in several different math textbooks. Evidence exists that US textbooks present a splintered vision of school mathematics in terms of content topic coverage and emphases (Schmidt et al., 1997). Moreover, the differences in school math curriculum within the U.S. might be even more dramatic than cross-national differences in content organization (Schmidt et al., 1997) and problem sets presented in textbooks (Li & Carter, 1998). The importance of reaching a broad consensus on providing the best for students learning mathematics indicated by Schmidt et al. (1997) is reinforced with this study. Reform efforts are needed in restructuring

Figure 1

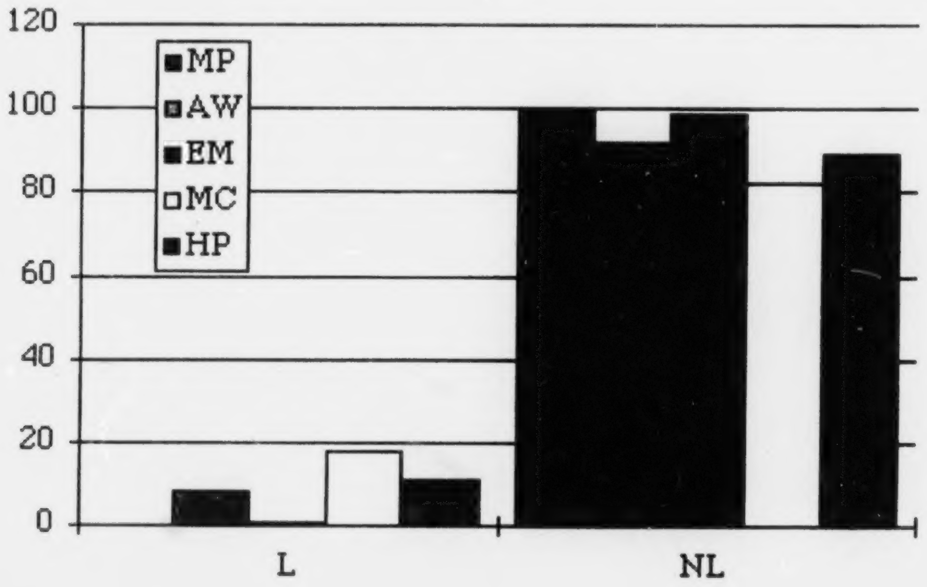
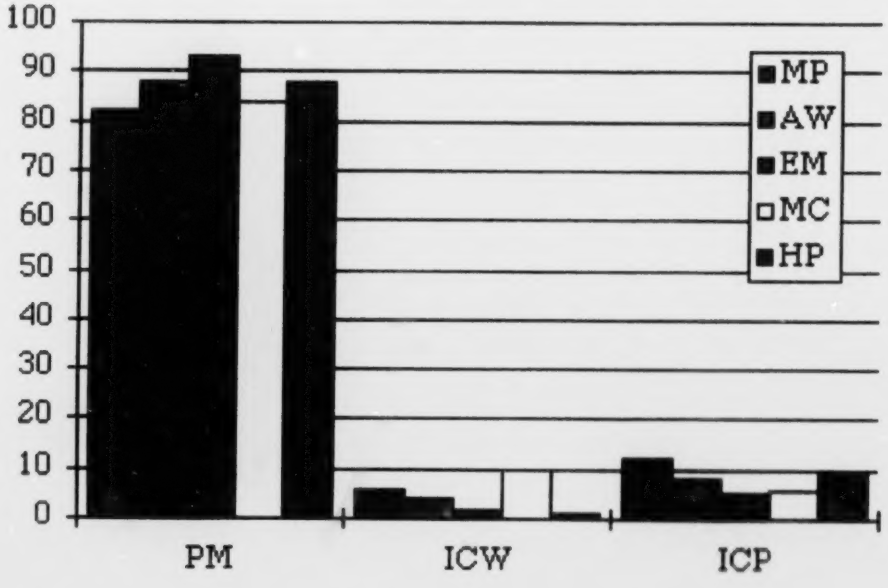


Figure 2



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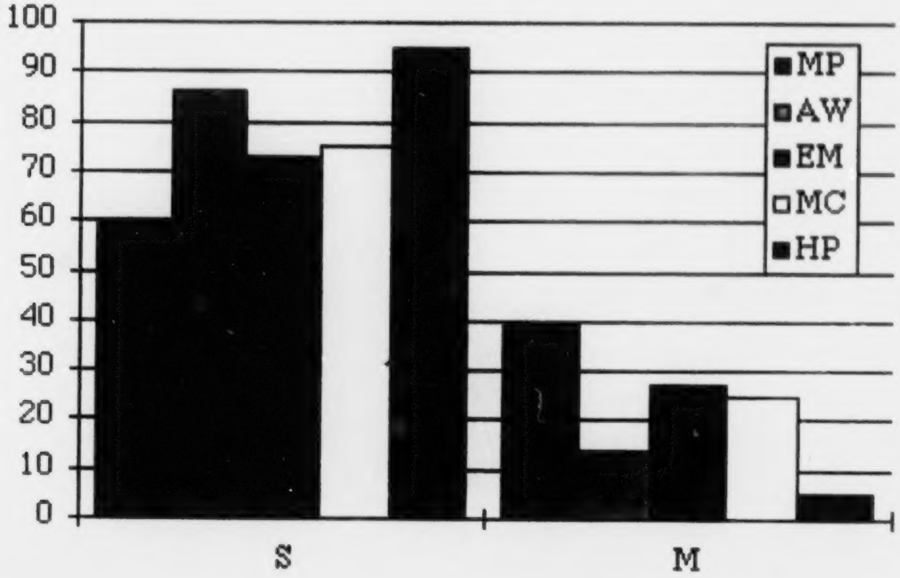

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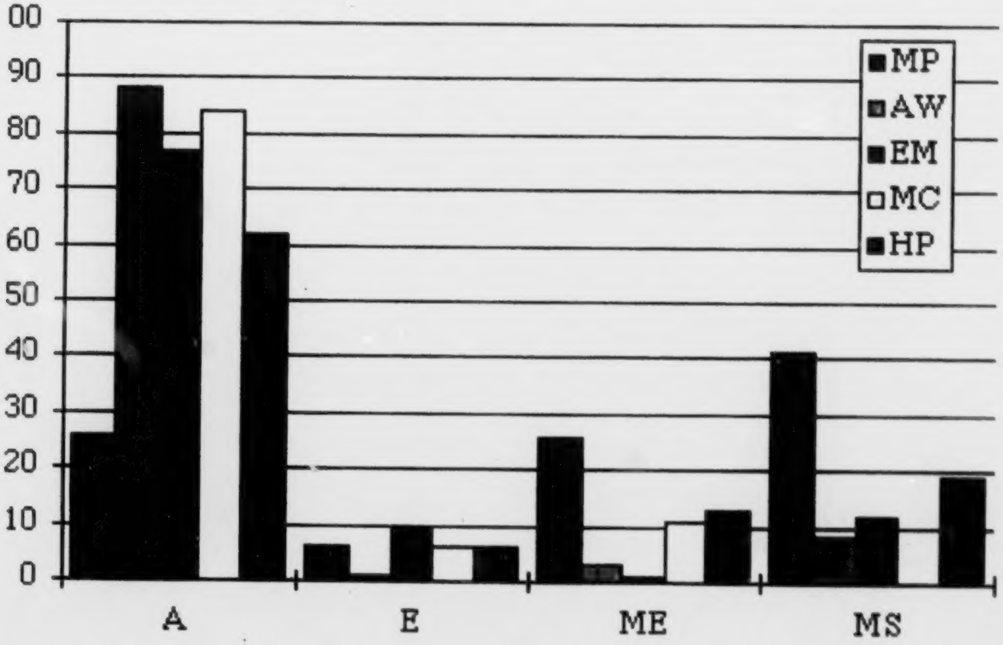

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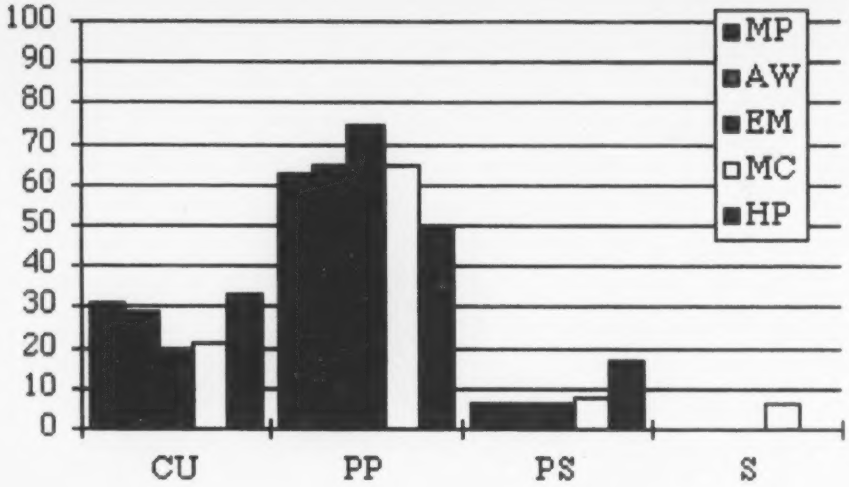

not only content presentation and organization but also problem sets in textbooks that envision a nation's educational expectations for a new generation.

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RECOGNIZING ISOMORPHISM AND BUILDING PROOF: REVISITING EARLIER IDEAS

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A focus group of students have been observed for ten years as part of a longitudinal study of how students build mathematical ideas. This paper examines the mathematical thinking of two of these students engaged in an exploration in combinatorics in grade 4 and then again in grade 10 in order to investigate the extent to which they retrieve and extend their ideas.

Objective

A longitudinal study has been in progress in the blue collar community of Kenilworth, New Jersey for over a decade.[1] Children, who are now tenth-grade young adults, have been observed working on combinatorial tasks over several years. Data from fourth grade videotaped sessions of investigations in combinatorics are available in which the students are working together to build a justification. Six years later the same problem was posed to a subset of these students. The purpose of this paper is to examine: (1) the nature of representations produced by students; (2) the process by which a solution is built; and (3) the extent, if at all, to which they retrieve and/or build on earlier ideas.[2]

Theoretical Framework

The research is based on the view that when children are presented with situations in mathematics classrooms that allow them to investigate problematic situations, opportunities often rise that enable them to build powerful and lasting mathematical images (Davis, 1984, 1992a, 1992b). The ability to formulate mathematical thinking and to build connections between representations must be nurtured as students mature mathematically through the elementary grades into the high school mathematics classes and beyond. Opportunities to build arguments and to produce justifications can arise out of carefully crafted classroom interventions in a curriculum designed to invite students to revisit, review, modify and/or extend earlier ideas (Maher, 1998). In this atmosphere, fostering the ability to think mathematically is not the objective of a single lesson, but becomes a long range goal. When students are then later challenged with similar tasks which are interesting to them, the earlier built images are retrieved, modified, and extended. It is the study of these modifications and changes to earlier built representations that is the focus of this paper.

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[2] See the accompanying paper presented in this volume by Kiczek and Maher: "Tracing Origins and Extensions of Mathematical Ideas."

The task. Variations of towers problems have been given extensively to many students (Maher & Martino, 1996a; Maher & Speiser, 1997). The task was to produce a convincing argument that all towers[3] , five-tall, using exactly two red cubes, when selecting from red and yellow cubes have been made. What is interesting to observe is that some students in providing a justification to their solution of the problem created a proof (Maher & Martino, 1997; Maher & Martino, 1996b; Maher, Martino, & Alston, 1993). However, not all students build a proof when encountering the problem in the earlier years.

Data Source

A subgroup of five students had the opportunity to investigate the task in grade 4 and then again in grade 10. Videotape records of the students as they work on explorations which document their earlier development of mathematical ideas. Teams of two or more graduate-student researchers validate, review, and analyze the videotapes and transcripts from the earlier years as well as from the current sessions, the children's written work, and the field notes from the senior researchers. The analysis begins with the identification of critical events, e.g., moments of insight, cognitive obstacles, or a conceptual leap. Each critical event folds back to past events (recent or distant) and is the basis for tracing the development of future ideas. The data are coded along the timeline, according to several categories. The data that are relevant to the critical event are used to explain the development of a particular image over time.

Results

Grade 4: Michael and Ankur. During the group sharing portion of an investigation of the task in grade 4, the teacher/researcher led the discussion to the question: "How can you be sure that all towers, five-tall with exactly two red have been built?" Following a short period of time during which the students worked together to find an answer, the teacher/researcher again questioned the class as a whole.

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[3] A tower is an ordered sequence of Unifix cubes, snapped together. Each cube is called a *block*. The height of the tower is the number of its blocks.

Tchr/Rchr: Now somebody at this table told me that when I looked at all the towers with exactly two red, there would be how many of them?

Ankur: Ten

Tchr/Rchr: How many got ten? Towers with exactly two red floors? How many? Okay. Now I want you to think [for] tomorrow is [about] how you can convince me that what you found the ten. That there can't be eleven, or twelve, or eight, or nine, or six.

The students responded immediately to the challenge and developed a proof by cases. They first determined that there were exactly four where the two red blocks were "stuck together," moving down one position from each tower to the next. When asked if "stuck together" were the only way that a tower five-high with exactly two red blocks could be built, they responded by finding the three towers which can be built with the two red blocks separated by one yellow block. The following explanation was offered by Michael to Ankur's assertion that only two towers five-tall with two red blocks separated by two yellow blocks.

Tchr/Rchr: I'm asking you to find me exactly two reds separated by two.

Michael: Here's a third one. Here's a third one. [*holding up a block tower*]

Ankur: There's only two.

Tchr/Rchr: I got that.

Ankur: There's only two. [*classroom noises*]

Tchr/Rchr: Yes? [*pointing to Ankur*]

Ankur: There's only two.

Tchr/Rchr: Why are there only two?

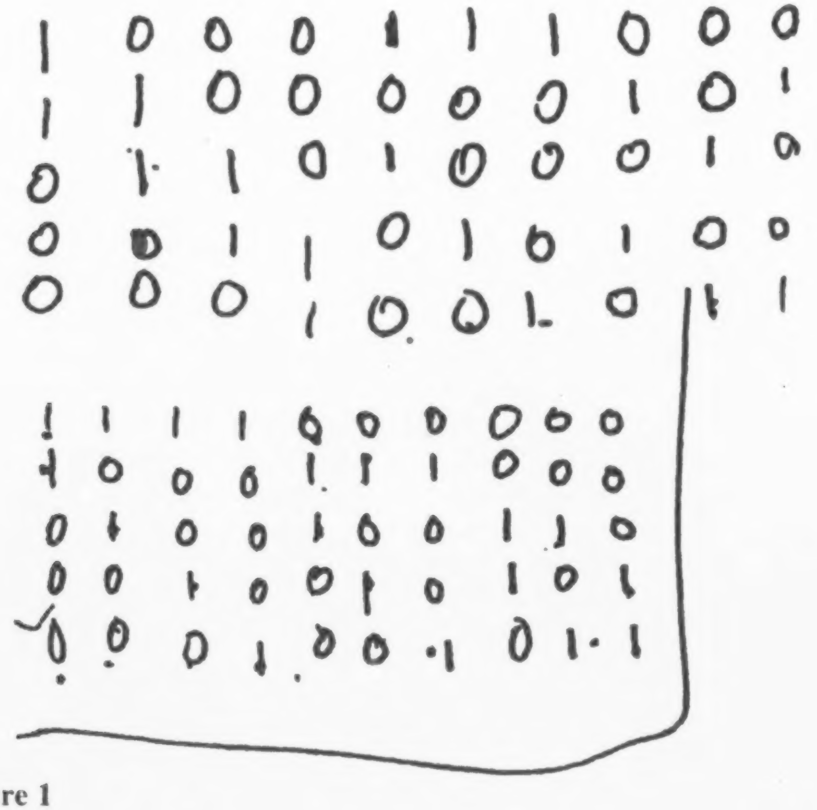
Tchr/Rchr: I see a lot of hands up here! [*indicates that Michael should answer*]

Michael: Because if you needed one more, you would need more than five, because you need another one.

Tchr/Rchr: Wonderful! You'd need another block. So let's put this here. Is there another way to have two reds?

The discussion concluded with students determining that there could only be one tower with the two red blocks separated by the three yellow blocks and that no further towers with this particular set of requirements could be built. They determined that exactly ten towers could be built with the condition of including exactly two red cubes and they made a convincing argument using a proof by cases—all towers with two red "stuck together," all towers with the two red cubes separated by one yellow cube, all towers with the two red cubes separated by two yellow cubes and all towers with the two red cubes separated by three yellow cubes. While the full argument was not voiced individually by either Michael or Ankur in this particular episode, it was convincingly put forth by several members of the classroom community in which the boys were active participants.

Grade 10: Michael and Ankur. During an after-school session, the fourth grade episode was shared with the students. In response to their interest, the problem was again posed. They quickly answered "ten" to the question on how many towers could be built. They were again challenged to explain the answer. Michael and Ankur used a coding scheme that Michael had developed to solve a pizza toppings problem during the after-school session the week before[4]. Using zero to represent a yellow block and one to represent a red block, they constructed an array (top array, Figure 1) that follows the arrangement developed in the fourth-grade activity. In his explanation of their solution, Ankur refers to the second (bottom array, Fig-



1	0	0	0	1	1	1	0	0	0
1	1	0	0	0	0	0	1	0	1
0	1	1	0	1	0	0	0	1	0
0	0	1	1	0	1	0	1	0	0
0	0	0	1	0	0	1	0	1	1

1	1	1	1	0	0	0	0	0	0
1	0	0	0	1	1	1	0	0	0
0	1	0	0	1	0	0	1	1	0
0	0	1	0	0	1	0	1	0	1
0	0	0	1	0	0	1	0	1	1

Figure 1

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[4] A detailed description of the isomorphic problem and the coding scheme can be found in the article by Kiczek and Maher in this volume.

ure 1) arrangement of zeros and ones. Here a red cube is kept fixed in the top position and then the second red block is moved into successively lower positions until it reaches the bottom most position. The fixed red was then moved into the second position and the process was repeated until Michael and Ankur had accounted for all of the possible towers fitting the requirements.

Michael: Well, I just like, like…

Ankur: …goes with the first number. It's a one there [*holding a one fixed in the top position*] and then you put the one here and then the rest are zeros.

Michael: [*elaborating on Ankur's explanation.*] See, this is, let's say, uh, the first, the top of the tower was the color.

Ankur: Red.

Michael: You would have the one and you could also have this one. Then you would go with um the second tower - I mean the second…

Ankur: second.

Michael: space from the top.

Ankur: is red.

Michael: is always that color.

Tchr/Rchr: Mhm.

Ankur: And then the third line. [*Ankur indicates that the third row is now held fixed and points them out.*]

Michael: … You could have them. [*Michael agrees that the third row is now held fixed and points them out.*] And then that one. [*Michael indicates the case were the ones are in the bottom two positions.*] That's all. I can probably make the little lines, probably, you know, with the one, two, three or something like that.

Tchr/Rchr: Mhm. That's another way you can probably do it. But this works you say?

Michael: Yes.

Ankur: Yes.

Tchr/Rchr: It's same thing. Okay.

Ankur: Are you convinced?

Tchr/Rchr: I'm convinced.

Michael and Ankur quickly retrieved a solution to the question posed which extends upon the solution built when they were fourth grade students by incorporating a coding scheme of zeros and ones introduced to the group of students by Michael in a previous after-school problem-solving session. This notation allowed them to represent their ideas more readily and to extend and generalize those ideas.

Conclusions/Implications

Videotapes provide rich data that enable us to view students from a fourth-grade setting and then the same students in a tenth-grade setting. It is possible to watch as fourth-grade students build an argument for a proof by cases to a combinatorial problem they have been investigating. When students are given the opportunity to revisit earlier investigations, we are finding that their previously built ideas are retrieved, modified, and extended (Maher & Speiser, 1997). As the students' thinking at each critical event is carefully considered and a tracing of the development of particular mathematical ideas is examined, there is a "folding back" to earlier ideas in a way similar to that discussed by Pirie and Kieren (1992). We see tenth-grade students utilizing arguments parallel to those built in the fourth-grade discussion. However, the representation of the solution is now expressed symbolically rather than with concrete objects and is extended easily by the incorporation of the coding scheme.

One implication which may be drawn from this, that is, children are able to form strong arguments at a young age and that these arguments are durable over time. If we believe that it is important for students to think abstractly, then we should provide them with early opportunities to begin to build the foundations upon which to advance.

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STUDENTS' STRATEGIES FOR SEARCHING THE INSTANTANEOUS FLOW IN A GRAPHICAL REPRESENTATION OF TIME VS VOLUME

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This paper describes what students did to find the instantaneous flow when they discussed problems that involved graphical representation. The students had not previously taken a calculus course. The analysis focuses on the students' interaction among themselves and with the teacher. Particularly, there is interest in documenting the students' concept evolution of instantaneous rate.

Introduction

Some approaches to studying the fundamental ideas of calculus have appeared recently. Kaput (1994) suggested that concepts as change and accumulation should be fundamental to understanding key calculus concepts, "those root problems associated with describing change and accumulation of continuously variable quantity, and especially the relation between change and accumulation as represented geometrically and kinematically" (p. 86). In the same vein, the National Council of Teachers of Mathematics (1989) recognized the importance of approaching informally ideas that include limit, the area under a curve, the rate of change, and the slope of a tangent line. Problem solving has also been identified as a main activity in the classroom (Santos, 1997). "The function/derivative relationship is embedded in contexts of daily life, which allows us to construct intuitions from an early age. These intuitions allow students who have never taken calculus to think about, predict, and discuss situations" (Nemirovsky, 1992, p. 1). What student's do when some of these ideas are implemented in classroom? This study documents what students did when they tried to solve a problem whose mathematical structure is the relationship between a function and its derivative, specifically, what strategies the students showed in order to find the instantaneous flow, using the graphical representation.

Conceptual Framework and Procedures

Thurston (1995) pointed out that people have very different ways of understanding particular pieces of mathematics. He said that the concept of derivative could be seen from different ways: as a ratio of small change (infinitesimal); as a symbolic manipulation of sign (the derivative of x^n is n x^{n-1}); as a formal definition which involve the concept of limit; as the slope of a line; as the instantaneous speed; and these ways do not include all the possible ways of thinking about derivative. What are some of the ways with which we are able to construct our ideas mentally? Lesh (1997) said that "Humans create models of the world ... constructors' abilities to interpret decision – making situations quickly and accurately depend heavily on the models that they develop to make sense of their experiences" (p. 399). Models embody explanations of how the facts are related to one another; give holistic interpretations of entire situations; and involve descriptions and explanations of patterns and regularities beneath the surface of things.

There are some characteristics in students' behavior when they solve problems:

> "(a) students' early interpretations are often quite unstable, obtuse, and biased; (b) students often neglect to notice information that later proves to be important; (c) superficial data often take attention away from deeper, but less obvious, patterns and regularities; (d) inconsistencies in reasoning are often ignored when attention shifts from one perspective to another; and, (e) inappropriate meanings are often rejected onto the situation on the basis of expectations from similar experiences." (p. 399)

As competing models are gradually introduced and tested, some are refined, whereas others are rejected, some are refined on the basis of private internal reflection, comments from peers or teachers, or mismatches between prediction and reality. Therefore, a series of modeling cycles tend to be required to establish more stable, unbiased, and high-fidelity interpretations that are useful for constructing, describing, explaining, manipulating, predicting, or controlling the behaviors of the given situation. So, the students' knowledge tends to be situated, unstable, and that it often exists in relatively disconnected pieces.

The study took place in a twelfth-grade introductory calculus class of 50 students. The group was divided into smaller groups of 3 or 4 students, in which there were discussions in order to reach a solution to the problems; after which the obtained results in the small groups were discussed by all the students in one large group. The small groups in the previous class had been given homework concerning the problem of a water tank (refer to problem found in appendix) in which this variable, volume of said tank, is expressed in a graphic representation of time vs. volume. These students were asked to find two points of reference in which the flow of the tank was the same. The aim for posing this problem was so that the students could investigate the relation between a function and its derivative in a context based on a real-life situation with data represented graphically. A class was videotaped as students from the small groups passed to write their answers to the problem on the blackboard to be discussed as a single larger group for a period of 25 minutes. The written answers of the work also were analyzed. We wanted to document and analyze the cognitive models (strategies) that the students used to find the instantaneous rate including the manipulation of the notions of function and limit when using a graphic representation and also the evolution of these models was owed to the discussion in the classroom.

Results

To illustrate the type of analysis realized we present some results from the video analysis. We interpreted the apparent mental model of student Octavio visually looking for equal parts on the graph (see Figure 1) AC=A'C', therefore inferring that the flows were equal. When asked to explain, he noted the run and the rise and the relation of the rise to the volume.

Octavio in his written answer to the problem wrote:

25 seconds—— 30 seconds
1370 liters —— 1300 liters = 70 liters/ 5 sec = 14 lts/sec
85 seconds—— 80 seconds
270 liters —— 200 liters = 70 liters/ 5 sec = 14 lts/sec.

He showed that he didn't have an appropriate relationship between the graphic representation with the numerical representation. He talked about graphically represented distances and what he calculated were the secants' slopes to respond to the question as to when the flows were equal. As well he couldn't demonstrate his concern over the process of approximation; what is more is that he appeared to be more concerned about parts of the graph where numerically the variation of volume and time was the same.

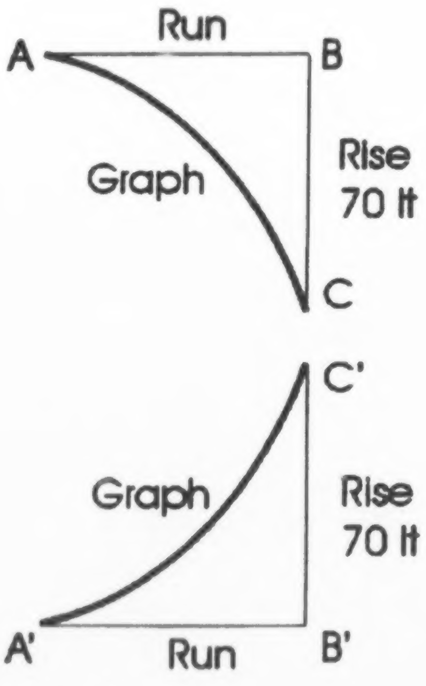

Figure 1

In the following episode, the student Armando, just like Octavio, concentrated on equal parts of the visual representation of the graph so that the flows would be approximately equal, but he added the same slope and shape in order to find equal flows (see Figure 2).

Nevertheless, in the homework turned in, Armando calculated the flow using tangent's slopes (see Figure 3). Armando in his written answer to the problem wrote:

> *The two moments with the same flow are: at 40 sec (40, 1100), (30, 1400), and at 100 sec (85, 100), (120, 1300); the flow is 34.28 lts/sec.*

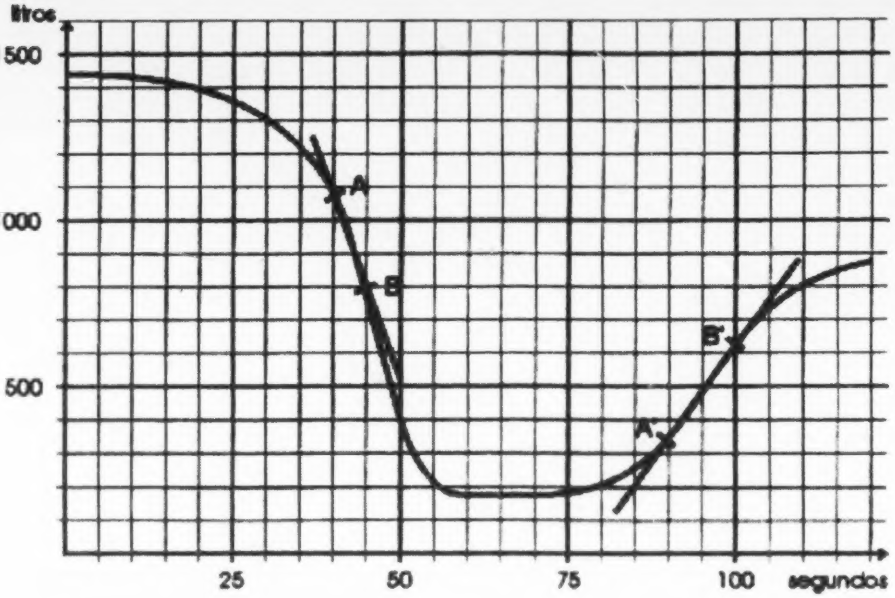

Figure 2

Just like Octavio, there wasn't a good relationship between the explanations he used to find the instantaneous flow.

The strategy used by another student, named Alex, to find equal flows evolved in class. In the beginning he had compared the run and rise from corresponding parts of the graph as being equal, therefore the flows were equal as well; later, he considered only the inclinations of the tangent lines to determine if the flows were equal. We think that this change appeared as a consequence of having listened to Armando in a previous episode relating the flow with the slope. At the beginning, Alex in his written answer to the problem had not demonstrated the processes of approximation, taking intervals of 5 seconds ([25, 30] and [80, 85]), and said the solution was 14 liters/sec in each case without specifying which moments the intervals were 14. He gave the same answer as Octavio (see Octavio's written answer). At the end when the teacher asked at which moments the flow had been equal, he hesitated a moment, and responded that whichever one you decide to use respectively between [83, 85] and [105, 107]. It appeared that before the question he focused on small intervals determined by the best approximated points and not the instantaneous point.

The strategy of Mauricio to find two different moments in which the flow had been the same was to find where on the graph there appeared a straight line ([42.5, 47.5]) and considered that in this entire interval the instantaneous flows were equal. Other students in the earlier episode had seen that there had been a horizontal line ([60, 70]) and this confused some students in reference to the possibility of a flow being zero, but others

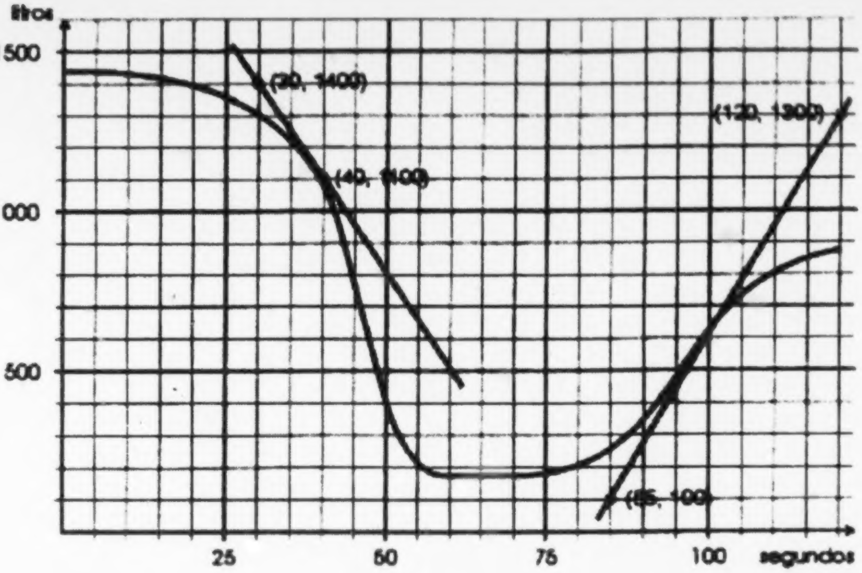

Figure 3

students said that this signified that water doesn't enter or leave the tank. In general, they took the variation of volume as an absolute value without mentioning if it increased or decreased.

Discussion

In general, students also didn't mention the relationship between sign of the flow or the sign of the tangent's slope with the exit and entry of water into the tank. We can say that the strategies employed to find the flow graphically are based on incomplete, unstable cognitive models, and include pieces of relatively disconnected bits of knowledge. Like the cases of Octavio and Armando, the graphs and verbal descriptions of the situation had little or nothing in relation to the processes of calculations employed. The models became more adequate each time that they were tested and discussed without ever arriving at the optimal solution. Like was the case of Alex, who, to find equal flows at first compared the run and rise on the graph, and then compared the equal inclinations. There were notable individual differences in the adequacy of strategies used by the students. Not many students related the instantaneous flow with the slope (inclination) of the tangent nor was it used as the first supposition. The teacher's knowledge of the strategies and difficulties of the students in solving these problems whose structure was the relation between function and its derivative could help in the design of their methods of teaching.

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AN INVESTIGATION OF THE ABILITY TO PROCESS AND COMPREHEND TEXT ON STUDENT SUCCESS IN SOLVING ALGEBRAIC WORD PROBLEMS

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Implementation of current reform initiatives in mathematics requires learners to deal with increased amounts of text. In order for students to reach an acceptable level of mathematical literacy, they must be able to comprehend and process text in order to integrate textual information into the language of mathematics. This study focuses on the impact which reading and comprehending text plays in solving what is commonly referred to as 'word' problems. The study involved 270 ninth grade algebra students enrolled in general and advanced courses. Tests were designed to measure student performance on computation, reading, and problem integration. While advanced students performed higher than general students in computation, both groups performed similarly when testing situations provided for the oral reading of the problems. The advanced group was successful twice as often as the advanced group when the testing situation required silent reading. These results indicate the importance of reading on student performance in solving algebraic 'word' problems and the benefit of informal measures of reading comprehension and interpretation.

Problem solving is a vital part of the mathematics curriculum. The National Council of Teachers of Mathematics (1989) includes problem solving as one of the educational goals which the curriculum must reflect in order to facilitate overall mathematical development. This emphasis upon problem solving comes at a time when there is considerable attention given to the importance of algebra in the mathematics curriculum. Many states have begun to require an introductory course in algebra as a condition for graduation. Part of this requirement is founded upon beliefs that no student should be deprived of educational opportunities which provide key skills necessary for survival in an ever increasing technological world (Lawson, 1990). Though reform initiatives stress the importance of multiple solution paths to a problem situation, the ability of students to successfully translate information into mathematical statements is an important skill. The ability to successfully translate problems into the 'language of algebra' is consequential in students' success with advanced mathematics courses which become increasingly abstract. Additionally, texts which model reform initiative place an increasing emphasis upon the students' capability to read and interpret written problems. Therefore, this study investigated the role of comprehending text in the performance of ninth-grade algebra students while solving word problems.

Perspective

Historically, both students and teachers have acknowledged the difficulty in understanding and solving 'word problems' (Knifong & Holton, 1976). Computation and reading skills have been widely noted in the literature as factors which are important to success in solving such mathematical situations (Balow, 1964). The interpretation of the problem situation, including the selection of the appropriate operations has been identified as variables which affect success in solving word problems (Kantowski, 1980). More research is needed to help us identify what facets of solving word problems become areas of difficulty for our students and why. Particularly, the relationship of reading text and successful problem solving must be clearly articulated if students are to gain the skills in algebra which will allow them to experience success.

Information-processing theorists have been deliberating how both skilled and unskilled learners process or "chunk" different modes of printed discourse; how they increase information-processing skills in different modes, and thus how they become more literate learners. Information-processing theorists like Goodman (1976) have developed models which illustrate how fluent readers make greater use of non-visual information (what is in the reader's head) than visual information (what is on the printed page) during most literacy tasks. Likewise, Kinneavy (1971) and Moffett (1983) have developed communication models which show how we make use of non-visual information as we compose in writing and comprehend in reading.

Success at tasks which involve mathematical problem solving involves not only reading word problems but setting up word problems, performing the necessary computations and integrating these aspects into a total and correct solution which assumes the recognition of thinking and oral language as powerful prerequisites in the comprehending, processing, and solving of word problems. An understanding of the complex and interrelated nature of these skills is essential in order to improve our practices in teaching mathematics. Particularly, one goal of this study is to articulate the degree to which reading skills anticipate a student's ability to translate problem situations into algebraic expressions and ultimately arrive at a correct solution.

Method

Data Source. The participants in this study were 270 students from two different suburban high schools in the Piedmont Region of North Carolina. From one school, 139 general ninth-grade algebra students participated in the study. From the other school, 131 advanced algebra students participated in the study. While the students in both schools could self-select algebra courses, for the most part students were placed in general or advanced courses based on academic ability and previous academic performance in mathematics.

Research Questions. This inquiry sought to answer the following questions:

1. How many word problems do general and advanced ninth-grade students solve satisfactorily on tests of (a) computation, (b) reading, and (c) problem interpretation?
2. What similarities and differences exist in mean test scores between general and advanced students on tests of (a) computation, (b) reading, and (c) problem interpretation?
3. What are some main sources of difficulty preventing both general and advanced ninth grade algebra students from solving certain word problems on all three tests: (a) computation, (b) reading, and (c) problem interpretation?

Procedures. Three tests were constructed for the project, each with 12 word problems. Each test consisted of four easy, four average, and four difficult word problems. The word problems included in each test were constructed to be comparable to those found in the University of Chicago School Mathematics Project, a national school reform program. The progression of problem difficulty was kept consistent with the sequence within the algebra texts developed as part of the Chicago program. Additionally, two professors of mathematics education and two high school algebra teachers read and compared the three tests constructed for the study. These four individuals agreed that (a) each test had three levels of difficulty and that (b) all three tests were comparable, or equally weighted, in relation to total ease/difficulty. All students in the study completed one test a week over a three-week period. Two algebra teachers, one from each school, administered the tests to students in this study using scripted directions for administration and prompts to generate later student discussion.

The first test was designed to measure pure computational skills. Each problem was derived from a set of word problems constructed for Test 1 during the test design phase. The problems were set up in computational form to measure each subject's computational skills. All tasks of reading and problem interpretation, as well as integration, were removed by virtue of each problem being set up in purely computational form.

Test 2 was designed to measure the influence of the teacher's oral reading of the problem on the student's ability to interpret and solve the word problem. The problems were read to the students and scores were based on whether or not the students set the problems up properly, that is the student indicated some type of representation of the information presented in the problem which could lead to a correct solution. The students had a copy of the problem in from of them while they worked in order to avoid unnecessary memory load.

Test 3 was designed to measure the student's silent reading, interpretation, and integration abilities. The third test yielded two scores - one computed by whether the problem had been set up correctly and the other by the correctness of the answer.

Data Analysis and Results

Mean scores were calculated on all three tests for all general and advanced students. In addition, students were asked directly after taking the tests to first write and later discuss the major sources of difficulty preventing them from solving specific word problems.

The mean number of correct answers on the 12 algebra word problem computation tests for the general students in the study was 5. The mean oral reading/problem interpretation was 6. The mean score for silent reading/problem interpretation was 4. The total problem/integration mean score was 4.

The mean number of correct answers on the 12 algebra word problem computation tests for the advanced students in the study was 9. The mean score for oral reading/problem interpretation was 6. The mean score for silent reading/problem interpretation was 8. The total problem integration mean score was 8.

Three common problems were identified in the analysis of student responses about what areas of the test gave them difficulty. They include (a) lack of notational discrimination, (b) lack of ability to set up problems algebraically, and (c) word problems with multiple tasks.

Educational Significance

Teachers and educators must attempt to understand, such as through the use of informal math inventories similar to the tests used in this study, how students with different reading abilities may be helped or hindered by oral reading during the solving of math problems. Secondly, students must be encouraged to develop a reading automaticity to math problem solving through concomitantly and interactively teaching computation skills, oral and silent reading skills with problem interpretation, and integration skills. Third, though mathematical word problems are subject to more than one interpretation and may lead to more than one right answer, the development of skills in problem interpretation and integration are essential prerequisites in student success at representing problems algebraically. Such skills are vital to the development of mathematical reasoning and essential in advanced courses in mathematics; therefore, students of algebra must be given every opportunity to become proficient in the use of such tools.

These findings emphasize the applicability of information processing models to assist in understanding the processing of text in solving algebraic word problems. The findings further highlight the need for perspectives from other disciplines, particularly from the fields of psychology and literacy, in developing appropriate learning models for mathematics problem solving. Our current models of mathematical teaching and learning must be expanded to address issues from learning theory and literacy. Such broadened perspectives are important in advancing efforts to reach all students and assist them in becoming the mathematically literate learners of tomorrow.

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VARIABLES INFLUENCING THE EFFECTIVENESS OF SMALL-GROUP WORK: A YEAR LONG STUDY OF COLLABORATIVE MATH PROBLEM SOLVING IN TWO FOURTH-GRADE CLASSROOMS

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Fourth-grade children in two classrooms worked in small, heterogeneous groups to solve a collection of problems. We videotaped one group of children in each classroom each week. We also observed the other groups of children as we assisted in the classrooms. Our goal was to capture the natural development of the children's interactions in these group settings in the evolving classroom climate as unobtrusively as possible. For this report, we focused our analyses on a group of two boys and two girls for two sessions. We examined the children's interactions in terms of their physical positions within the group, their mathematics achievement levels, and their gender. We compared their discourse in terms of mathematics, group management, and off-task behavior. We found that both gender and position in the group had an effect on interaction.

A commonly-held belief is that the use of collaborative groups creates an environment in which students can develop their abilities to solve problems, reason, communicate (Artzt, 1996) and develop confidence in their ability to do mathematics. This belief is supported by the contextualist view of cognitive development, in which learning is embedded within a social context in which children's thinking advances through interactions with skilled adults (Vygotsky, 1978) or through collaboration with peers (Miller, 1993). Theoretically, small groups of peers engaged in shared problem solving are actively learning mathematical concepts when they challenge one another to use mathematical reasoning to explain strategies and to defend answers. In practice, collaborative learning can contribute to students' self-esteem and peer acceptance (Slavin, 1989), and presumably their confidence to do mathematics.

Yet, the use of collaborative learning as a method for elementary mathematics instruction is not common practice (Good, Grouws, & Mason, 1990). This is perhaps, in part, because research reports offer inconsistent conclusions about the learning gains of students of different ability levels (Davidson, 1985). For example, there may be a positive effect for low-ability (Leiken & Zaslavsky, 1997; Webb, 1985) and for high-ability students (Webb, 1985), or there may be a negative effect for low-ability (cited in Good, Mulryan, & McCaslin, 1992) and middle-level-ability students (Webb, 1985).

Another issue is whether boys and girls profit equally from collaborative learning. Gender equity in mathematics has long been a concern of educators (see the AAUW report, How Schools Shortchange Girls, 1992). Ambrose, Levi, and Fennema (1997) speculate that girls may be less likely than boys to participate in a classroom in which mathematics discourse and debate are encouraged, either due to a lack of confidence in their mathematical ability or to their dislike of in conflict.

In light of these concerns, Good et al. (1992) argued that we do not understand the actual interactions that occur among students in collaborative learning groups. They issued a call for more observational studies that examine these interactions in detail.

Yackel, Cobb, and Wood (1991) engaged in such a year-long study of a second grade class in which small groups of children worked together and then shared their strategies with the whole class for all of their mathematics instruction. The researchers described ways in which the teacher communicated expectations and ways in which the children communicated strategies. In a review of observational studies across grade levels, Webb (1991) found that students who were expected to provide explanations tended to have higher achievement scores then students who were simply told answers. More specifically, Cobb (1995) suggested that children must be engaged in "genuine argumentation" (p.125) for learning to occur.

Research questions. The focus of our study was the identification of factors related to group composition that may influence these interactions. We selected ability level, gender, and physical position within the group as factors for investigation.

Method

Sample and Context. We observed two fourth-grade, heterogeneous home-room classrooms in which students engaged in a weekly problem-solving session throughout the school year. After a brief whole-class discussion in which the teachers focused on understanding the problems, the students in small groups would work on a set of approximately 8 problems. Typically, the teachers concluded the weekly sessions with groups of children presenting their strategies and solutions to the class or with the class engaged in writing their explanations for a selected problem. Most of the children had experienced cooperative groups and mathematics problem solving in third grade. The teachers used a variety of techniques to form heterogeneous groups of three to five students. The teachers indicated that they were more concerned about the development of students' problem-solving skills than in the acquisition of specific mathematics concepts, although they believed that learning mathematics concepts was a "natural by-product" of the program.

We used a computerized coding system to score the videotapes. We coded every statement and then used a software package to calculate the frequency and duration of specified variables. Each comment was scored as belonging to one of three categories: mathematics problem solving, group management, or off-task behavior. We also coded who made the comment and whether it was directed to the group, a specific person in the group, or self-directed.

To address the question of gender interactions, we selected videotaped sessions of groups of two boys and two girls. This restricted our data to 13 sessions. We then were able to select two sessions in which the groups differed by only one student. We identified the students as G1L, G2H, B3H, B4H. "G" or "B" indicates girl or boy, the numeral indicates the position at the table from left to right, and the last letter indicates the ability level of low, middle, or high. In the second session, B4M was replaced by a high-ability boy.

The transcribed sessions were coded independently by two of us, and then the two coders resolved any differences by jointly viewing the video-taped portions and consulting with the code definitions. These codes were then entered into the computer system.

Results and Conclusions

In general, interactions for the two sessions were similar (see Table 1). The largest proportion of discourse was directed by students to the entire group (*M*=44%). The next largest proportion was between individual students (*M*=36%). The remaining comments were either self-directed (*M*=17%) or directed to an adult facilitator (*M*=3%).

Table 1
Percent of Speech Time

Session Comments	Speaker							
	Girls		Boys		Adult		Totals	
Directed To	1	2	1	2	1	2	1	2
Girls	4	6	5	10	1	2	10	18
Boys	8	10	9	11	3	3	20	24
Group	21	18	21	17	5	5	47	40
Self	10	8	9	7	0	0	19	15
Adult	1	1	3	2	—	—	4	3
Totals	44	43	47	47	9	10	100	100

When we examined comments that were directed to individual students, we found different patterns of interaction for boys and girls. Table 2 illustrates these interactions for all three categories of comments combined. In both cases, boys contributed slightly more comments than girls did. This sex difference was greater for session 2 (13% more total discourse for boys; 20% more mathematics discourse) than for session 1 (8% total discourse; 8% mathematics discourse). There was an even stronger effect for sex when we considered to whom comments were directed; considerably more discourse was directed to boys than to girls. This difference was greater for session 1 (30% total discourse; 50% mathematics discourse) than for session 2 (13% total discourse; 12% mathematics discourse). The session differences may have resulted from the single change in group membership (medium-ability boy in session 1 switched for high-ability boy in session 2). The new high-ability boy in session 2 overall spoke more and directed more comments to the girls than the original medium-ability boy in session 1.

The three groupings in session 2 reflect changes in the students' positions during that session. In session 1, the amount of interaction for same sex pairs was almost double that of opposite sex pairs. The pattern was more variable for the second session, indicating a possible link between sex and seating position. Same sex pairs engaged in more interaction than opposite sex pairs when same sex pairs were seated next to each other (BBGG), yet opposite sex pairs engaged in more interaction when same sex pairs were not seated next to each other (BGGB). The findings for adjacent and nonadjacent pairs also point to a possible link with sex. Adjacent pairs engaged in substantially more interaction than nonadjacent pairs when same sex pairs were seated next to each another (GGBB in session 1 and BBGG in session 2), while nonadjacent pairs engaged in more interaction when same sex pairs were not seated next to each other (BGGB in

Table 2
Percent of Total Interaction Time by Gender

Session	Speaker					
Comments	Girls		Boys		Totals	
Directed To	1	2	1	2	1	2
Girls	16	16	19	28	35	44
Boys	30	28	35	29	65	57
Totals	46	44	54	57	100	101

session 2). Only the initial GGBB segment of session 2 does not follow these patterns, and there the differences for gender and position were minimal.

The data indicate that students' ability levels also plays an important role in student interactions. High ability students interacted most with other high-ability students. It is not the case that high-ability students are simply talking more. Middle- and low- ability level students direct more comments to the entire group and to themselves than do high ability students.

This report reflects a project still in progress. We offer a preliminary examination of students' contributions to the process of solving mathematics problems in small groups. The focus so far has been on how group composition variables such as sex, seating position, and mathematics ability level relate to quantitative measures such as how much each child contributes to discussion and to whom they direct their comments. Further work will involve similar analyses for additional groups participating in sessions throughout the school year. We are also working on developing a more qualitative system of analyses that will describe the actual problem solving strategies and solutions used by the students. We hope that this research will help us to better understand the dynamics of small group collaborative learning, and will also provide guidelines to help teachers organize groups that function more effectively.

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PROBLEM-SOLVING SHORT ORALS

CREATIVE THINKING: A MENTAL FUND AND FUNDAMENTAL OF MATHEMATICS EDUCATION

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Two years of research in a second-grade classroom incorporating contemporary theories of learning indicated that children's creative thinking underpinned meaning-making during mathematical experiences. In contrast with logical positivism, our study reconstructed what it means to "do" and "know" mathematics. Along with constructivist perspectives the study incorporated an eco-feminist ethic (Nelsen, 1996) and a problem-centered approach (Reynolds & Wheatley, 1996). Such an approach replaced competition with cooperation, fostered collaboration in solving social as well as academic problems, promoted individual voice by respecting each person's attempt to express an opinion, and decried coercive authority. As a community of learners children's intellectual, social, cultural, physical, and emotional development were cultivated, akin to what is portrayed in the writings of Dewey as "personal fulfillment and social well-being...the spiritual aspects of human experience" (Jackson, 1990, p. xxxvi). As children solved daily math problems they also focused on social issues of cooperation and mutual respect. The interaction of these dimensions coalesced as a view of what it meant to "do" mathematics. The synergistic coalescence of methodologies (Geoghegan et al., 1997) highlighted how meaning-making became as much a creative as a constructive experience. Children engaged "playfully" with their developing mathematical ideas within a setting that was respectfully attentive to their "needs." They became increasingly creative as if liberated from the conformity, passive compliance, and objective heteronomy that typify "traditional" classrooms.

Data from our study indicated that fostering cooperation and individual voice supported children's capacity to take risks, make novel suggestions and invent unconventional, yet effective, ways of "doing" mathematics, which in turn, fortified their confidence to explore and generate even more creative mathematical thinking. When provided with experiences based on mutual respect and problem-solving children played with what they created and created with what they played. Devlin (1997) asserts that "original thought and the ability to see things in novel ways" (p. 3) are exactly what mathematicians require.

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PROBLEM-SOLVING POSTER

CHARACTERISTICS OF SIGNED ARITHMETIC WORD PROBLEMS

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There is reason to believe that the visual/spatial nature of American Sign Language (ASL) and its unique features may facilitate the solution of word problems. The structure of ASL and how it derives meaning from symbols is very different from Western languages (Bryant, 1995). For example, by using the space in front of the signer to establish locations of objects and using directional verbs to indicate the relations between these objects, the signer creates a mapping that may model the problem's solution. The study reported here investigated the through-the-air communication of mathematics to deaf students.

Thirty-eight teachers of K-3 deaf and hard of hearing students from 5 schools for the deaf translated 15 standard arithmetic word problems of 9 semantic types from written English into sign. The signed versions of the problems were compared to the original written text and analyzed with respect to problem characteristics established by studies of hearing students to be related to problem difficulty (e.g., semantic structure, order of events, explicitness of action, problem wording). A second analysis inspected the visual representations and identified characteristics that have potential to influence the problems' solutions. Among factors related to the sign characteristics were teachers' hearing status and sign proficiency, teaching experience and grade-level taught.

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PROBLEM SOLVING STRATEGIES INFLUENCED BY CONCEPTIONS

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Purpose of the study was to study students´ mathematical problem solving strategies and conceptions. Senior high school students in two math classes on functions and the derivative ($n_A = 15$; $n_B = 9$), who had taken algebra and analytical geometry, were given pre- and post-tests on an optimization problem (maximum volume of a rectangular box. The derivative was discussed over twelve hours in a three-week period. Both groups considered definitions and exercises, and group A used word problems and graphic calculators. The post-test was given two-weeks after the derivative was taught. An acceptable solution (AS) was defined differently for each of the tests. Actions taken, and results obtained, by each student were identified and classified. On the Pre-test three different strategies were identified:

- *adding* walls to the metal sheet in the problem
- *partition* of the sheet into mostly unequeal subareas
- *cutting* the sheet in different ways

Most students worked out only one case and took information in the problem as *data*, not *variables*, all indicative of an algorithmic view of mathematics. It was expected students would use previous knowledge , do the cutting, and consider at least two cases, compare them and determine the largest volumen. Most of them did not make use of any of this. On the Post-test, all but two students (one from each group) followed the *cutting strategy*. Most used algebraic operations and *variables*, but only one student from group A used the derivative and obtained an AS. In group B it was three and one. Though an improvement over the pre-test, *procedural* difficulties are clear.

CREATIVE MATHEMATICAL EXPLORATIONS IN NON-TRADITIONAL ENVIRONMENTS

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It is not uncommon to find students who clearly possess ingenuity and mathematical intuition, but are not good at more structured algorithmic learning. More emphasis on the creative aspects of mathematics may enhance the success of such students. This presentation will focus on the creative path taken by one such student in her problem solving effort.

This research took place in the after school enrichment program called Kindermath, (See Kajander, In Press) in which seven to ten year olds explore and create in mathematics. Using a modified version of the tunnels problem described by Zack (1995), students were asked to find a pattern between the number of nodes in a network and the number of "roads" between them. Two students, a boy aged 8 and a girl aged 10 had made a chart listing the number of dots and the number of lines, for one to six dots. They were then asked if they could predict generally what the number of liens would be given a certain number of dots, without drawing the picture.

While the female students was in a modified program in math and could only add single digits using her finger, she obviously grasped the essence of the problem easily because she pointed to the six dots problem and said, "Well, this one has to be six time something because there's six dots." When the teacher prompted by asking, "How many lines are at each dot?", she began counting them, marking them off in colour as she did. She wrote (vertically) 5+4+3+2+1 and after some prolonged finger counting, the sum 15. She was then able to verify that the method worked for the previous cases on the chart. When asked if she could write out her solution, she wrote, "You start with the number of dots subtract one. Then write out all the other numbers going down to one, and add them up. That's the number of lines."

The environment clearly plays an important role in facilitating creative mathematical behavior, as this was not the first example of good problem solving displayed by this students. Yet this girl's classroom teacher described her as "very poor in math and she doesn't even seem to care." Her mother reported, however, that it was with great enthusiasm that the student shared her results with her astonished teacher.

Perhaps further research into and emphasis on such small group creative problem solving environments would allow students such as this girl to develop to their fullest potential.

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INVESTIGATING STUDENTS' EVOLVING MODELS OF MOTION

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This analysis focused on major shifts in the ways of modeling motion that evolved in a mathematics class of preservice teachers. The pedagogical goal was that through the process of modeling, in which students discuss the advantages and disadvantages of various models, general notions of meta-representational knowledge (diSessa et al., 1991) would become taken-as-shared. The goal of this research was to investigate the degree to which the pedagogical goal was attained and to investigate what aspects of classroom microculture supported this.

Students were engaged in situations that involved modeling motion and that were intended to encourage the development of increasingly sophisticated models for describing quantities that change over time. A starting point of activity was a scenario involving motion for which students were asked to develop as many models as they could.

What ensued may be characterized as a progression from pictographs to models of discrete events to models of continuous recordings over time. The first round of models included cartoon-like drawings of the specific event. The second round contained several conventional graphs such as bar graphs and histograms that appeared to "fit" their prior knowledge of mathematical representations to the present situation. The shift from the pictograms to the more mathematical models was based on negotiations for what constituted a useful model: efficiency, ease of understanding, and ease of figuring out what questions the models could answer. In the third phase the models resembled the traditional line graphs, where time is represented along the x-axis and distance or speed is along the y-axis. These discussions involved: 1) a push toward showing where a character was at any time and 2) an effort to identify one quantity changing over time. Students developed a sense of the conventions that are embedded in speed and position graphs.

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THE STRENGTH OF GROUP–CONSTRUCTED (MIS)CONCEPTIONS

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This research study investigated the construction of mathematical (mis)conceptions by college remedial mathematics students working in small collaborative groups. Theories of symbolic interactionism (Bauserfeld, 1993) and constructivism (Yackel & Cobb, 1996) were used to guide both data collection and analysis. The symbolic interactionist perspective was particularly useful because it emphasized individual sense-making as well as social processes.

Two groups of remedial mathematics students (n = 8) were observed as they attempted to approximate solutions to an algebraic *work* problem. Both groups developed algorithms that produced reasonable approximations—but not exact solutions—for this particular problem. The students, however, believed that their solutions were exact and viewed their efforts as *knowledge* rather than *exploration*. In an attempt to highlight their misconceptions, a new *work* problem was developed such that the students' faulty algorithms would produce ridiculous results. The students were again observed as they attempted to resolve this new conflict.

Data (observational notes, student written work, examination responses) indicate that the students were reluctant to revise/abandon their group-constructed algorithms. These results underscore the importance of close observation/guidance by instructors during group activities.

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RATIONAL NUMBERS RESEARCH REPORTS

CREATING THE CONDITIONS FOR CONCEPTUAL CHANGE

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This paper reports on exploratory research conducted jointly by a mathematician and a mathematics educator. Our research investigates the conditions that facilitate change in the rational number concepts of preservice elementary school teachers. We examine how mathematical conversations can be used to strengthen rational number concepts and the relevance of historical development of these concepts. The purpose of this work is to enhance both our own understanding and that of our students and to inform teaching at both levels.

To modern mathematicians, a fraction is a number no more or less legitimate than a positive integer. But to children, as well as to many adults and the ancient Greek mathematicians, "number" is reserved for the positive integers and fractions have a somewhat different status. Our research explores the extent to which this is true for college students preparing to be elementary teachers. We are looking at the process of revision of the number concept to include fractions both in historical and in individual development. The central focus for our ongoing research is the conditions under which change in our students' rational number concepts are affected through dialogue with peers and teachers. Questions we are investigating include: How does the transition occur from seeing fractions as attached to concrete objects to fractions as abstract numbers? How do analogies and concrete models facilitate or interfere with abstraction? How can knowledge of the historical development of mathematical concepts inform our work with students? In this paper we will focus on preliminary work with respect to students' understanding of the density of rational numbers.

Perspectives

While multiple perspectives inform our thinking and research, the most immediate and important influence has been the ongoing mathematical conversations with students in our classes and our investigation into their thinking. In addition, the following theoretical frameworks for understanding and promoting concept development inform our research: Vygotsky's (1962) theory on the social construction of knowledge; Kuhn's (1970) view of the nature of the history of science; work relating Kuhn's ideas to the history of mathematics (Gillies, 1992); and the models posed by Carey (1991, 1995), Posner, Strike, Hewson, and Gertzog (1982), and Strike and Posner (1992) for conceptual change in individuals.

Our work is also informed by research on college students' learning of scientific concepts. Cavicchi's (1997) research showed similarities and differences between a college student's learning about magnetism and the original experimental investigations of Faraday. This work investigates the roles of confusion, doubt, and analogy in the development of understanding. In their study of college students' learning about special relativity, Posner et al. (1982) observed that a radical or revolutionary conceptual change need not be abrupt and is more likely to entail a series of gradual changes leading to a substantial reorganization of central concepts; it "involves much fumbling about, many false starts and mistakes, and frequent reversals of direction" (p. 223). The conditions for conceptual change described by Posner et al. (1982) are used in the analysis of our interviews. Their focus is on fundamental changes in a person's central organizing concepts from one set to another, incompatible one. The conditions they describe are:

1) dissatisfaction with existing conceptions - an individual must have collected a store of unsolved puzzles or anomalies and lost faith in the capacity of his current concepts to solve these problems.
2) intelligibility of the new concept - this is aided by use of analogies and metaphors.
3) initial plausibility of the new concept - this is aided by consistency with other currently held ideas.
4) fruitfulness of the new concept - it should have potential to be extended to new areas of inquiry.

Methodology

We investigate our students' conceptions of rational numbers and changes in those conceptions through use of the "extended clinical interview" as developed by Eleanor Duckworth (1987) and used by Elizabeth Cavicchi in her work on student learning about magnetism (1997). This method "encourages the researcher to analyze interviews interactively during sessions and reflectively afterward" (Cavicchi, 1997, p. 869). In our interviews, questions and problems are posed to individual students or pairs of students who are invited to explore and discuss their responses. Students are asked to reflect aloud and to describe their strategies and methods for finding solutions to problems posed.

From April 1997 to April 1998, 24 University of Massachusetts Boston students preparing to be elementary school teachers were interviewed. We began by interviewing individual students and then moved to interviewing students in pairs. During the pair interviews, each student was able to both observe and interact with the other's learning process. In an effort to extend this research to classroom practice, we also conducted sessions in which students previously interviewed by the investigators led conversations between pairs of students who had not yet been interviewed. During this period, there were four sessions with individual students (one each), fourteen with pairs (including two students who had been interviewed individually), and three with students interviewing other students. Three pairs were interviewed twice and two three times. Each session lasted approximately ninety minutes.

Interviews were flexible in structure but also included a predetermined set of initial and follow-up questions. We structured the interviews around a series of questions of the form, "Can you find a number that...?" We began with questions about the density of rational numbers, size comparisons, mathematical operations, and questions probing the differences between operations on fractions and on ratios. We wanted to examine not only student conceptions but also the way in which mathematical conversations might advance those conceptions. Interviews were recorded on audio tape for later transcription and analysis. Transcription was done by graduate research assistants with a strong background in mathematics. Three interviewers, the authors and a graduate student in mathematics education, examined the audio tapes and transcripts for evidence of students' conceptions of rational numbers and changes in these conceptions during the interviews, with particular attention to the conditions for conceptual change discussed by Posner et al. (1982).

In summer 1998, a small group of students will participate in the analysis of their interviews through discussion of observations and conclusions about the processes of change in their understandings. We feel that this work is a collaboration not only between us but also with our students.

Preliminary Findings and Discussion

Of the 24 students interviewed, only 6 were initially certain that there are infinitely many rationals between any pair of numbers. Two others expressed this idea initially but were not sure of it. Sixteen students thought that the number of fractions between successive integers was finite. Some thought this number was as small as nine (4.1, ..., 4.9). Others said that there were many such rationals; in some cases this number was "almost infinite".

All the students we interviewed who did not initially realize that there were infinitely many rational numbers between any two numbers were able to come to this conclusion with the help of their interview partners or with some probing by the interviewers, and were able to generalize quickly once they grasped the concept of density. We have found that pair interviews are more effective than individual interviews because they facilitate conversations between students as they work on problems, articulate their concepts, and explain to each other their approaches to the problems. We also found that we were more able to serve as facilitators rather than directors of the conversation in the pair situation. When the conversations worked best, we offered a starting point and general framework and the students themselves found a path through the material.

Vygotsky (1962) and others have pointed out the strong social aspect of cognitive development. Our interviews were intellectual and social interactions between pairs of students and among the students and ourselves. Although the pairs we interviewed were made up according to scheduling constraints rather than prior knowledge or learning styles of the students, we found that the members of each pair had differing strengths and that they were able to help each other integrate their strengths. These strengths included the ability to calculate with fractions vs. intuitive, concrete understanding, facility with decimal vs. fractional representation, and algebraic vs. arithmetic skills.

The interviews highlighted the working of partial and fragile understanding. Some of the students we interviewed went through a discovery process in one interview, and then had to repeat the process, although more quickly, in a subsequent interview. Of the four criteria defined by Posner et al. (1982), three were clearly present in the interviews: dissatisfaction, intelligibility and plausibility.

Dissatisfaction with existing concepts was created through questions that raised inconsistencies or conflict with prior knowledge. Students who posited a finite number of rationals between two integers were asked if they could name them all and if there were a smallest one. They were also asked to develop strategies to find all such numbers. If a pair of students had different answers or strategies, they were asked to try to convince each other.

The density concept was clearly intelligible even for some students who did not initially accept it. Most of the students were aware of the infinitude of the natural numbers and could use this notion to understand the idea of density. For others, intelligibility was present only after they worked through the infinitude of natural numbers. Clearly this awareness did not immediately lead all of them to accept density. For example, one student said, "I know there are many numbers between four and five but not infinitely many."

The concept gained plausibility through students finding or being guided to strategies for finding an infinite number of rationals between two given numbers. These strategies included taking reciprocals of larger and larger numbers, repeatedly dividing small fractions in half, and adding another digit to a decimal, either an initial zero (e.g., .0001 to .00001) or a final non-zero digit (such as .1111 to .11111). For student pairs who did not develop these strategies on their own, some questions that helped them were: "How can you find other numbers?", "Can you find a strategy for finding other numbers?", and "Can that process go on indefinitely?"

Several students were reluctant to accept density even after this plausibility was established. For a number of students, one difficulty seemed to be interference between the physical models used to represent fractions and the abstract notion of density. For example, one student clearly articulated the difference between physical and abstract objects and the conflict between the two classes of concepts. When asked about dividing numbers into smaller and smaller units, she pointed to the lid of her coffee cup and said that if you try to cut it up, you will eventually get to a point at which it can not be cut any further. But when we asked if the same is true for numbers in the abstract, she replied that "you can cut up something that's abstract as much as you want." She was able to appreciate the density of rational numbers after making a distinction between concrete and abstract objects.

A related difficulty seemed to echo Zeno's Dichotomy paradox, which states that is impossible to move from 0 to 1, because first a distance of 1/2 and then 1/4 and then 1/8, and so on, must be covered. This presented a conflict for several students. For example, one student said that there could not be infinitely many numbers between 4 and 5, "because you would never get to 5." On the other hand, she realized that if there were only finitely many fractions in an interval, "there would be an end to the whole numbers and there isn't." Another student said "I ... bounce back and forth because I am stuck into like two of my firm beliefs."

The fruitfulness of the density concept for students' ability to form a more complete understanding of rational numbers is not yet clear. What is clear is that most students showed that they were quickly able to generalize this concept. For example, one student who came to understand density through consideration of decimal representation said "since there are infinitely many numbers between eight tenths and nine tenths, there must be infinitely many between eight elevenths and nine elevenths." On the other hand, some students verbalized that they understood the concept but could not "make it mean anything." Further work will be required to see how such meaning can be established. In particular, we plan to investigate how an understanding of density can help in the transition from strictly concrete to more abstract conceptions of rational numbers.

It has become clear to us through these conversations that many students long for mathematical understanding and express great excitement when they find it. Since our students were adults and the concepts we were discussing were not being introduced to them for the first time, they had ideas about them that, in many cases, they had not previously found an opportunity to express. Students have felt empowered by the interview process. When encouraged to develop strategies for reasoning about mathematics, they were often delighted to find that they are able to do so. Several students have told us that they feel that such interviews should be available to all students in mathematics content and pedagogy classes and that the conversation has changed their relationship to mathematics.

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RATIONAL NUMBERS
SHORT ORALS

EQUIVALENCE AND ORDER THROUGH EQUAL SHARING

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Unlike apparently universal patterns in the development of childrenís thinking about whole-number problem solving, the development of rational-number thinking may be much more sensitive to instruction. It is possible that the use of different kinds of problem contexts in instruction can result in different patterns of strategy development. Because the semantic origins of rational-number concepts and processes can be linked to a variety of contexts, researchers need to investigate a variety of instructional approaches and studentsí resulting thinking.

Equivalence and order are the lynchpin concepts of the domain. Prior studies of childrenís thinking about equivalence and order, and related proportion problems have shown that children have informal knowledge of certain proportional relationships; that they are initially able to construct nonstandard strategies for solving equivalence and order problems that depend on doubling, halving, and repeated adding; and that the context in which a problem is set has an impact on children's thinking. Much of this work has focused on locating transitions from (often incorrect) additive thinking to multiplicative thinking, the basis for proportional reasoning.

There has been much less focus on the kinds of strategies students are able to invent given powerful contexts within which to reason. It is feasible, for example, that incorrect additive reasoning might be largely avoided. Examples of problem contexts that hold promise for developing rational number concepts and processes include splitting, measuring, folding, and equal sharing. The question is not whether there is one best context, but what kinds of concept and strategy development each context affords.

The present study analyses 48 fourth graders' strategies for solving equivalence and order problems set in equal sharing contexts. The analysis extends previous work in the field by looking at how equal sharing supports the use multiplicative thinking in problem solving, and by considering how information about children's thinking might inform the creation of communities of discourse in classrooms.

INTRODUCING FRACTIONS ON THE BASIS OF MEASUREMENT

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Approaches to introducing fractions are needed that help children to accommodate everyday notions of numbers as things you count with to include fractional numbers, and to consider fractions as separate entities (Carpenter, Fennema, & Romberg, 1993). While measurement activities provide a natural context for developing these notions, developing fraction concepts out of measurement is not the norm. This teaching experiment introduced six 4th graders to fractions on the basis of measuring quantities using Davydov and Tsvetkovich's (1991) fraction curriculum. The study was designed to describe changes in the children's thinking, and relate these changes to the instructional activities. Some problems frequently reported in the literature on fraction learning were not observed. For example, the children did not exhibit difficulties in treating fractional unit measures as separate entities. Division of fractions was independently developed by the children; Davydovís curriculum emphasizes activities where children determine how many unit measures of size b fit into a quantity A. The children solved 3(3/5, for example, by determining how many unit measures of length 3/5 fit into the length that corresponded to the number 3 on the number line. The children also spontaneously referred to the inverse relation between the size of the unit measure and the number obtained as a measure, but did not quantify the relationship. Instruction assisted them in doing so, although three children vacillated between describing these relationships in multiplicative and additive terms. When instruction moved to fractional unit measures, they interpreted 2/4 as 2 unit measures of length 1/4, and 4/8 as 4 unit measures of length 1/8; and since they had already established that when the unit measure is 2 times smaller, the number fitting into the quantity is 2 times bigger, this explained why the lengths were equal. While five of the six children were capable of giving this kind of conceptually-based explanation of equivalent fractions, it required a great deal of coordination. Developing fraction concepts out of measurement was intellectually demanding for the children; the effort would seem to have substantial benefits, however (see e.g., Kamii & Clark, 1995).

RATIONAL NUMBERS
POSTERS

THE INFLUENCE OF PAST EXPERIENCE ON PATTERN RECOGNITION

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One of the heuristic strategies students use when solving problems is to "look for patterns" (Davis, 1984). This investigation examined fifteen groups of fourth grade students working in small groups (4-5 students) in a classroom setting. Students were from low income families in Rio de Janeiro, Brazil. Data were collected from videotapes, audiotapes and students' written work. Students compared groups of bill amounts and their related 10 % tips. They were asked to find a rule to explain how the bills were related to their tips. In doing so, they first had to recognize a pattern. Then one student from each group explained how the pattern worked.

Data showed they each used a different starting point, what one brings from one's past experience. Different starting points produced different explanations. Starting points differed among individuals as well as among groups. Explanations that students gave for their rules included examples, descriptions of what was going on, or the reason the rule worked. Although the students could understand other rules, they continued to use their own rules. This behavior demonstrates an individualist aspect of a social community.

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INTRODUCING RATIONAL NUMBER THROUGH THE TEACHING OF PERCENTS: TWO EXPERIMENTAL STUDIES

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It has been suggested that children need a deeper and more overarching sense of how the various representations of rational number relate to each other. Keeping these suggestions in mind, Case and I have designed a new approach for the introduction of rational number. Using developmental theory as a guide, we propose that a core conceptual organizing structure for rational number understanding is formed by the coordination of children's intuitive understanding of proportion and their splitting schemas (Confrey, 1994). In order to support this coordination we have designed an experimental program that introduces this number system through the teaching of percents in a linear measurement context. The initial props that are used, such as cylindrical beakers filled with water, allow students to make ratio and proportional judgements of the fullness of these containers relative to the whole using the language of percents. This context also promotes a spontaneous use of invented strategies for calculating percents that uses benchmarks and halving. In this curriculum decimal and fractions are taught later and are grounded in students' acquired knowledge of percents. (For information please see Moss & Case, in press.) So far I have conducted three formal teaching studies with 37 fourth-grade and 16 sixth-grade students in which this curriculum was implemented and assessed. The results of the first study have already been reported. Posttest results and analyses of classroom lessons of Studies 2 and 3 reveal that after instruction students show flexible movement among representations, resistance to misleading cues, successful ordering of numbers by magnitude, and, that these results are shown by both high- as well as low-achieving students. In the poster I will describe both the measures and the curriculum that we designed as well as present qualitative and quantitative analyses of the results of the studies.

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RESEARCH METHODS
RESEARCH REPORTS

ASSESSING STUDENTS' ATTITUDES TOWARD MATHEMATICS FROM A MULTILEVEL PERSPECTIVE

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Affective variables such as Attitudes Toward Mathematics (ATM) play a major role in the learning of mathematics. Since students' encounter with mathematics in the classroom play a major role in whether a student will have a positive or negative ATM, there is the need for research models that could guide researchers in identifying classroom processes associated with ATM. This study demonstrates the use of multilevel statistical models, Hierarchical Linear Model (HLM), to assess classroom effects on ATM. Classroom instructional practices such as discussion of practical problems and using real life situations in problem solving seem to have positive effect on students' ATM.

Introduction

Affective issues continue to play a very significant role in the teaching and learning of mathematics. The discussions of the cognitive processes of learning are almost always associated with students' feelings about learning. Current initiatives to reform mathematics education in many education systems have placed special significance on the role of effect on learning. In North America, the National Council of Teachers of Mathematics (NCTM) through its publication of the standards for curriculum and evaluation (Commission on Standards for School Mathematics, 1989) has advised her members to help students understand the value of mathematics as well assist in the development of students' confidence and interest in mathematics.

Attitudes toward mathematics (ATM) are among the affective variables that have received considerable attention from both educational researchers and educators. This is primarily due to the significant role the affective variables such as ATM play in students' achievement in mathematics (AIM) (McLeod, 1992). Research on ATM has usually emphasized three constructs: students' interest in or enjoyment of mathematics, the usefulness of mathematics to students' life , and the confidence of students in their ability to learn mathematics.

ATM is not an innate characteristic of students. Students develop these attitudes through their experiences with school mathematics. According to McLeod (1992), ATM arise out of ones' responses to difficulties while carrying out a task in mathematics. He argues that when the difficulties in a mathematical task leads to a blockage, individuals attempt to assess the meaning of this unexpected or otherwise troubled situation and evaluate the situation cognitively. The repeated interruptions in the same or similar context often results in less intensity emotionally and less cognitive evaluation of the interruption. Students therefore develop positive or negative ATM through a repeated encounter with mathematical problems.

Studies indicate that gender effects on ATM is substantive with males generally having more positive ATM than females. Students with low socioeconomic backgrounds (SES) also tend to have negative ATM. Most of these studies have relied on traditional statistical models which tend to analyze data from a particular unit of analysis (individual or school). The problem with such analysis is that it ignores the individual/context (school) interaction effects.

My substantive interest in this paper is to demonstrate the use of multilevel linear models, the Hierarchical Linear Model (HLM) by Byrk, Raudenbush and Congdon (1996) in assessing the variation of ATM in schools. The multilevel technique allows for the estimation of parameters using simultaneously data from both student and school or classroom levels (Bryk & Raudenbush, 1992). In school effects research, this involves the estimation of a separate regression equation for each school, providing a set of intercepts (i.e., levels of outcome for each school) and slopes (i.e., gradients for each school). The set of intercepts and slopes become the outcome variables at the second level of the model, that can be regressed on variables describing school processes. Gradients are measures of the gap between males and females in school outcomes and could also refer to the relationship between school outcomes and independent variables like SES. SES is a measure of a student's social class.

Research Questions

The following research questions are addressed in the study.

. To what extent do classrooms of grade 8 mathematics students in Canada vary in their levels of ATM, their SES gradients, and Sex differences in ATM?

. Are school/classroom differences in ATM associated with a particular group attribute (e.g., gender, SES)?

. Are some of the variations among schools/classrooms in levels of ATM, SES gradients, and Sex differences in ATM attributable to teacher instructional practices?

Data and Variables

The Canadian grade 8 data from the Third International Mathematics and Science Study (TIMSS) are used in this study. TIMSS was conducted under the auspices of IEA (International Association for the Evaluation of Educational Assessment) in 1994-95 academic year in more than 40 countries. The grade 8 population for each country is represented by grade 8 students from a random sample of schools within the participating countries. Data collected in the study included variables describing students background characteristics, their attitudes towards mathematics (ATM), and classroom processes.

The ATM construct is an aggregated measure of students' response to five questionnaires describing the extent to which they enjoy learning mathematics, feel mathematics is boring, feel mathematics is easy, feel mathematics is important to everyone's life, would like a job involving using mathematics. The aggregated measure was on a 4-point Likert type scale ranging from most negative to most positive coded as 1 = strongly negative, 2 = negative, 3 = positive, and 4 = strongly positive. The educational level (measured in years of education) of mother and father of students along with some selected items, including books, TV and a computer in a student's home were aggregated and scaled to have a mean of 0 and standard deviation of 1 and used as a measure of SES. Two variables "life" and "practical" describing instructional practices were also used in the analyses (see Table 1).

In Table 1, the descriptive statistics and the description of all the variables used in the study are outlined. The table shows that the average score on the attitude scale was about 2.9. This indicates that in general, the ATM

Table 1
Means, Standard Deviations, and Descriptions of Variables

Variable	Mean	SD	Description
			Student-level variables (N = 8362)
Sex	0.50	0.50	Dummy variable coded as 0=male, 1=female.
SES	0.00	1.00	Socioeconomic status of students' family.
ATM	2.86	0.70	Students' attitudes toward mathematics. Scale coded as 1=strongly negative, 2=negative, 3=positive, 4=strongly positive.
Life	1.55	.93	How often teacher solves problems with everyday life things. Likert type scale coded as 0=never, 1=once in a while, 2=pretty often, 3=almost always.
Practical	1.42	1.00	How often teacher discusses practical problems in class. Likert type scale coded as 0=never, 1=once in a while, 2=pretty often, 3=almost always.
			Classroom-level variables (N=364)
Mean-Life	1.48	0.41	Mean Life of a class
Mean-Practical	1.79	0.24	Mean Practical of a class

of grade 8 students in Canada is between negative and positive on the ATM scale. Students' response to the questionnaire on instructional practices in their classrooms indicate that on the average their teacher do not often solve problems with everyday life things or discuss practical problems in class. Mean for "life" and "practical" are 1.55 and 1.42 respectively.

Models and Analyses

In the final analyses all the variables were standardized (mean=0, S.D.=1) at the student level. The analyses involved 3 models. The first model (model 1) usually called the "null" model did not include any independent variables. The model simply partitioned the total students' variation on the dependent variable (ATM) into within class and between class variations (similar to what is done in Analysis of Variance). In the second model, I included the SES and sex measures into the equation to determine the extent of the effect of SES and Sex on ATM. The school variables (measures of teacher instructional practices) were added into the equation in Model 3.

Table 2 shows the final estimates of the models. The null model indicates that about 9.4% of the variance on ATM was between schools while the rest of the variance, 90.6%, was within schools. However, the between schools variance (0.089) was statistically significant ($p<0.01$). This suggests that schools differ on their average ATM. Males have more positive ATM than females, while students from high SES families have more positive ATM than those with low SES backgrounds. In both cases the difference is about 5% of a unit on the standardized ATM scale. The variance(.008) of the SES slope was significant at $p=0.01$ which suggests that differences in ATM associated with SES differ from school to school. SES slopes was negatively correlated with average school ATM. This means that SES gap on ATM is smaller in schools with positive ATM.

When variables describing instructional practices of teachers were introduced into the model, the between school variance was reduced by 25% (0.088 to 0.066), and the between SES gradients variance was also reduced by 25% (from 0.008 to 0.006). The two instructional practice variables had positive effect on the average school ATM. The effect was .323 and .154 for "life" and "practical" respectively. This means that a unit increase on the standardized "life" scale could increase the average ATM in a classroom by 32.3% on the standardized ATM scale while a unit increase on "practical" scale might increase the average ATM by about 15.4%. "Life" had a negative effect (-.081) on the SES gradients. This means that a unit increase on the life scale could decrease the gap on ATM between high and low SES students by about 8.1% on the standardized ATM scale.

Conclusion

The extent of Canada's grade 8 students' ATM within classrooms vary from school to school. Female students and students from low SES fami-

Table 2
Hierarchical Regression Coeffecients, Variances and Correlation Between Parameters

		model 1	model 2	model 3
Student-level Equation			Regression Coeffecients	
Constant		.084**	.084**	.082**
Sex			.054*	-.050*
SES			.051**	.055**
Classroom-level Equation				Regression Coeffecients
Effects on Adjusted ATM	Mean-Life			.323**
	Mean-Practical		.154**	
Effects on Sex differences	Mean-Life			———
	Mean-Practical			———
Effect on SES gradients	Mean-Life			-.081*
	Mean-Practical		.005	
			Variation	
Within Classrooms		.859	.846	.850
Between Classrooms	ATM score	.089**	.088**	.066**
	Sex differences		.014	———
	SES gradients		.008**	.006**
		Correlation Between Parameter Variances		
Average ATM (1)		1	2	3
		1.00	0.11	-0.38
		0.11	1.00	-0.58
		-0.38	-0.58	1.0
Sex Differences (2)				
SES gradients (3)				

Note: ** means $P<0.01$ and * means $P<0.05$.

lies have lower ATM than their counterparts. Classroom practices such as using real life and practical mathematical problems during class lessons are likely to, not only reduce the variation between classroom ATM averages but also the difference in ATM between low SES and high SES students. Using multilevel models, further research could be done to explore other classroom and school processes that affect students' ATM.

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STUDYING THE IMPACT OF REFORMED MATHEMATICS CURRICULA[1]

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This paper supports the use of a dynamic structural model as a viable means for testing a theory of reform in mathematics education. This paper outlines the implementation of the structural model in designing and organizing both quantitative and qualitative data analyses for a longitudinal/cross-sectional study on the impact of a particular reform-based middle school mathematics curriculum.

Numerous mathematics programs for K-12 students have recently been developed to address standards for the content, pedagogy, and assessment practices recommended by mathematics educators and organizations such as the National Council of Teachers of Mathematics (NCTM, 1989, 1991, 1995). As reformed mathematics curricula are becoming commercially available, however, a common concern raised by educators, parents, school officials, and the community at large is the lack of evidence that these new programs do significantly improve students' mathematical performance. This paper (a) examines the impact of a reform-based middle school mathematics curriculum as a means of testing a theory of reform in mathematics education and (b) discusses the implementation of a research design for examining the impact of a reform-based middle school curriculum in a longitudinal/cross-sectional study currently in progress.

The reform-based curriculum for Grades 5–8 used in this study is *Mathematics in Context* (MiC) (National Center for Research in Mathematical Sciences Education & Freudenthal Institute, 1997, 1998). MiC was designed to support the recommendations of the NCTM curriculum, pedagogy, and assessment standards. MiC includes the study of number, algebra, geometry, probability and statistics and encourages students to deepen their understanding of significant mathematics emphasizing connections among mathematical ideas developed in various lessons, units, and grade levels. MiC provides ample opportunities for students to solve experientially real problems designed to stimulate mathematical thinking. Students use problem-solving strategies to construct solutions in which they must recognize, understand, and extract embedded mathematical relationships. Students are expected to explore mathematical relationships, develop and

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explain their own reasoning in solving problems, and listen to, understand, and value each other's strategies. Assessment of students' understanding focuses on the use of mathematical concepts and procedures and provides attention to various levels of reasoning students apply when solving problems. The three levels of reasoning addressed in MiC are: conceptual and procedural knowledge (facts, definitions, and efficient application of standard procedures); connections (within and across content strands, integration of information, and selection of appropriate mathematical tools for solving problems); and analysis (interpretation and mathematical argumentation, development of the student's own strategies, and generalization).

MiC was also designed to reflect the principles of Realistic Mathematics Education developed at the Freudenthal Institute in The Netherlands (Gravemeijer, 1994). Students are given the opportunity to reinvent significant mathematics under the guidance of their teachers and through interaction with their peers during instruction. Specific problem contexts are selected to reflect situations that (a) make sense to students and (b) aid student development of mathematical concepts and procedures. Mathematical models allow students to solve problems at different levels of abstraction, and students can always fall back to more concrete, less abstract strategies. These models also act as bridges between concrete real-life problems and abstract formal mathematics.

Initially, students develop a model of a situation in which they use the problem context and informal strategies. Through instructional activities that allow students to solve problems using a variety of strategies, teachers encourage students to discuss interpretations of the problem situation, express their thinking, and react to different levels and qualities of solution strategies shared in the group. Through instruction and discussion, more elaborate models and strategies based on students' informal reasoning are introduced. As students use a model in a variety of situations containing the same concepts and mathematical structure, they begin to generalize the model across situations. As a result of exploration, reflection, and generalization, abstract formal mathematics develops through a process of progressive formalization from a context-specific model to a model for abstract reasoning.

The purposes of the longitudinal/cross-sectional study described in this paper are: (a) to determine the mathematical knowledge and understanding, attitudes, and levels of student performance as a consequence of studying MiC over 3 1/2 years and (b) to compare student knowledge and understanding, attitudes, and levels of performance between students using MiC and conventional curricula. The research model for this study is an adaptation of a structural model for monitoring changes in school mathematics (Romberg, 1987). The model was designed to capture the impact that the dynamic, complex interaction of social context for learning, curricular content and materials, classroom experiences, and pupil motivation has on student performance and further pursuit of mathematics. This model includes

14 variables in five categories: prior, independent, intervening, outcome, and consequent variables.

Various data collection instruments were designed for this study. District and school profiles, teacher and student questionnaires, and interview protocols for principals and teachers were designed to identify differences among students, teachers, and school contexts (prior and independent variables). Standardized test scores (provided by each district) and the Collis-Romberg Mathematical Problem Solving Profiles (1990) were also used (prior variables). Classroom observation scales, daily teacher logs, and protocols for teacher interviews were designed to characterize instruction and identify differences between reform and conventional classrooms (intervening variables). Three instruments were designed to measure outcome variables: a student attitude inventory; a problem-solving assessment system (examining three levels of mathematical thinking in relation to number, algebra, geometry, probability and statistics), and an external assessment system (providing comparison to both national and international tests of educational progress). (See summary on the external assessment system by Webb & Romberg in this volume.) An additional questionnaire and interview protocol will be designed to gather information about students' further pursuit of mathematics (consequent variable) as they begin Grade 9.

This structural model for monitoring change in mathematics provides the basis for gathering and interpreting information about the impact of reform-based mathematics curricula. Through the development of appropriate instruments designed to capture the fluctuation associated with each variable, useful information can be available for policymakers, school personnel, and researchers. In addition, the various instruments designed for data collection in this study can be used for the evaluation of other reform-based curricula, extending the ways in which researchers look at the complex interactions that affect student performance in mathematics.

Methodology

The longitudinal study features a multiple-cohort prospective panel design: Data are gathered at multiple distinct periods throughout the same periods on the same cohorts of individuals in relation to the same variables (Menard, 1991). Beginning in the 1997-1998 school year, data were gathered on three cohorts of students (one cohort beginning the study in Grade 5, one beginning in Grade 6, and one beginning in Grade 7). The study also features a nonequivalent control-group, quasi-experimental design: The assignment of subjects to the groups is nonrandom and pretests and posttests are administered to both groups. This design allows researchers to distinguish whether observed group differences on posttests were caused by a particular curriculum rather than preexisting group differences on some of the variables in the research model (Borg & Gall, 1989; Campbell & Stanley, 1963). The cross-sectional research (completed at the end of the first year) provides a basis for exploratory and preliminary investigations of the research questions using the structural model (Menard, 1991). Both quantitative and qualitative research methods are used in the analysis of data.

In order to attribute research findings to the reformed curriculum, the study includes a comparison of both reform and traditional classes. Seventy-five percent of the approximately 1700 students and 50 teachers use MiC while the remaining 25% use conventional curricula already available in the schools. These students and teachers are from 17 schools located in four research sites from both large urban and small suburban school districts.

Conclusions

First-year data from the longitudinal/cross-sectional study are currently being analyzed. The results will be reported for both reform and conventional groups. A summary of the research analysis and anticipated dissemination of first-year results is presented in Table 1. Although extensive analyses are currently in progress, preliminary results of data obtained from the observation scale, student attitude inventory, and student questionnaire are described in other research papers in this volume. (See summaries by Davis & Shafer on the observation scale, Wagner, Shafer, & Davis on the student attitude inventory, and Arauco & Shafer on the student questionnaire.) Preliminary analyses support that the use the dynamic structural model outlined in this paper is a viable means for studying the impact of reform-based curricula.

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Table 1
Research Analysis and Anticipated Dissemination of 1997-1998 Results

Variable in Research Model	Purpose Addressed	Data Collection Instrument
Prior: • Student Background • Teacher Background • Social Context	Identification of differences among students, teachers, and school contexts	District and school profiles Teacher and student questionnaires Interview protocols for principals and teachers District standardized test scores Collis-Romberg Mathematical Problem Solving Profiles
Independent: • Curricular Content and Materials • Support Environment • Teacher Knowledge • Teacher Professional Opportunities	Identification of differences among teachers and school contexts	Daily teaching logs Teacher questionnaires Interview protocols for principals and teachers
Intervening: • Pedagogical Decisions • Classroom Events • Pupil Pursuits	Characterization of instruction and differences between reform and conventional classrooms	Classroom observation scales Daily teacher logs Interview protocols for teachers
Outcome: • Knowledge and Understanding • Application • Attitudes	Examination of the mathematical knowledge and understanding, atti-tudes, and levels of student performance Comparison between students who have used reform and conventional curricula	Student Attitude Inventory Problem-Solving Assessment System (three levels of thinking in relation to four content strands) External Assessment System (comparison to both national and international tests of educational progress)
Consequent: • Further Pursuits	Identification of students' further pursuit of mathematics in Grade 9	Student questionnaire Interview protocol for students

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RESEARCH METHODS
POSTERS

DEVELOPING A MODEL FOR RESEARCHING CURRICULUM DEVELOPMENT

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Undergraduate mathematics curricula are often developed with little consideration to research how the materials achieve results. Some projects do field test and this feedback can be valuable, but time and distance constraints limit this model's usefulness (Gravemeijer, 1994).

We are developing a number/algebra class for preservice teachers. The course is grounded in research-based theory about mathematics teaching and learning (e.g., Krutetskii's [1976] model of mathematical abilities). However, we also research the use of materials.

Our research and development model (see Rachlin, 1989) involves the following players: university mathematics educators and mathematicians, graduate students, middle grades mathematics teachers, and students enrolled in the class. The model of instruction, problems posed, questions asked, and so on are informed by task-based interviews with students conducted before a class session. Classes are then conducted in a research classroom equipped with cameras to capture teacher and student work and record classroom discussion. This class is piped live to members of the development team who can then make decisions concerning modifications to be implemented in a parallel class the next day. Regular team meetings held directly after a class session provide time to reflect on what happened, particularly with regard to modifications in content or instruction. This tight cycle of development-research-modification has led to a number of assertions and questions of interest to the research and development team.

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A PLACE FOR STORYTELLING IN MATHEMATICS EDUCATION RESEARCH

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Storytelling was the major source of data collection for this research study designed to better understand and analyze elementary teachers' knowledge about mathematics teaching. This study was conducted at the four-year mark of a school district's participation in mathematics and science education project focused on alignment of teaching practices with the national mathematics and science standard. The data collection process was a modified version of an exercise called "Writing Your History" (or "Herstory"). The exercise has been successfully used for awareness of oneís self as mathematics teachers (as contrasted to viewing one's self exclusively as an elementary teacher). Teachers participating in the project took turns in telling their stories about teaching mathematics to each other. Storytelling has become more than a rhetorical device for expressing sentiments about teachers. Story is a mode of knowing that captures in a special way one's meaning in human affairs (Carter, 1993). Through storytelling teachers can tell about classroom experiences and practices; search out and construct reasons for outcomes of their interactions with learners; talk about their success and failures, and recognize the questions they have been unable to answer and able to answer. Storytelling is a vehicle for understanding, explaining and comparing classrooms cultures (Hansen & Kahnweiler, 1993).

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SOCIAL AND CULTURAL FACTORS
RESEACH REPORTS

CONNECTING STUDENTS' EVERYDAY MATHEMATICS AND SCHOOL MATHEMATICS

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This paper is based on a research project that has as a main goal the development of mathematics teaching innovations in primarily minority, working-class classrooms. In these innovations, units that promote students' mathematical learning while capitalizing on their knowledge and experiences from everyday life are developed. Three such units will be presented. In particular we address the mathematical potential in the units, issues related to the assessment of what students learned, and the role that values and beliefs about mathematics play in that assessment.

This paper is based on a research project that explores the creation of teaching innovations that promote students' development of mathematical knowledge while capitalizing on students' (and their families') knowledge and experiences from everyday life. The foundation for these innovations emerges from the mathematical and cultural resources found in the students' households and community, in particular, minority working-class families. The key research question underlying this paper is how to develop effective and meaningful teaching innovations that combine in-school and out-of-school mathematics. Three examples of theme units that have connected formal and informal mathematics (games, architecture, and garden) will be discussed in this paper. We use findings from students' questionnaires, interviews, and performances on mathematical tasks to explore the undergirding research question. Special attention will be given to the potential for development of students' mathematical knowledge through the implementation of these units.

Theoretical Framework

Our current work builds on a prior research project — The Funds of Knowledge for Teaching Project— grounded on a sociocultural approach to instruction (Moll, 1992). The premise in the Funds of Knowledge project is that qualitative studies of households of language minority children will uncover familiar and community knowledge bases that can serve as strategic resources for classroom practice.

Our claim is that by capitalizing on household and other community resources, we can organize classroom instruction that far exceeds in quality the rote-like instruction that these children commonly encounter in schools (Moll, Amanti, Neff, & González, 1992, p. 132).

Our previous work with teacher-researcher study groups has enabled us to lay the foundation for working with teachers as they develop the skills to research the households of their students for their mathematical potential. By drawing upon previous work, and by building upon congruent research in mathematics education, we hope to contribute further to this body of research, and to develop exciting innovations in the teaching of mathematics to language and other minority students.

The research in mathematics education includes work on the development of classroom communities where mathematics is socially constructed (Cobb, 1991; Lampert, 1990; Schoenfeld, 1991). A common thread in this research is the restructuring of classrooms as learning environments that try to model a community of practice in which students engage in working on mathematics tasks as mathematicians would. According to Rogoff (1994), the development of classroom communities should focus on students' learning through their participation in community-based activities and projects, because "learning and development occur as people participate in the sociocultural activities of their community" (p. 209). Van Oers (1996) stresses the importance of participation in socio-cultural activities that are both personally meaningful to the students and "recognized as 'real' by the mathematical community of our days" (p. 106).

Another body of literature that is particularly relevant to our work addresses the apparent gap between in-school and outside-school mathematics (Abreu, 1995; Frankenstein & Powell, 1997; Lave, 1988; Nunes, Schliemann, & Carraher, 1993). These researchers claim that the mathematics used in everyday life situations is both rich and culturally based. Furthermore, children bring with them an understanding of mathematics before entering school from their cultural and community-based experiences. The research, however, shows that a gap exists between the mathematics used in everyday life situations and the mathematics that is taught in school. Schools and students are failing to make the connections between mathematics learned in school and the mathematics that is used outside of school.

Method

The goal of the research project is to develop mathematics teaching innovations in which students and teachers engage in mathematically rich situations through the creation of learning activities that capitalize on students' (and their families') knowledge and experiences in their everyday life. To accomplish this goal, we rely on a model that has four key components: (a) analysis of students' interests, experiences, and prior mathematical knowledge; (b) development of a curriculum unit based on collected information; (c) classroom implementation of unit; and (d) reflection and evaluation about the mathematics embedded in the unit.

First, student questionnaires, informal interviews, household interviews, and brainstorming sessions with the students are used to gather information about the students' interests, hobbies, family labor history, and prior mathematical knowledge. Teachers conduct fieldwork in the households of their students, with a specific focus on the mathematical potential residing in the homes (e.g., in everyday activities such as construction, sewing, budgeting, and gardening). Teachers also elicit information from their students (via journal writing, conversations, etc.) in regard to their out-of-school practices, specifically looking for mathematics-related activities and instances of mathematizing.

Common themes emerge from the data collected through the above methods which serve to determine the mathematical unit. Of course, the potential for mathematical discovery and teachers' attitudes toward the unit also serve as factors in determining the unit. University researchers and teacher-researchers participate in biweekly study group meetings to discuss the findings from the household and student interviews and develop mathematically rich instructional units.

The third component is the execution of the mathematical unit. Classroom observations conducted both prior to and during implementation are used to document the teaching innovation. The focus of these observations is on mathematical discourse, patterns of interaction, and the kinds of tasks being posed.

The last component of the model involves the discussion of mathematics from the theme unit in relation to the school curriculum. Students' attitudes and mathematical understanding are assessed through a variety of means: interviews, journal writings, classroom observations, tests, and formal presentations of their projects to wider audiences.

Figure 1 summarizes the four components of the model for each of the three units discussed in this paper:

Results

The three learning units that are the focus of this paper range over three schools, (fourth / fifth graders in two of the schools; seventh / eighth graders in the third one), with three different teachers (having different mathematics backgrounds, different teaching experiences). However, the three units shared several points in common, in particular with respect to the students taking responsibility for and pride in their group projects; developing negotiating skills; persistence on the task. But there were also marked differences among the three projects in terms of their mathematical richness. There are a variety of reasons for this disparity, including school constraints and teachers' mathematical background. But the main reasons, in our view, have to do with our thinking and how it is continually evolving in the process. For example, the notion of "mathematical richness" has to do

Math Units	Information Gathered	Development of Unit	Unit Implementation	Reflection/ Evaluation
Games	Informal student questionnaire; Whole class web	Teacher-researcher discussions	Students analyze math rich games and create their own game	Small group discussions; informal interviews; presentation of games
Architecture	Student questionnaires; Household interviews	Teacher creates math contextual problems around construction	Students develop math problem solving skills to design their dream home	Team discussions; Written assessment; Project display; Reports; and Presentations
Garden	Student questionnaires; Brainstorming sessions; Household visits	Teacher involved in study groups with researchers; creates gardening unit	Students build their own potted garden	Portfolios (their data, observations, and journal writings); Student interviews; Written assessment; Dramatization

Figure 1

with our values and beliefs about what we consider mathematics to be. In the same way that students as they work on their projects have to negotiate many aspects of that project among themselves, we (where "we" may be the teacher and one or more university researchers, or the teacher and the students) have to negotiate the content and the pace of the mathematics learning activities.

Games

The curriculum unit on games represents some of the early stages of our process (see Civil, 1994, for a more detailed description). Our main goal was to explore the development of a mathematics classroom community in which students would engage in doing "mathematicians' mathematics" while building on students' knowledge, experiences, and interests in their everyday life. The topic of games appeared to be a natural way to accomplish this goal. Games give us access to the children's real world. Ainley (1990) points out that with games "there is a context for using some mathematics that you have learned, and that context is real for children because they can engage with it and the outcome matters to them" (p. 86). Bishop (1991) includes playing as one of the six mathematical activities present across cultures. He remarks that playing "stimulates the 'as if' feature of imagined and hypothetical behaviour" (p. 23).

Prior to implementing this unit the teacher and two researchers met to discuss games that would address some of the topics required in the curriculum (e.g., looking for patterns; writing in mathematics; introducing probability concepts). Thus, we started by having the children play and analyze mathematically rich games (e.g., strategy games such as Nim; probability based games) with a definite school mathematics agenda in mind. During this part of the process, we focused on whole class discussions of the mathematics in the games, and on trying to develop an atmosphere in which all students felt comfortable contributing.

Then, students worked in small groups to develop their own games. Their games were very informative of the students' interests (e.g., sports, airplanes, board games they were familiar with), but they did not seem to have much mathematical content. This raises the question of how our values and beliefs influence what we count as being mathematical. How can we convince someone (and ourselves) that in doing this kind of work students advanced in their learning? Maybe the question to ask is not "did they learn?", but "what is it that these children learned from their participation in the games module?" (Lave, 1994). There was ample evidence as they worked on their games of problem-solving (in an everyday sense of the word), cooperation, persistence, interest in their peers' games, critical reflection on those games (through many "what if" questions they raised during the presentations). Maybe these come close to being indices of learning in the "establishment of a community of practice—with a common communication system, norms, values" (Forman, 1996, p. 128).

Architecture

The architecture module was developed with middle school students by a teacher with a solid background in mathematics (including graduate level courses) and extensive teaching experience. This teacher emphasizes four key points in the development of this unit. First, through informal interviews with his students he realized the wealth of knowledge they had about building and construction. Then, he outlined the mathematical content embedded in these practices, such as 3-D visualizations, measurement, scaling factors, and cost estimation. A sequence of learning activities were devised that led them to be able to construct their dream house. The third point was the engagement of the students in the development of elaborate architectural models that reflected an array of mathematical concepts. The fourth point was his continuous assessment of a wide range of attitudes (classroom behavior, perceptions of their own abilities, and confidence), communication skills in mathematical contexts, and students' understanding of the mathematics.

The first noticeable change as the unit began was in the students' positive attitude towards working on and completing assignments. At the same time students' mathematical competencies improved to a degree that they were able to estimate construction costs, amount of materials needed, volume, capacity of heating and cooling (the diagnostic evaluation on basic skills given prior to the start of the unit had painted a very dismal picture, that was altered by the students' performance on their projects). Students gained knowledge on scaling factors and spatial visualization. Furthermore, they improved their ability to describe their project using mathematical language through their team discussions and presentations.

Container Garden

The third teaching innovation involved a garden curriculum unit. This teacher is a firm believer in developing a learning community in her classroom and she is experienced at doing this. The garden project is largely a result of the students' expressed interest through interviews and brainstorming sessions. In collaboration with other researchers in the project, the teacher integrated the required mathematics curriculum into the garden theme. This theme has provided not only a theoretical basis for the students to learn mathematical concepts such as measurement (area, volume, and perimeter) and graphing but also an opportunity for the class to create a school environment similar to a kinship or community consisting of students, teachers, and parents. The students began to use appropriate mathematical discourse and express their understanding of the mathematical concepts through their discussions, presentations, and journal writings. Through regular brainstorming sessions on the many aspects of the gardening project, the teacher facilitated students' reflection on what mathematics they thought they were learning in the process. Parents contributed not only as project supporters but as academic assistants, and from this perspective they are viewed as cultural resources. Through this process the students have gained insights and developed an appreciation for the knowledge that their parents possess.

Through this project, students were given real problems that required mathematics to reach solutions. The exploration of mathematics began out of a genuine need, and through these in-class learning experiences the students developed proficiency with mathematics concepts. For example, their need for more gardening space (due to the growth of the plants and limited wiring to enclose the plants) led them to the exploration of varying area and fixed perimeter. The students' understanding of skills in measuring area and perimeter were a direct result of needing to care for something that had relevance to them.

Conclusion

In summary, we are exploring a model to improve the mathematics education of students, in particular of minority, working-class students since for them mathematics is often the gate-keeper for success. This model draws on our theoretical orientation that views learning as an interactive process, and centers around mathematics, including a discussion of what mathematics is and what beliefs and values of mathematics are held by the project participants (teachers, students, parents, and researchers). Reciprocally, in the process of change, the teachers are more aware of what the families know and what mathematical knowledge the parents possess. They may then draw on this knowledge for classroom instruction.

Our work on the three units outlined earlier has shed some light on how to uncover students' "mathematical world" and how to build on it for school mathematics purposes. However, these theme units also underscore the key role that the teachers' backgrounds and expertise play in this process (whether it is a flexible understanding of mathematics, or an ability to develop a learning community in their classroom) as well as how much time and resources go into the development of these modules (e.g., in the garden theme, teacher's visits and calls to local gardening consulting centers are only one example of the many outside-school activities she engaged in for this project). Thus, an issue that we are currently examining is how to adapt the research project to the varying needs, interests, expertise, and circumstances of each individual teacher.

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A VYGOTSKIAN ACTION-RESEARCH MODEL FOR DEVELOPING AND ASSESSING CONCEPTUAL MODELS AND INSTRUCTIONAL MATERIALS INTER-ACTIVELY

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U.S. mathematics curricula have serious design limitations. This "underachieving curriculum" that is "a mile wide and an inch deep" dramatically underestimates what most children can learn. In prior work, we have described more ambitious classroom interventions for K-3 mathematics that build on individual experiences, interests, and the practical math knowledge of children. In this paper we describe not a particular curriculum, but a general model for the research process that we call "Classroom Conceptual Research." The central feature of this model is that it involves a tight interaction among model building, design work, and classroom-based action research, with a strongly "conceptual" emphasis. The work is carried out collaboratively by an interdisciplinary design team of university faculty, teachers, and research staff. We believe that the wider use of this model by the research community can lead to significant improvements in U.S. mathematics curricula.

U.S. mathematics curricula have serious design limitations that limit student learning. The grade placement of topics is delayed relative to other countries, and excessive spiraling (returning to topics every year) leads to too much reviewing at the cost of learning time on new topics. This "underachieving curriculum" that is "a mile wide and an inch deep" dramatically underestimates what most children can learn because curricular placement allows so little time for any one topic (Fuson, Stigler, & Bartsch, 1986; McKnight et al., 1989; Peak, 1996; Stigler, 1997). Deep development of student and teacher understanding of a given topic requires time. This time can be obtained by identifying core grade-level topics and concentrating deeply on these.

Significant theoretical and empirical research on mathematics learning and teaching is also severely limited by these curricular issues and by the fact that research and the development of instructional materials are often independent endeavors with relatively little coordination. NSF funding is separate, so projects must focus on research or on materials development. Traditional textbooks in the past rarely used much research on student's conceptual structures, developmental progressions of concepts or methods, types of word problems, or conceptual supports for learning (e.g., see the analyses in the chapters in Leinhardt, Putnam, & Hattrup, 1992). Ascertaining what kinds of learning are really possible, and identifying progressions of understandings and solution methods within a domain, require designing teaching-learning materials that will potentially support students through a more ambitious learning trajectory (Simon, 1995) of activities. Undertaking such a coordinated effort is complex but can be very productive. Understanding and naming this kind of work would seem to advance our current way of thinking about both research and development. Toward this end, we describe here a preliminary model in which the development of conceptual models and empirical action-research are tightly interwoven with the on-going design of teaching-learning activities.

Thus, the purpose of this paper is not to describe a particular curriculum, but instead to argue for a general model for the research process. We should note that we are not alone in advocating programs that integrate fundamental research, design, and enactment (e.g., Brown, 1992). However, we believe that articulating the particular features of our approach will facilitate dialogue about this class of approaches and thus begin to sharpen this paradigm. We use the term "Classroom Conceptual Research" to describe our model, to highlight what we broadly refer to as a "conceptual" emphasis in our classroom research. This emphasis is manifested in a focus on the following four kinds of issues: (a) discovering, enabling, and articulating learning trajectories of conceptual structures children use for certain kinds of problems, (b) creating conceptual learning supports of various kinds, (c) uncovering and creating models of affective, conceptual, cultural, and social aspects of classroom interactions, and (d) developing pedagogical models of ways in which teachers and peers can support children's constructions of concepts in a given mathematical domain. In what follows, we briefly overview the success of the Classroom Conceptual Research approach in prior work. We then describe the research model in more detail.

The Classroom Conceptual Research model was developed and used in a 6-year action-research project directed toward designing a conceptually complex and challenging K-3 math curriculum that builds on the individual experiences, interests, and practical math knowledge that diverse children bring to our classrooms. In order to ensure that our work generalizes across socioeconomic boundaries, our collaborative research project is carried out in urban schools of underrepresented minorities, schools in which most students are Latino English-speaking and Latino Spanish-speaking children, as well as in English-speaking upper-middle-class schools. We have higher grade-level expectations than in most present U.S. curricula, and have had success across the populations studied. In our formal assessments, we used a range of whole-class and interview items assessing single-digit and multi-digit numerical, word problem, and place-value competence (Fuson, Smith, & Lo Cicero, 1997; Fuson, 1996; Fuson, 1998).

The items were taken from other studies to provide comparison data. Highlights of outcomes include:

- Though over 90% of our urban children meet federal guidelines for the free-lunch program, they considerably outperformed heterogeneous and middle-class samples of U.S. children who received traditional mathematics instruction. On many items, they outperformed children from Taiwan, U.S. children using the reform curriculum Everyday Mathematics, and, on some tasks, equaled or exceeded the performance of Japanese children.
- On standardized tests, 90% of the first- and second-grade urban children were at grade-level on computation and 65% on word problem solving. Class means on standardized overall math scores were above grade level, some children were 3 years above grade level, and no child was more than one year below grade level.
- No child used a unitary strategy in multi-digit subtraction problems in contrast to children using the reform curriculum *Everyday Mathematics,* where 45% of average- and low-achieving children still used unitary methods for subtraction (Drueck, 1996) or did not have effective multi-digit addition or subtraction methods (Murphy, 1997).
- Results for suburban children were even stronger, though on many items the typical urban-suburban gap was decreased.

The activities of our project are similar to the developmental research process described by Gravemeijer (1994) that was used in developing the Realistic Mathematics instructional materials in the Netherlands. That effort began with a substantial theoretical base of the work of Freudenthal and participants in the Freudanthal Institute. It then contributed to more articulated and detailed theoretical perspectives in several areas as well as to the initial and the newer instructional materials. These instructional materials have had remarkable commercial success in the Netherlands, holding a considerable amount of the market. The long period of development, and the intertwining of the theoretical model-building and design of instructional materials, produced a coherent product whose pedagogy, domain analyses, and developmental progressions in children's thinking could be described for and used by teachers and by other researchers. In describing our research model we have not borrowed the Dutch term "developmental research" because, in this country, that term implies "research about children's development."

Our Vygotskiian Model of Classroom Conceptual Research

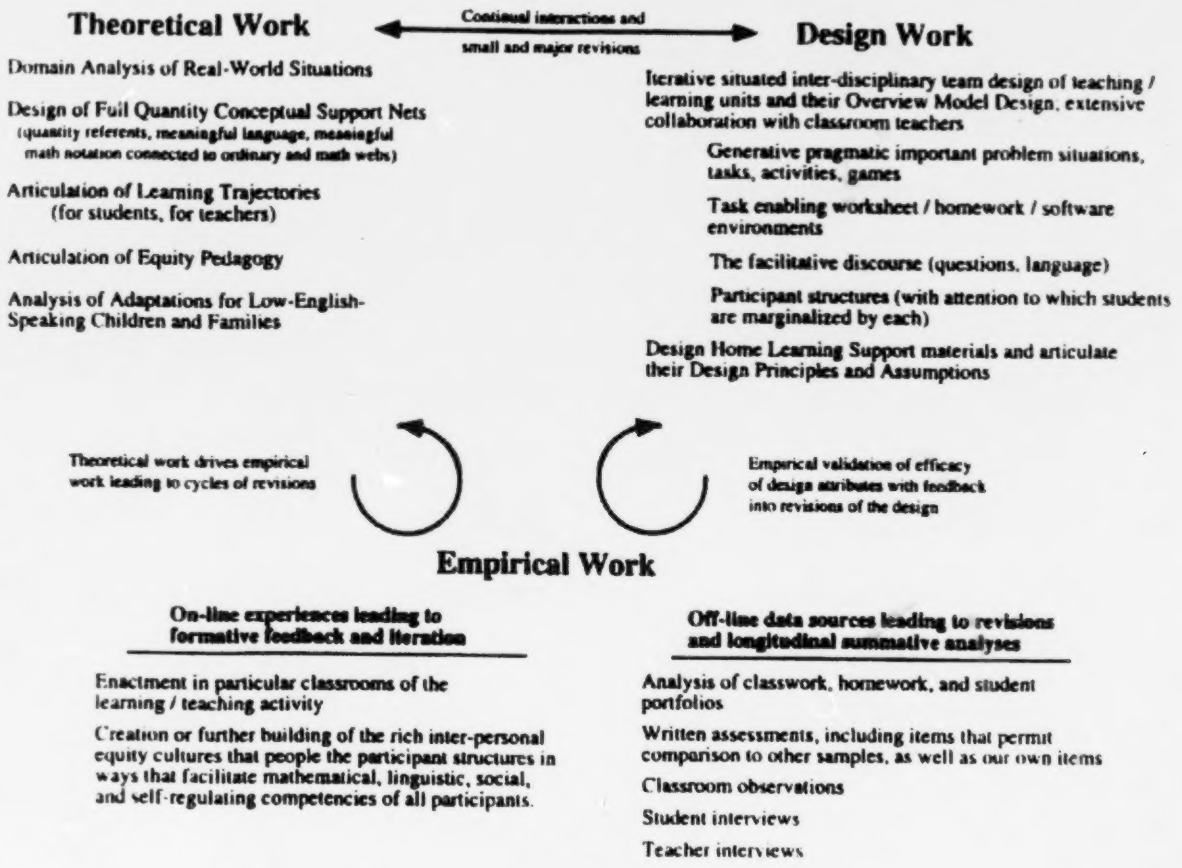
Our Vygotskiian Model of Classroom Conceptual Research is shown as Figure 1. Although our purpose here is to present a generally applicable model, we will mix high-level description with details of our own implementation in order to provide a more grounded account, and to give a feel for the scope and depth of our approach. The model shown in Figure 1 provides an overview of how we concurrently integrate the design of instructional materials, enactment in the classroom (empirical action research), and the development of conceptual models. Our project efforts in these areas are interwoven in continuing cycles of mutually adapting reflection and revision of the model building, instructional materials design, and classroom implementation work.

In the first of these three areas, our development of models focuses on four major activities. First, we undertake domain analyses of real-world situations that can help children build meanings for and uses of mathematical concepts. Second, we create Full Quantity Conceptual Support Nets of learning supports for particular concepts (Fuson & Smith, 1997). These use physical quantity referents (e.g., penny strips of ten pennies on one side and one dime on the other), drawn quantity referents (e.g., ten-sticks and one dots), meaningful language (e.g., 3/4 said as "out of 4 parts, take 3"), and meaningful math notation connected to ordinary (5 dimes, 3 pennies) and to math meanings (5 tens 3 ones). Third, we articulate our pedagogical approach in a model of an Equity Pedagogy (Fuson, De La Cruz, et al., 1997). This model, which builds on prior Vygotskiian work (Fuson, Lo Cicero, et al., 1997), outlines ladders of support to help students build on their initial personal meanings and experiences to create advanced and ambitious mathematical concepts, notations, and methods. Finally, the fourth model-building activity is to specify Learning Trajectories for students and for teachers that describe developmental progressions through which learners advance. Our emphasis in these models on inter-psychological phenomena (Equity Pedagogy) and semiotic tools (Full Quantity Conceptual Support Nets, Domain Analyses of Real-World Situations) reflects our Vygotskiian perspective.

Moving to the second part of our three-part model, the instructional materials design work focuses primarily on four aspects of classroom learning, as well as on the design of home learning-support materials. We identify problem situations that both occur frequently in the real world and are mathematically clear and generative (e.g., using money calculations with dimes and pennies to extend understanding of place-value concepts). We design worksheet-based activities that facilitate children's approach to a learning activity. We consider features of classroom discourse (questions, language) that will support understanding and clear communication in a co-constructing environment. We create participant structures with attention to which students might be marginalized by each structure. All of these features are subject to modification in action in the classroom.

The empirical action-research work in the classroom begins from the initial instructional materials, with enactment in the classroom informed by the theoretical models. These models suggest adaptations to unfolding student thinking that extend and modify the initial teaching plan. More generally, the mutual adaptations that occur among enactment, the developing models, and the design of instructional materials operate at multiple

Figure 1: Our Vygotskiian Emergent Conceptual / Pedagogical / Empirical Action Research Model



Theoretical Work

Domain Analysis of Real-World Situations

Design of Full Quantity Conceptual Support Nets (quantity referents, meaningful language, meaningful math notation connected to ordinary and math webs)

Articulation of Learning Trajectories (for students, for teachers)

Articulation of Equity Pedagogy

Analysis of Adaptations for Low-English-Speaking Children and Families

Continual interactions and small and major revisions

Design Work

Iterative situated inter-disciplinary team design of teaching / learning units and their Overview Model Design, extensive collaboration with classroom teachers

Generative pragmatic important problem situations, tasks, activities, games

Task enabling worksheet / homework / software environments

The facilitative discourse (questions, language)

Participant structures (with attention to which students are marginalized by each)

Design Home Learning Support materials and articulate their Design Principles and Assumptions

Theoretical work drives empirical work leading to cycles of revisions
Empirical validation of efficacy of design attributes with feedback into revisions of the design

Empirical Work

On-line experiences leading to formative feedback and iteration

Enactment in particular classrooms of the learning / teaching activity

Creation or further building of the rich inter-personal equity cultures that people the participant structures in ways that facilitate mathematical, linguistic, social, and self-regulating competencies of all participants.

Off-line data sources leading to revisions and longitudinal summative analyses

Analysis of classwork, homework, and student portfolios

Written assessments, including items that permit comparison to other samples, as well as our own items

Classroom observations

Student interviews

Teacher interviews

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time-scales: repeatedly during the design work, several times while teaching a lesson, daily in revising tomorrow's lesson, several times yearly as new teachers try the newly designed unit, and over years as full conceptual support models and full developmental learning trajectories of student thinking are developed and adapted. These many different kinds of feedback loops, and the sustained prolonged efforts, facilitate the development of coherent and powerful theoretical models and teaching-learning units and curricula based on these models.

It is worth emphasizing that, at its highest level, enacting this research model can be seen largely as a project in orchestrating a complex process of collaboration. What we propose is a broad social design that includes not only what happens in the classroom, but also a set of interactions that includes the design team, the school, and the classroom. Our interdisciplinary team includes people with strengths in teaching, mathematics education, developmental psychology, and linguistics. Individuals lead design efforts in a particular area and grade level, with repeated consultation from two to five other people. A unit is sometimes taught by a staff teacher-researcher, often in active collaboration with a classroom teacher. One or more team members may be present at any of the teaching efforts to gather empirical data on classroom activity. In addition, interview data are gathered from children and teachers, frequently during initial development and summatively for more final versions of units.

Particular team members also assume intellectual and management leadership roles in articulating and directing the theoretical model-building and writing. These then are adapted to the thinking of team members in successive reflective discussion cycles. This collaborative research method stimulates a continuing flow of good ideas while enacting our units in the classroom (a productive interaction of teacher, researchers, and students), during project meetings, and in individual work (through voices and perspectives of our fellow collaborators).

Conclusion

In conclusion, we have described a model for research that we call "Classroom Conceptual Research." We have introduced and named this model in order to initiate a dialogue in the field about new research methods focused on conceptual teaching and learning. We believe that the adoption of this model can enhance the quality of research-based curricular reform efforts, as well as the usefulness of the research on which it is based.

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RELATING EQUITY TO CLASSROOMS WHICH PROMOTE UNDERSTANDING: IDENTIFYING RELEVANT ISSUES

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In this paper, I describe issues of diversity which relate to mathematics classrooms which promote understanding. These issues are described in the context of a seventh-grade classroom teaching experiment and grounded in relevant literature. The purpose of the seventh-grade classroom teaching experiment, from the perspective of diversity, was to identify salient issues to be investigated further in a follow-up classroom teaching experiment to be conducted fall of 1998. The paper indicates a direction for future studies examining diversity issues in mathematics classrooms which promote understanding.

Clearly, students come to school with a variety of backgrounds and prior experiences. Many complex issues are involved when considering students of various backgrounds coming together in any educational setting. I use the term diversity to refer to these broad issues, and here in this paper, I focus on one facet of issues within the broad area of diversity. In writing this paper, my purpose is to highlight issues of equity as they relate to mathematics classrooms which promote understanding. Within diversity issues, conversations centered on equity usually involve the fairness in how opportunities are presented to students, which may involve funding and policy considerations and may include access to learning opportunities in the classroom. Mathematics classrooms which promote understanding may also be described as inquiry-based classrooms or classrooms consistent with reform recommendations (*NCTM Professional Standards for Teaching*, 1991; *NCTM Curriculum and Evaluation Standards*, 1989). More specifically, my goal in this paper is to examine ways of conceptualizing issues of equity as they "play out" in these mathematics classrooms which promote understanding. To accomplish this, I refer both to relevant literature and to events from a classroom teaching experiment.

As a member of a research team, I conducted a review of the literature related to diversity in order to interpret events in a classroom teaching experiment conducted during the fall semester of 1997. The literature review guided my interpretations of the project classroom in light of diversity. In this way, the seventh-grade classroom teaching experiment served as a pilot study in which the purpose was to identify relevant issues which would warrant further, detailed study during the eighth-grade classroom teaching experiment to be conducted during the fall semester of 1998.

Theoretical Framework

My conceptualization of participation and learning ultimately guides how I make sense of the events of the classroom. The perspective I employ includes a coordination of the social and psychological processes of learning which are consistent with the theoretical framework discussed by Cobb and Yackel (1996). This framework views the social and psychological aspects of the classroom as inseparable and reflexive. From this perspective, individual students' thinking is both constrained and enabled by class interactions and similarly, whole-class discussions are constrained and enabled by the psychological learning process. Moreover, learning is described in terms of students' participation in mathematical practices established by the classroom community.

I use the elaborated framework of Cobb and Yackel (1996) as a means of conceptualizing events of the classroom as they are situated within the larger context of the school, the local community, and the wider society. The framework emphasizes the practices of institutions and the wider society, and places individuals' activity within a context. The relationships among the classroom, school, and society may be viewed as reflexively-related. Cast in these terms, individuals are seen as bringing their ways of participating in these different communities into the emerging classroom microculture. In this sense, classroom events are not isolated, but can possibly be accounted for by examining students' out-of-classroom lives.

In sum, the theoretical framework (a) describes learning in terms of students' participation in mathematical practices established by the classroom community, and (b) situates the events of the classroom in a broader context. From this perspective, student comments during whole-class discussions may be viewed as aspects of a negotiation which contribute to the emergence of mathematical practices, or in other words, student learning. In addition, the framework points to situations outside of the classroom when attempting to account for events observed in the classroom.

Data and Methodology

Data were collected during the fall semester of 1997 in a seventh-grade classroom and consisted of dailyvideo-recordings for a period of twelve weeks. The purpose of the classroom teaching experiment was to design and enact an instructional sequence that supported students' development of increasingly sophisticated statistical reasoning. Field notes were also taken to record observations of specific areas of interest such as diversity and argumentation. In addition, data included observations from shadowing two students from the mathematics class throughout the school day. The overall purpose of the shadowing was to understand aspects of the school culture from the students' perspective as well as students' diverse ways of participating in school practices. The shadowing situated the classroom within the school context so that the events in the classroom could be understood from a broader perspective. Data also consisted of video-recordings of student interviews conducted following the classroom teaching experiment. Interview questions addressed several themes including the school culture, the classroom culture, and students' perceptions of mathematics.

Issues which Emerged

It is important to reiterate that the purpose of the seventh-grade classroom teaching experiment, from a diversity perspective, was to identify the possible issues that would warrant further investigation in the eighth-grade classroom teaching experiment. At this point, I would like to highlight three of the relevant issues which emerged.

During the classroom teaching experiment, there existed an irreducible tension between the mathematical agenda of the research team and the contributions of the students. More specifically, the tension related to the teacher's mathematical agenda and including and building on particular students' comments. The example that follows involves discussions concerning the data collection process. Cobb (1998) describes the research team's intent of having students talk through the data creation process so that students might come to view the data as having a history and as a result of a series of arguments and decisions. During the initial weeks of the class when discussing how the data were collected, many students seemed to make extraneous comments which did not relate to the data collection process. These comments often involved personal narratives or opinions about the topics at hand. These extraneous comments made by students were not included and built upon as part of the whole-class discussions. However, towards the end of the twelve weeks, most of the student contributions seemed to clarify the data collection process. In other words, it was the researchers' mathematical agenda to have the students talk through and clarify aspects of the data collection process, but as a result, not all student contributions were included as topics of discussion. From this perspective, equity issues of silencing and marginalizing emerged.

The term silencing may be used as a broad term to describe individuals or groups of people being marginalized through educational practices and policies. However, when interpreting events of the classroom, a more specific definition is helpful. Fine (1987) writes of silencing at the school and classroom level. Her use of the term silencing refers to students having no voice, their opinions not heard or issues being silenced and not discussed. She describes one type of silencing in which the topics which really matter to the students do not become topics of conversation in the classroom. Her descriptions are consistent with the discussions involving the data collection process. The example of possible silencing from the statistics class builds on the work of Fine in that it describes a situation of possible silencing that is specific to mathematics content. Though the mathematical agenda was for the students to talk through the data collection process, from an equity standpoint, not including or building on the students' comments might have been interpreted by the students as silencing. I acknowledge that this tension between the mathematical agenda and students' contributions was an inherent part of our project classroom and is an inherent part of classrooms which promote understanding, but it is important for researchers and teachers to be aware of how these situations may be interpreted by students.

A second issue is that of how student contributions in the classroom were dealt with by other students. From the classroom teaching experiment, it was apparent that students responded to individual student comments differently. Possible conjectures to account for these different responses included differences in students' narrative styles and the students' membership in different groups constituted within the school. The differences in narrative style might explain students' different responses to each other. In her study, Cazden (1981) writes of children having narrative styles which differ in organization and logic, making some narrative styles more easy for some individuals to understand than others. This might have been the case in the statistics classroom teaching experiment. In this sense, perhaps students in the classroom sharing particular narrative styles could more easily understand one another. The different student responses might also be accounted for by how students viewed themselves and others as members of particular groups constituted within the school. The idea of a student's membership in a particular group relating to how that particular student participates in school is reflected in the studies of Ogbu (1988), Matute-Bianchi (1986), and Fordham and Ogbu (1986). Though these studies deal specifically with how membership in groups may relate to students' views of academic achievement, they point to the possible relationship between student membership in groups constituted within the school and student participation in the mathematics classroom.

The third issue is related to the approach used in the classroom teaching experiment to induct the students into a particular type of statistical argumentation (Cobb, 1998). From this perspective, the students might be viewed as learning what Delpit (1988) terms the "culture of power." The students' induction into this type of argumentation may be viewed as assisting students in becoming participants in public policy debates which are often presented in terms of data-based arguments (Cobb, 1998). Since the claim is that the students are being taught this type of statistical argumentation, it would be important to examine, from their perspective, if they are indeed learning this particular way of participating. As a result, a key issue which emerged from the classroom teaching experiment was how the students perceived themselves as students in the statistics class and how they perceived themselves as students in their regular mathematics class.

In interviews conducted during the spring semester following the classroom teaching experiment, students were interviewed regarding their perceptions of the project class and their regular mathematics class. Though the purpose of the interviews was not to understand their specific perceptions of themselves as mathematics students in these different classes, the student comments did indicate that the students viewed aspects of the classroom microculture as being very different. The following comments of two students are in response to a question which asked the students to compare their individual participation in the project class and in their regular mathematics class.

> S1: Like now she [current math teacher] has on the board we're going to have a class discussion over this and you have like a ten minute discussion and then you just work and you don't get to talk to other people in your group about it, but in [the project class] you got to discuss with the whole class, you got to discuss within your group on the computer and at your desk. You got like three different discussions for every problem. And it's not one discussion which often gets interrupted by people talking to their friends than learning about what we're actually learning.

> S2: [Referring to the current mathematics class] I don't think the ones that we have now are really class discussions because if you don't understand something and you ask, sometimes other people, the people who understand might be like "you don't understand this" and they'll start screaming at you. And sometimes it's stuff that we've learned already, but she puts it on the board and we'll have to, have to sit there and write it down. And a lot of times people are just trying to write it down as fast as they can so they'll have it. And they're not really paying attention to what they're writing. It's not a class discussion any more, just her talking...

These comments were significant in that they presented the students' different perceptions regarding the two mathematics classes in which they participated. Furthermore, the comments indicated that the students viewed the two classroom microcultures as being different in terms of the opportunities for class discussions and the nature of the class discussions which occurred in each class. Since the students viewed the two classes as being different, its seems possible that they may have viewed themselves as students differently in the two contexts. Clearly, further investigation involving discussions with students will be necessary in understanding the students' emerging perceptions of themselves in the project class during the eighth-grade classroom teaching experiment. This focus will have to be introduced as specific topics of conversations with the students during the follow-up project.

Conclusion

All three of the issues described relate to the theme of students' emerging perceptions of themselves as mathematics students in the project classroom. From the first two issues, silencing may occur according to how student contributions are dealt with by both teacher and students. A student's view of himself/herself as being silenced relates to how the student perceives himself/herself as a mathematics student in the class. In addition, the third issue focuses directly on student perceptions, but relates these perceptions to the students' learning a type of statistical argumentation. This paper highlights the need for further investigation of these three issues in addition to other issues which examine mathematics classrooms which promote understanding from the perspective of equity.

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THE POWER OF ONE IN A MATHEMATICS CLASSROOM: LISA AND DIVISIBILITY BY FOUR

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The impact of student actions on the mathematics in a classroom is documented in this paper. A classroom episode on rules for divisibility is analyzed in detail to reveal the impact of one student's actions on the direction of the mathematical discussion of the class. The teacher's role in capitalizing on student conjecture is acknowledged, but the importance of the student's role is emphasized. In this episode, a student conjecture leads the discussion away from a teacher established routine of state the rule, verify it on a few known examples, and write the rule on the board. Instead, the class is drawn into a search for a counterexample and the need to extend the book's definition of *multiple*. The discussion resulting from the conjecture includes justification and pattern examination. In this episode, the student's role is crucial to the departure from the routine. Careful examination of student instigated mathematical exploration is critical to a more complete picture of the complexity of mathematics classroom life.

The role of the teacher in effecting mathematics education reform is universally recognized as important. The significance of the roles of students in mathematics education reform, however, is not as agreed upon. Yet, students' beliefs about their roles, about the teacher's role, and about the nature of mathematics are important when examining classroom interactions (Balacheff, 1990). How those beliefs are translated into actions in the classroom is particularly critical and not well documented. This paper examines the impact of one student's beliefs and actions on the content and organization of one episode in an eighth grade beginning algebra class.

Theoretical Framework

This paper adopts the theoretical framework of *philosophical world views* as discussed by Pepper (1982, 1942). Pepper describes four basic world views: 1) formism, 2) mechanism, 3) contextualism, and 4) organicism. He hypothesizes that each world view has a root metaphor that is used to explain the workings of the world. For example, mechanism has a root metaphor of a machine. This particular world view has dominated much of Western scientific thought. We have used the machine metaphor to describe the movement of the stars, the workings of the mind, and even basic mathematical concepts such as function.

Within the framework of world views, an individual's way of experiencing the world is revealed in her way of talking about the world. The metaphors or descriptions used can reveal the basic mode in which the person experiences the world. Furthermore, the actions of the individual can also be compared to a world view in order to reveal patterns within those actions.

Modes of Inquiry

Qualitative methodologies were used in this study because of the nature of the questions investigated (Eisenhart, 1988). In particular, data collection and analysis were guided by traditions of ethnography of communication and holistic ethnography. Ethnographers of communication are interested in patterns of social interaction within a particular group (Erickson & Mohatt, 1982; Heath, 1983; Mehan, 1979), while holistic ethnographers study the entire culture of a group (Jacobs, 1987). It is my contention that a classroom represents a bounded group that creates a unique way of living in that classroom. How that way of life is created depends on how the individual parts—teacher, students, and curriculum—interact and fit together. One goal of the larger study was to show that the beliefs and attitudes of the students in this classroom fundamentally influenced the way life in the classroom was constructed. Using these two modes of inquiry, I studied students' actions in the classroom in detail through analysis of videotapes of individual work and group work. I also analyzed videotapes of student interviews. These detailed analyses revealed some principles of organization of behavior and actions of individual students.

Data Sources

Because this study considered both the interactions in the classroom and the beliefs of the individuals who made up the class, a diversity of methods were used to ensure a large body of data from which to work. This study was conducted over the course of one school year in an eighth-grade beginning algebra class in a middle school in the Northwest. The study class consisted of 30 students, 18 boys and 12 girls. The students showed a broad range of ability, prior knowledge, and interest in mathematics. Mr. Scott, who taught all of the eighth-grade mathematics, had taught for three years. He was enthusiastic and interested in helping his students to learn mathematics. He had an obvious love of mathematics which he tried to share with his classes. Prior to the beginning of data collection, informed consent was obtained from the teacher, each student, and the parents of each student. One student chose not to participate and was not included.

Three primary methods of data collection were used to create a large, varied body of information: participant observations including video and audio taping, key informant interviews, and document collection (Bogden & Biklen, 1992). Participant observation was used initially to help focus the study and to aid in identifying key informants, and later to help refine and test the analysis. Daily observations were made from the middle of September through the end of October, and again from the beginning of

February through the middle of March. During the rest of the school year, observations were made two or three times each week. All observations were videotaped and audiotaped to allow for detailed analysis of classroom interactions, and field notes were recorded during the observations. Ten students were identified as key informants through purposeful sampling. These five boys and five girls included both quiet and assertive students, and students of apparently high and low levels of ability and interest in mathematics. The key informants were the focus of whole-class and small-group observations and were interviewed individually three times during the course of the study. The first interview concentrated on the student's beliefs and attitudes about mathematics, mathematics classes, and school in general. A second interview of each student focused on the student solving several problems aloud. The third set of interviews was conducted at the end of the school year to gain the students' reflective perspectives on the entire year. All interviews were audiotaped and transcribed for further analysis. The teacher was formally interviewed at the beginning of the school year to ascertain his expectations and goals for the class. Additionally, informal conversations with the teacher were a continual part of the study. Reports of these conversations were included in the field notes generated during observations. During the summer after the school year ended, the teacher and the researchers met to review and discuss some of the videotapes and our preliminary analysis. This type of member-checking continued throughout the next school year on an irregular basis, as schedules permitted. All of these meetings were audiotaped and formed part of the corpus of data. The teacher's perspective was particularly useful in determining the extent to which individual students influenced the instruction in class. Examples of student work, classroom rules, handouts, tests, and other assignments were also collected and analyzed.

This paper will focus on only one episode from this classroom and will concentrate on one student's impact on the mathematics of that episode. For a more complete discussion of the students in this classroom and their actions, see Ivey (1994).

The Classroom Episode: The Rule for Divisibility by Four

This paper examines in detail the actions of one student during a class on divisibility rules which occurred on September 30. On that day, Mr. Scott reviewed rules for determining divisibility by two, three, four, five, nine and ten. Overall, this review lasted 18 minutes and 23 seconds, with 8 minutes and 20 seconds devoted to the rule for divisibility by four. Later, Mr. Scott, while viewing the videotape of this lesson, attributed the disproportionately large amount of time spent on the rule for four to Lisa's actions. The lesson had proceeded in the following routine. Several multiples of the number under consideration would be listed on the board. Mr. Scott or one of the students would recall the rule for divisibility by that number.

Mr. Scott would apply the rule to the examples listed, verify that the rule worked in each case, and then list the rule in a chart on one end of the board.

After this routine was followed for the rule for divisibility by four, a student whom I will call Lisa conjectured a different rule based on the examples shown on the board. The following numbers were listed on the board as known multiples of four: 4, 8, 12, 16, 2016, 292, 2808, 652.

Lisa: Only on some of them, it's not an all of them, if you take like, you take the first number and the third number, like on 2016, that'd be eight and it's divisible by eight, and if you took the second one, two plus. . ., yeah, two times two is four, and on the third one, two times eight is sixteen, on the fourth one six times two is twelve, and if you take the first number and the last number and you multiply them together.

Mr. Scott: Oh, OK. The first number and the last number, you're multiplying together. OK? That's very interesting, it seems like Lisa has.

Lisa: But it doesn't work on all of them. It doesn't work on twelve and sixteen.

Mr. Scott: OK, it doesn't work on twelve and sixteen. Maybe it just works for numbers that are larger than two digit numbers. Let's see if we can come up with a three digit number or a four digit number that that does not work on. Kevin, go ahead and multiply four by 82. Let's just see what happens. Maybe it works and maybe it doesn't.

Mr. Scott's response to Lisa's conjecture was quite different from the established routine. He did not know whether her rule was valid; so he was not satisfied by verifying that her rule worked in the cases shown. In effect, he searched for a counterexample. Lisa's conjecture had moved the class into an exploratory mode. Mr. Scott had a student produce additional multiples of four using a calculator; each new multiple was tested using Lisa's rule. During this exploration for a counterexample, a different issue arose, which created a need for mathematical justification. Following Lisa's rule, the class tested the number 320 and obtained a value of zero which, if it were a multiple of four, would indicate that the number was also. Both Mr. Scott and the students had to extend the textbook definitions of *multiple* and *divisible* to a case previously unconsidered.

Mr. Scott: 320. Well, that's kind of an interesting one. Three times zero is zero. Is zero a multiple of four?

Students: Yes. No. (5 second pause)

Mr. Scott: Why is it? Who can argue that it is? Why is it? Lisa?

Lisa: Because zero times any number is zero.

Mr. Scott: OK, because it's four times zero is zero, so you could say that zero is the zeroith multiple of four. We said the first multiple of four is four. So the zeroith one would be zero. OK, so that could work.

To extend this meaning, Mr. Scott asked for justification, which Lisa supplied. The form of her reasoning was based on the definition as it had been given in class earlier. Mr. Scott even coined a new term, *zeroith multiple*, to further place this extension into the language used to describe other multiples. The class finally found a counterexample when they tested 316 using Lisa's rule. (3 x 6 = 18 which is not divisible by four, but 316 = 79 x 4 and is a multiple of four.)

Lisa's Role in the Interaction

Lisa's actions in this lesson are consistent with her world view—contextualism. Through analysis of her interviews, Lisa's contextualism is clear. She believes that you learn mathematics in the world, and that in school, you learn the names for what you do. She further believes that it is important to challenge others and to examine examples for patterns. Generally, when Lisa answered a question about what students or teachers are supposed to do in the classroom, she answered as if she were quoting someone within the situation. Many of her answers were given as hypothetical monologues or even dialogues.

The lesson on divisibility was memorable to her. About seven and one-half months after this lesson took place, I asked Lisa about it. Not only did she remember the lesson, she very nearly reconstructed her rule from memory. She also said that she had made the conjecture because "he had asked us if anybody else knew any more rules." The videotape of that lesson does not confirm her memory of this request. The only comment which I could construe as the source of this part of Lisa's reconstruction of the event occurred about eight minutes before she offered her rule. Before having someone state the rule for three, Mr. Scott said "I don't want the answer yet. I just want some examples. Sometimes we can see a pattern from examples." Thus, in what was primarily a mechanistic lesson—rules were stated with no justification, and examples were given of their uses—Lisa was able to act in a contextual way.

Clearly, Mr. Scott was responsible for pursuing the investigation of Lisa's conjecture. In that sense, it was his actions that led to the richer classroom discussion. However, without Lisa's willingness to make a conjecture, the class would have continued in the pattern that Mr. Scott established early in the lesson. Without her search for connections between the examples listed on the board the class would not have explored for a counterexample, nor would the need to extend the book's definition of multiple have arisen. The class also reacted to Lisa's rule by raising their heads and appearing to follow the action more closely than they had been. The justification, reasoning, and interest that this episode reveals are directly related to Lisa's conjecture. In this way, the power of one student to change the direction of the class is evident.

This particular classroom episode shows the strong effect that a student can have on the education process. In particular, Lisa's beliefs about mathematics and about her role as a student were translated into actions that changed the type of mathematics presented in the class, at least for a few minutes. Her willingness to conjecture a new rule based on examples and to extend definitions when the need arose moved the class, with the teacher's acquiescence, into a different mode of instruction. Instead of recall and verify, the class explored, extended, and justified on their way to disproving the new rule.

Implications

One way to improve our understanding of the teaching and learning process is to concentrate on individual as well as whole class interactions. This paper seeks to deepen our understanding of the role of student beliefs in the teaching and learning of mathematics through examination of the effect of one student's mathematical thinking. Within the latitude allowed by the teacher, one student can have a positive impact on the type of mathematics in the classroom. Similarly, students can thwart the best efforts of a teacher to create an exploratory environment. While there is no doubt that the teacher has a primary responsibility for creating the atmosphere of the classroom and for the type of mathematics presented, this episode documents the power of one student to also dramatically affect the type of mathematical activity undertaken. The scope of the study that underlies this analysis lends credibility to the story of Lisa and her impact on the classroom. Other stories of students and their impacts, positive and negative, could provide important detail in our efforts to capture the complexity of classroom life and the mathematics encountered in them.

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STUDENTS' USE OF FUNCTION IDEAS IN EVERYDAY ACTIVITIES

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The study of students' out-of-school mathematics practice has been a research arena that holds promise for insight as to how to help learners build connections between mathematics and settings that are relevant to middle school students. There is also a substantial body of research on the conception about functions that are held by older students. In this paper, we seek to bring these two research strands together by analyzing and identifying the function ideas which are expressed by middle school students in their out-of-school mathematics practice. We have found evidence that students use rich language to describe their ideas of co-variation as two related quantities change and their ideas about direct dependence (or causality) as one quantity is determined by another.

Research on cognition and learning has pointed out the need for closing the gap between learning and doing mathematics in and out of school (e.g., Saxe, 1991). Furthermore, recent proposals for teaching and learning mathematics in school have encouraged educators to connect mathematics with other subjects and out-of-school mathematics practice. The following challenge is laid out in the *Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics (NCTM), 1989):

> *Problem situations that establish the need for new ideas and motivate students should serve as the context for mathematics in grades 5-8. Although a specific idea might be forgotten, the context in which it is learned can be remembered and the idea re-created. In developing the problem situations, teachers should emphasize the application of mathematics to real-world problems as well as to other settings relevant to middle school students. (p. 66)*

However, in order for teachers to help students make these connections we need to know much more about how students use mathematics in out-of-school settings. This is relatively unexplored territory, as Pea (1991) noted: "Even though that field [mathematics education] calls for relevance of mathematics learned to everyday settings, there has been remarkably little ethnographic investigation of mathematical activities by children in settings outside classrooms" (p. 490). This study is part of a larger research project, entitled *Connecting In-school and Out-of-school Mathemat-*

ics Practice (a National Science Foundation-sponsored grant, RED-9550147), whose primary purpose is to examine middle school students' everyday activities and see how they make sense of things mathematically. In this particular study, we examine how students' ideas about functions and variables arose in the context of their out-of-school activities.

Perspectives and Guiding Frameworks

Students' conceptions about functions have been (and continue to be) studied by a large number of researchers in a variety of secondary and post-secondary settings (c.f. Leinhardt, Zaslavsky, & Stein, 1990; Romberg, Fennema, & Carpenter, 1993). Much is known about student conceptions and misconceptions from this work. The perspectives guiding this body of research include the representational view which suggests that students' understanding develops through a coordination of multiple, often linked and dynamic representations. Others are guided by an epistemological view that suggests students' understanding proceeds from a process view of functions to a deeper view of functions as mathematical objects in their own rights (Sfard & Linchevsky, 1994). Much of this research would suggest that the function concept is appropriate for the mathematical thinking of older students. Very little research has examined function thinking among younger students at the middle school level. In this paper, we hope to contribute to filling that gap. In our analysis, we will be guided by a view of functions as arising out of a need to interpret and explain meaningful situations and where both direct, causal, dependency relationships are posited or exist and where co-variational ideas are used in the students' interpretation of the context.

Methods and Data Sources

In June 1995, we visited a middle school teacher with whom we worked later in the project and discussed the project and asked for volunteers. Six students (five females—Kristen, Robin, Amy, Jessica, and Nicole; one male—Linus) returned the permission forms and volunteered for the project. We met with all the students and their parents before data collection began to explain the purposes and procedures of the project. Throughout the summer and fall of 1995 we collected data on their mathematics practice out of school through (a) activity sampling with electronic pagers and logs, (b) observations of each student in a number of out-of-school activities, (c) interviews with students about logs and observations, (d) logs kept by students and parents, and (e) interviews with students and parents about logs and their activity.

We met with the respondents prior to the activity sampling and explained the procedures. The students carried electronic pagers for one week and completed a sampling form whenever they were signaled. This method has been used by other researchers with success (e.g., Schiefele & Csikszentmihalyi, 1995). Each respondent received seven to nine signals per day. The sampling form consisted of several open-ended items: (a) Describe what you are doing.; (b) Are other people involved in what you are doing? If so, describe who they are; (c) List any objects or tools that you are using.

We met with the respondents following the activity sampling for debriefing and to collect the pagers and sampling forms. We also discussed what kind of activities the students would be doing in the coming weeks. We analyzed the sampling forms and, using this information and the information about their activities that we obtained through our conversation with the respondents, we selected several activities in which to observe the students. Later in the summer, we also asked each respondent to keep a log of how they used mathematics. The log consisted of pages for four days, and the respondents were asked to "describe how, where, and in what activities you used mathematics today." for all four days. In the fall we asked each respondent and a parent to keep a log for four days. The directions were the same as described above and the students and parents each recorded descriptions about their own activities. Every time data were collected—through the sampling form, the log pages, field observations—we met afterward with the respondents to discuss what they had written and to ask questions that we had.

Students' Use of Function Ideas

In our analyses, we have observed a number of areas of mathematical concepts that we have categorized as function, including the concept of variable. The concept of function—and closely linked to that, the concept of variable—has been suggested as an organizing theme for K-12 mathematics, but as yet there has been little work to suggest how younger students might connect their experiences to function ideas. There is a large body of research, especially at the secondary level and beyond, concerning functions and how the concept of function can be thought of and used in different ways (e.g., as process or object, as a causal relation or direct variation, as co-variation). In our observations with middle school students, we saw function ideas being used as causal relationships, that is, variables represent quantities that change and functions describe a direct relationship between the quantities. We saw instances of co-variation, where the focus of the students' thinking appeared to be on the relationship between the change in one quantity with respect to the other quantity.

We observed the students using function ideas in (a) planning when they scheduled activities and when they made decisions about preparing a meal; (b) working when they did chores; (c) creating when they made craft items and when they created music; (d) investigating when they tried to determine the salinity of sea water, the weight of certain coins, and how what they saw changed according to the lenses used by the optometrist; and (e) playing in organized activities, such as soccer camp, and recreational pursuits, such as miniature golf. It is important to note that it is our perception that the students are using function ideas during these activities. However, the students seemed to always understand that there was a relation between quantities in a situation and they used their own language to describe this relationship.

Students' Use of Correspondence Ideas

A common relation we found in many of the play activities involved the use of angles. In general, the relation was that the angle that was to be used to accomplish something depended upon another variable or several variables. In one of our observations, Kristen, Robin and Jessica were playing miniature golf together. In the interview after the game, the girls discussed using the wooden blocks and walls in the course to bank the ball into the hole.

Researcher: So ricochet, or bank, or hit it off the wall—how does that help you?

Kristen: 'Cause you go farther without having to bump into a block.

Researcher: Without having to hit twice?

Kristen: Yeah, like the thing (block) might be at an angle but the ball will go right by it if you hit it at an angle; it will bounce off the wall and go into the hole. (Observation and Interview, 1995)

The girls also discussed how the location in which they wanted the ball to end up, and the obstacles that were in the way, determined the angle at which they hit the ball with the putter.

Robin: Well, it depends on how you want to angle it.

Kristen: It depends on how you gotta hit it. If the hole is over here and you have to get by the bend, then you have to hit it off the wall so it will go in the direction of the hole (see Figure 1A).

Robin: But if you angle the putter this way, the ball will go toward the hole without bouncing off of anything.

Researcher: What if you decided to bank it, or ricochet it? What if there was a block?

here, then . . . (see Figure 1B)

Jessica: You'd want to hit it off the block...

Kristen: Well, you'd want to hit it on the block near the edge so. . . actually you'd want to hit it over here so it would ricochet here . . . (see Figure 1C). (Observation and Interview, 1995)

The girls understood that the path of the ball depended upon the angle at which it hit an obstacle (wall or block). They tried to have the ball hit the

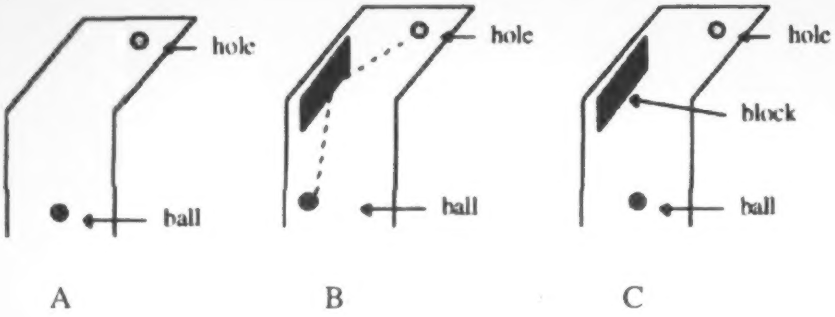

Figure 1. *Location of Ball and Block*

wall or the block at a particular angle so the path of the ball after it hit the obstacle would take the ball closer to the hole. Thus, the function ideas we observed at use in the miniature golf context were that the rebound path of the ball was a function of the angle at which it hit an obstacle and the angle of their putt was a function of the location of the obstacles and the hole.

Students' Use of Co-variation Ideas

We also saw our respondents using ideas of co-variation. For example, Linus described the relation between the gas consumption of his lawn mower and the amount of time he spent mowing.

Linus: I have to fill the gas tank and I sort of have to know how long a tank will last.

Researcher: How do you know how much to fill?

Linus: Well, since I know about how long it will last, I just watch the time and when it gets close I just look in the tank to see if it's full and if it isn't, I fill it.

Researcher: Do you know how much gas fits in the tank?

Linus: No, because it doesn't matter; I just fill it up. (Interview, 1995)

Linus understood that the amount of gas that is used by the lawn mower depends upon how long he runs the mower. He also appears to measure the amount of gas the tank will hold not by a capacity quantity, but by a time quantity.

Before going to swim camp for a week, Amy went to the store with her mother and was allowed to spend $2.00 to buy some candy to take with her to camp. In discussing her purchases in an interview following the shopping, Amy described how she made her purchases.

> Amy: When I picked out items, I had to check to see how much it costs. Some were 5¢, some were 10¢, and a bunch were 20¢. I had to decide if I wanted lots of little candies, or fewer bigger ones, or to trade a 20¢ for two 10¢ ones or for four 5¢ ones . . . . (Interview, 1995)

As Amy discussed how she made choices of candies, it appeared to us that she was weighing the number of each type of candy (i.e., 5¢ candy, 10¢ candy, 20¢ candy) while needing to stay within her limit of $2.00. We saw that she was considering the relationship between the change in the number of candies and the change in the total cost.

In another situation, we watched Linus as he demonstrated how he manipulated his remote-controlled boat, and how he learned to have it spin around while remaining under his control.

> Linus: I want the boat to be going fast and straight for awhile and I turn it sharp and it spins around and comes back, or it does a loop and keeps going. I learned to do that by trial and error—I had to keep turning it a little and see how the boat turns, then I turn it a little more. (Observation and Interview, 1995)

We noted that when learning to manipulate his boat, Linus has focused on the relationship between the change in how much and in what direction he moved his control and the action of the boat.

We continue to analyze the data from these respondents to understand (a) how they are making sense of phenomena they encounter in their out-of-school activities, and (b) how early thinking about functional relationships may arise.

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MATHEMATICS POWER: DEVELOPING THE POTENTIAL TO DO WORK*

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In this paper I investigate the relationship between students' interpretations of their environment and their inclination to use mathematics as a means to organize their interpretations. It is argued that students make conscious choices about what to attend to when interpreting a situation and that these choices may influence their potentials for and inclination toward the development of mathematical power.

Mathematical power is the potential to do work. For human beings, work is a series of actions directed at accomplishing a particular goal, and so the need for mathematical power is developed through and structured by the socio-cultural activities that determine what goals and purposes are adopted. Of course the mathematical power that a person develops or needs is not limited to the mathematics they need in any given situation. The generalizability of the work done with mathematics adds to its potential to do more work. Studying this generalizability in the abstract develops more mathematical power and prepares a person to handle work that may confront them in the future in more efficient ways.

How much mathematical power does a person need, and how much extra power can he or she be expected to develop? The last question, I believe, is a matter of personal need and preference. Just as some people will develop the power to lift 500 pounds, while others will not, some people will develop much more mathematical power than they may ever need and others will not. The first question is more complex and involves both questions concerning the nature of the person's environment, their interpretation of their environment and questions concerning how much generalizability is required to accomplish the work in an efficient manner. In this paper I will deal with the question of how a person's interpretation of their environment may impact the need to develop mathematical power. The argument is that the way a situation is interpreted will determine what work is appropriate and so how much mathematical power may be needed and developed.

To investigate this issue data collected from interviews with thirty-nine students were analyzed to determine how their interpretations of a situation might or might not support the development of mathematical power. Results indicate a wide range of interpretations and so potentially a wide range in the amount of mathematical power that would be needed. Before discussing the interviews, analysis, and results, I will provide a brief discussion of the theoretical framework that guides this research.

Theoretical Framework

Mathematics is developed as a result of generalizations of actions taken to deal with our environment. This fact is evident in the work of Piaget (1970) and Steffe (1991) who take a meta-level look at the cognitive processes involved in developing units, pluralities, and collections through interactions with elements in the environment. The connection between interactions with the environment and mathematics is also the central theme of research in ethnomathematics (D'Ambrosio, 1990). In addition, ethnomathematics also emphasizes the influence of culture on our interactions with the environment and the development of mathematics. One implication of this research is that if mathematics is generated through interactions with the environment then this is also the way it should be taught and learned. Freudenthal (1983) presents a theory of teaching based on the idea that mathematics organizes experience in particular ways. The way a situation is organized will depend on how the situation is interpreted. In any situation there may be better and worse ways to organize information and so mathematics may play a greater or lesser role in that organization.

A second implication of research that investigates the relationship between the development of mathematical ideas and interactions with the environment is that mathematics learning and mathematics instruction should remain close to the students' experience. Dewey (1902) puts forth these arguments. It is the child's interaction and investigation of their environment that sparks the desire to learn. Dewey (1989) also discusses three stages of curiosity that can lead to a sustained investigation of the environment. The first stage, the organic stage, involves general surveying of the environment. This is not a focused activity, instead it involves becoming aware of the things in ones environment in a more or less random order. The second stage of curiosity is more social and involves the asking of questions such as "What is that?" and "Why?" Dewey states that these questions are not a demand for rational explanation but simply an attempt to enlarge one's experience. The organic and social stage of curiosity lead to an intellectual stage of curiosity where a sequence of questions and inquiry are bound by some distinct end. Each stage represents a different level of interaction with, and level of interpretation of the environment. Thus, each stage represents a different potential for developing a need to do or use mathematics.

The first two stages of Dewey's three stages of curiosity can be seen in the interpretation of the environment or a specific situation. Interpretations deal with the central question of "What is this?", and may lead to initial questions concerning surface level relationships between objects in the environment. These questions might lead to intellectual curiosity that supports the development of mathematical work. Thus, an analysis of students' efforts to interpret events may provide insights into the ways that their individual interaction with the environment may shape their mathematical potentials, which would in turn shape their inclination and need to develop mathematical power.

Methods

To investigate these issues thirty-nine subjects were asked to provide several different interpretations of an event shown to them on a video tape. The subjects were asked to describe a drag race as if (1) describing it to someone who did not see the race, (2) from the driver's perspective, (3) from an historian's perspective and (5) from a mathematician's perspective. Interviews were conducted at three different times. The data analyzed here includes three students that were interviewed in the summer of 1996, sixteen students who were interviewed in the summer of 1997 and twenty students who were interviewed in the fall of 1997. All of the students were between the ages of thirteen and fifteen from a variety of schools and cultural backgrounds. The students' interviews were video taped and transcribed for analysis.

Analysis

What gets noticed in a given situation is the first stage of curiosity and determines what kinds of processes and concepts will be necessary or useful in organizing the experience. Taking a perspective on the situation is the second stage of curiosity, and focusing that perspective toward a specific goal is the third stage. The interviews discussed here were not intended to reach stage three. Instead the intent was to understand and identify what students, individually and as a group, noticed in the video, and also how their focus changed when they were asked to take a different perspective.

Interviews were analyzed to identify the specific concepts mentioned by the students when describing the drag race form each of the different perspectives. An initial list of the subjects and predicates of each utterance was developed. Items in the list were then placed into ten general categories. The categories included *Distance, Speed, and Time, Event* (included utterances that identified the event as a drag race), *Describing the Competition* , *Referencing People,_Process/Function* (engines, brakes, stopping etc.), *Physical Features, Emotions/ Cognition, Mathematical Operations, Narrating the Race*, and, *Miscellaneous*. The perspective that generated the response was also coded. This made it possible to identify differences in the things referenced by the subjects from each of the different perspectives. A sample of the coding categories is contained in Figure 1, which summarizes the number of references per category by perspective for students 1A through 2E. The numbers in the cells represent which perspective generated the utterance.

The resulting charts were analyzed to identify patterns and characteristics of the subjects' descriptions that could be associated with the change in perspectives they were asked to make. Finally the resulting patterns and

Response Categories	1A	1B	1F	2A	2B	2C	2D	2E
Distance, Speed and Time	125	135	5	15	5	125	15	
Event		1		1		1	1	1
Describing the competition	15	5	23	1	13	1	13	
Referencing People	3	1	1	1	13		1	
Process / Function	5	135	35	5	3	5	1	1
Physical Features		1	1		5	1	1	
Emotions / Cognition								
Mathematical operations*							5	
Narrating the Race							2	
Misc	1	25	3	3	35	13		325

Figure 1. *Sample of response coding into ten major categories*

characteristics were used to make inferences concerning the mathematical potential that might exist in the subjects' description of the drag race.

Results and Discussion

There was great variation in the responses generated by individual students. Within the ten major categories described above no fewer than 73 different referents were addressed by the thirty-nine students that were interviewed. This is significant because it means that as a group there are very few if any aspects of the drag race that were not mentioned by the students. This would suggest that had all thirty-nine students been present at the same time they would have the potential of producing a comprehensive description of the drag race. This means that as a group they could complete the first stage of curiosity in a manner that would hold great mathematical potential. However, it is not clear that a group discussion of the video would have generated the same robustness in the students' responses. The results here emphasize the importance of maximizing student input by encouraging divergent responses and individual thinking during initial classroom discussions.

Individual subjects varied greatly in their interpretations of the drag race, and across perspectives subjects varied their responses to a lesser or greater extent. The comprehensiveness of the sum of each of the subjects' responses also varied greatly. Again, this variation between subjects' responses emphasizes that all students will not interpret a situation in the same way, even when asked to use the same perspective. This might suggest that very different kinds of mathematics would be useful in organizing the situation for different students. The variation within the subjects' responses from each of the different perspectives suggests that they made conscious choices about the things they referred to when taking each of the different perspectives. This implies that their account of the drag race from any one perspective was incomplete. Thus it is a combination of perspectives that provides the most detailed description and so the best foundation for the development of the second stage of curiosity. More importantly is that there were students who made comments from the mathematical perspective that they felt were inappropriate in the other perspectives. This result has implications for how useful students found a mathematics perspective in describing the event.

While there was great variation in many of the subjects' responses there were some subjects whose responses were more consistent across perspectives. For those subjects whose responses showed this consistency and that responded to the mathematical perspective, mathematics may be a more integrated part of their general interpretive frame work. For example, some subjects' descriptions of the drag race from all perspectives included a focus on the *Distance, Speed, and Time* category. As they switched perspectives, they simply changed the way they thought about distance, speed and time. One student responded in the following ways to each of the different perspectives.

General: "Well, they were racing cars, drag racing-they were comparing how fast the cars were going, who was a head and come of the race track showing a picture of an engine. I am guessing that it would be like a really fast engine they put on display."

Driver: "In my opinion scary! I would be afraid of getting hit by the car. It would be neat though. You are really low to the ground and you go really fast."

Historian: We have an historian at our school. I ran for it - but I didn't make it. I would say it was a fast action race - crowds were yelling, a lot of excitement. That's about it basically.

Mathematician: The car is doing 174.6 miles an hour, which is twice as fast as #3 who is going half the speed of #54.

Here we see that speed plays a role in all four responses. This subject integrates a mathematical perspective in her general description, "Comparing how fast the cars were going", which she makes more explicit in the mathematical description giving numerical values for speeds and making accurate measurements. For this subject, using mathematics is a good way to organize the experience from all perspectives. Fourteen of the thirty-nine subjects discussed speed in their general description and again from the mathematical perspective. However, their general description mostly involved stating the speed or comment on how fast the race was. They did not suggest comparisons as in the last example. Eight subjects failed to mention speed time or distance in any of their four descriptions. Three students referenced speed time or distance from one of the perspectives but then not from a mathematical perspective, while seven students mentioned speed time and distance only from the mathematical perspective.

The implication of these findings is that students vary greatly in the way that they interpret a situation or event. The degree to which mathematics becomes an efficient way to organize information also varies greatly from student to student. Thus the mathematical potential of their interpretations and so the amount of mathematical power they may be incline to develop may also be reflected in their descriptions. The implication for teaching mathematics is that students must learn how to see the world in mathematical ways which will require using multiple perspectives to describe their environment. Teachers must be more sensitive to how students currently see the world since it is their views and interpretations that will lead to the development of work that requires mathematical power.

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Notes

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USING THE METAPHOR OF VOICE TO INVESTIGATE THE MATHEMATICAL EXPERIENCES OF AFRICAN AMERICAN STUDENTS

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This paper presents the results of a research study that used the metaphor of voice to investigate the mathematical experiences of two African American female college students. The study examined the students' perceptions of and responses to their mathematical experiences. The study particularly sought to determine how these students viewed their experiences in mathematics classrooms, how they dealt with barriers and obstacles in their mathematics education, and what factors contributed to their succeeding in mathematics.

Several research studies document the low mathematics achievement of African American students while ignoring or silencing African American students' voices. This is problematic because it places an emphasis on unsuccessful African American students which has the propensity to lead to stereotypes or generalizations (Matthews, 1984). Operating on the premise that African American students' voices have been ignored or silenced much too often in the literature, the metaphor of voice was used as the basis of this study. "Voice refers to the discourse that is created when people define their own issues in their own ways, from their own perspectives, using their own terms—in a word, speak for themselves" (Secada, 1995, p. 156). The African American students' voices in this study denote their perceptions of and responses to their experiences with school mathematics.

The objectives of this study were to: (a) identify the African American students' perceptions of their mathematics classroom experiences, (b) identify the students' perceptions of how their ethnicity affected their mathematical experiences, (c) determine how the students dealt with barriers, stereotypes, and/or obstacles in their mathematical experiences, and (d) identify factors that contributed to their succeeding in mathematics.

Theoretical Perspectives

Since race and ethnicity are categories laden with cultural beliefs and biases that are often unconscious, it is essential to consider cultural orientations when studying the role of race and ethnicity in students' mathematical experiences. How students perceive and respond to their experiences in mathematics classrooms is influenced by their cultures, orientations, and their social realities.

Ogbu (1986) argued that African Americans tend to act in various ways (e.g., resistance to school) that are in opposition to dominant culture. He termed this resistance, *cultural inversion*. "Cultural inversion may be defined as a tendency to regard a cultural behavior, event, entity of meanings as *not* [African American] because it is characteristic of Whites or vice-versa" (Ogbu, 1986, p. 48). Ogbu asserted that cultural inversion is manifested in education in the sense that academic success is perceived by some African American students as characteristic of White culture. Thus, those African American students who are successful in school are condemned as "acting White." Ogbu stated that cultural inversion is a coping mechanism that some African American students use to coexist with dominant culture.

Somewhat analogous to Ogbu's notion of cultural inversion, Fordham (1988) found the anthropological concept *fictive-kinship* useful in studying the social identity and cultural frame of reference among African Americans. Fictive-kinship refers to a "kinship-like connection between and among persons in a society, not related by blood or marriage, who have maintained essential reciprocal social or economic relationship" (Fordham, 1988, p. 56). Fordham asserted that the fictive-kinship extends beyond skin color and incorporates a particular mind-set or world view of those persons who are considered to be African American. Further, the fictive-kinship symbolizes a particular "people-hood" in opposition to the prevailing White society. The collective ethos of the fictive-kinship (or indigenous African American culture) is challenged when African American children enter school and are faced with the individual ethos and competitiveness of dominant culture. This causes some African American students to experience ambivalence in their schooling. There is a struggle between the two systems (fictive-kinship and dominant culture ethos), and African American students' conformity to one of the systems challenges their loyalty to the other. Thus, some African American students remain affiliates of the fictive-kinship, and to legitimate their membership in the indigenous African American culture, they ensure their failure in school. Conversely, those African American students who assimilate into the school culture and minimize their connection to the indigenous African American culture are more likely to succeed in school. These African American students minimize their relationship to the African American community and the stigma attached to being African American to improve their chances of succeeding in school. They take on a persona of "racelessness," which is "the desired and eventual outcome of developing a raceless persona, and is either a conscious or unconscious effort on the part of such students to disaffiliate themselves from the fictive-kinship system" (Fordham, 1988, pp. 57-58).

In this study, the authors found Fordham's (1988) theory of *racelessness* and Ogbu's (1990) notion of secondary educational strategies useful in conceptualizing particular factors that contributed to the African American students' succeeding in mathematics. Ogbu (1990) defined "secondary educational strategies" as strategies African American students use to achieve some measure of school success. These strategies consist of African American students disaffiliating themselves from African American culture in favor of White culture, camouflaging academic striving, and relying on parental support and the support of educators.

Methodology

This study employed a phenomenological research strategy. Phenomenological research describes subjective experiences of individuals (Tesch, 1984, 1987). It is aimed at interpretive understanding and describes individual experiences from the viewpoint of the individual (Tesch, 1984). Phenomenological research explores the personal construction of a person's world through in-depth, unstructured interviews and other data sources (Tesch, 1987). Data were collected in the form of initial surveys, autobiographies, and interviews to explore life histories of the participants in the context of their mathematical experiences.

Phenomenological research involves a back-and-forth movement between a phase of thinking and analyzing and a phase of data gathering, which is analogous to constant comparative analysis (Strauss, 1987). Surveys and autobiographies were used as data sources, were analyzed, and were then used as stimuli to gather more data during interviews. The first stage of the analysis occurred parallel with and informed subsequent data collection. During this first stage of analysis, particular themes were sought that were preeminent in the participants' voices.

The second stage of analysis received most attention at the end of data collection. The objectives of the study guided the search for invariant themes and patterns that emerged from the data. Particularly in this second stage, the authors sought to explicate, interpret, and make sense of the invariant themes in terms of particular theoretical constructs.

Results—Dissonant Voices

Ashley's Voice

Ashley's perception of her mathematical experiences can be characterized as a *struggle*. When Ashley talked about her mathematical experiences, she emphasized fighting battles and dealing with conflict. Rather than dealing with any difficulty of the mathematics itself, Ashley's struggle was couched in interacting with students, teachers, and school administrators and in contending with peer criticism. Frequently, Ashley talked about her mathematical experiences and racism in the same breath. Ashley wrote in her autobiography about her experience in a White mathematics teacher's class:

> My best friend [Amber] and I were the only Black students in [Mr. Miller's Algebra I] class. We could tell that Mr. Miller was burning with anger because we were smart enough to be in his class. When Amber asked him for help, he would just say, "Go figure it out; you have a book." On the other hand, I refused to ask him for anything because I was determined to be successful without his

> help. His racist jokes, ugly glares, and superior feelings only gave me the power I needed to defeat him. What I mean by "defeating" him was proving that I could make good grades and learn algebra despite his feelings. (Autobiography, 7/11/96)

Ashley used the strategy of taking on the persona of racelessness (Fordham, 1988) to succeed in mathematics. Employing the raceless persona meant that Ashley disaffiliated herself from and minimized her connection to the fictive-kinship or indigenous African American culture. Ashley stated," I guess I just did what I had to do to achieve my goals. That means that sometimes I would be someone else. I mean not really change, but sometimes you wear a mask. . . . I had to do that in order to be successful and I don't even know if I should have done that, but that was my main goal then." (Interview 1, 6/27/96)

Ashley's notion of wearing a mask was prolific in how she talked about succeeding in mathematics. She commented that in high school most of her African American peers were in lower level mathematics courses while she followed an advanced college preparatory track. She stated that she felt as if she was disassociating herself from African American culture to succeed in mathematics. Consequently, Ashley was ridiculed by her African American peers as acting White.

Ashley used secondary educational strategies (Ogbu, 1990) such as parental support and caring educators to help her contend with the negative aspects (i.e., racism, peer criticism) of her mathematical experiences. Ashley stated that her mother and grandmother pushed her to do well in mathematics, set high expectations for her, and encouraged her when she encountered racism and peer criticism. Ashley commented that caring educators also pushed her to do well in mathematics. Moreover, African American mathematics teachers helped her remain in advanced mathematics courses by acting as role models who had succeeded in mathematics themselves.

Sheilah's Voice

Sheilah's perception of her mathematical experiences can be characterized as a *challenge*. Sheilah's challenge was couched in the mathematics itself rather than in the social milieu of schooling. When she talked about encountering difficulty in her mathematical experiences, she specifically referred to grasping the mathematical content of her abstract mathematics courses.

Sheilah used secondary educational strategies (Ogbu, 1990) or support systems such as parental support and caring educators to succeed in mathematics. Sheilah's mother represented a model of someone who had succeeded in mathematics since she had received a master's degree in mathematics. Sheilah stated that when she was faced with difficulty in her mathematical experiences, her mother constantly reminded her "not to quit and to stick with it." For example, Sheilah contemplated dropping Real Analysis during her graduate studies, but her mother intervened and convinced her that she was capable of succeeding in this course as well as any other mathematics course.

Caring educators also played a significant role in Sheilah's success with school mathematics. Sheilah stated, "I had several good Black female mathematics instructors, so I knew that *we* [emphasis hers] could handle mathematics" (Autobiography, 7/9/96). These educators seemed to have served as role models and perhaps *evidence* that African Americans *can* become successful with school mathematics.

Concluding Remarks

Using a phenomenological approach to conducting this study was key in this research reflecting research *with* African Americans rather than research *about* African Americans. As a consequence, this study adds voice—a missing and valuable commodity—to the mathematics education literature. It is important that we as mathematics educators not only listen to African American students' voices but that we also frame issues and pose questions based on what we hear.

Ashley's and Sheilah's voices indicate that their perceptions of their mathematical experiences and their responses to these experiences were paramount in their succeeding in mathematics. Ashley perceived her experiences as a struggle and believed that the ideologies of school culture were imposed upon her. However, her response was not to resist these imposed ideologies but to conform to them in order to succeed in mathematics. By contrast, Sheilah perceived her mathematical experiences as a challenge, and her response was to work hard and persevere to reach an achievable goal. The key for Sheilah was that her goal was always achievable.

The dissonance of Ashley's and Sheilah's voices teaches us that we can no longer afford to misconstrue notions about African American students in general. The more we know about different voices and understand the complexity of voice, the better equipped we will become to improve the mathematical experiences of African American students.

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SOCIAL AND CULTURAL FACTORS
SHORT ORALS

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YEAR-ROUND EDUCATION: AN AID FOR OVERCOMING PEDAGOGICAL CONSTRAINTS?

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In 1983, the National Education Commission recommended reallocating time for learning within the school day or year. Accordingly, we need to investigate the impact of alternative organizations of time (e.g., longer school days, block scheduling, or year-round education [YRE]) on teaching and learning. YRE is defined here as a school calendar that arranges instructional and interim periods so as to use school facilities during virtually every month of the year to teach the majority of the student population. Proponents of YRE cite potential benefits such as increases in students' learning and retention, a decreased need for reviewing content, improved standardized test scores, and increased time for teachers' professional development.

A case study of Sharon, a middle school mathematics teacher in an economically disadvantaged community, was used to gain insight into her interpretive lens for making sense of her educational experiences. Specifically, I investigated both her constructions of experiences that she described as sustaining or constraining to her efforts to implement nontraditional instructional practices and her justifications for those constructions. I identified constraints such as student resistance, uncreated mathematical connections, and time limitations. It is the constraint of time, and the potential for YRE to help alleviate such constraints, that I will explore herein.

Sharon strove to frequently utilize activity-based instruction. However, she asserted that planning nontraditional activities takes a lot of time, and her time seemed "to be eaten up with other things" (e.g., behavioral management, grading, and other managerial details). Other factors such as student resistance, Sharon's construction of her role as an educator, uncreated mathematical connections, and limited engagement in critical reflection also contributed to the inconsistencies between Sharon's vision of ideal teaching and her practice. However, YRE could offer teachers such as Sharon more frequent and extensive periods of time to investigate resources, plan instruction, and learn subject matter. Likewise, YRE could provide instructors with more frequent opportunities for critical reflection and participation in support networks to stimulate such reflection, as well as support for implementing nontraditional teaching methods essential to sustaining such efforts. Accordingly, a shift to YRE may benefit both teachers and students.

INSTRUCTIONAL REFORM IN MATHEMATICS AND SCHOOL CHANGE: A CASE STUDY OF AN ELEMENTARY SCHOOL

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Reforming mathematics teaching and learning has been addressed by the mathematics education community.

However, how to lead and what impact such a wide-ranging, complex reform would possibly have on school change needs more investigation. Therefore, the purpose of this case study is to investigate the complex nature of mathematics reform in a elementary (K-4) school were the educators are leading and implementing major reforms. The primary research questions are (a) What did initiate the mathematics reform and how did it evolve?, and (b) What strategies were developed to facilitate, guide, and sustain this reform process?

The initial instructional changes in mathematics were mechanical, for although new materials and content were integrated into mathematics classrooms, there was little evidence of fundamental change in teachers' beliefs and practices. As the reform evolved, many teachers struggled to make mathematics teaching and learning relevant to students' experiences. Some teachers discarded their dependency on mathematics textbooks, wrote mathematics instruction, and designed performance task assessments to monitor students; growth over a period of five years. The teacher and principals researched and collaborated with university and secondary mathematics educators to better understand mathematics content and to develop their own pedagogy grounded in constructivism. Beyond instructional change, the reform ignited major changes in school structure and leadership (roles and relation).

CULTURAL DIFFERENCES IN EIGHTH GRADE MATHEMATICS: A CASE STUDY OF A JAPANESE LEP STUDENT

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Diversity exists among students in today's classrooms. When cultural differences exist between student and teacher, misunderstandings in learning may occur. This study examined the effects of cultural differences in a mathematics classroom. The general questions were: What factors influence the Japanese Limited English Proficient (LEP) students's adjustment to the new school setting? What factors influence his mathematics learning?

The primary method of study was participant observation, conducted over five weeks. The triangulated data sources included observational field notes, interview transcripts, and a content analysis of artifacts. Three broad categories emerged: teaching difference between cultures, communication, and assessment/evaluation.

The study found that the cultural differences between American and Japanese perspectives of learning mathematics and the ways the subject is taught, influenced the student's adjustment to the new school and his mathematics learning. Communication difficulties existed between student and teacher, and school and family. The traditional teaching and assessment methods discouraged the diverse ways of thinking which he was accustomed to in the Japanese classroom, thus limiting his expression of mathematical knowledge and thinking. As a result, little "new" mathematics learning occurred; instead, he learned the teacher's procedures for doing mathematics.

Although this study reports the case of one student, it is reasonable for educators to be concerned about its implications for other LEP students' mathematics learning. In what ways do cultural differences inhibit students' mathematics learning?

Note: Preliminary analysis of ninth grade data indicates that "new" mathematics is learned in his weekly Japanese Saturday School class rather than in the local school mathematics class.

SOCIAL AND CULTURAL FACTORS
POSTERS

NCTM STANDARDS: AN IMPETUS FOR A NATIONAL CURRICULUM?

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The National Council of Teachers of Mathematics (NCTM) has proposed national standards in mathematics which include content standards, performance standards, and opportunity-to-learn standards entailing three of the four categories of standards given by Lewis (1995). The remaining category is world-class standards or those described in Goals 2000. Do standards, in general, promote a national curriculum? Does a national curriculum create a more effective public education system? This paper seeks to illuminate these questions.

Allen and Brinton (1996) contend that American public schools would benefit from a national curriculum in four areas: (1) mobility, (2) equity, (3) obsolescence, and (4) accountability. Disadvantages of a national curriculum include: (1) lack of local control, (2) loss of parentsí ability to control family values instilled into their children, (3) de-emphasis of basic skills through emphasis of critical thinking, (4) extensive teacher development, and (5) a perpetuation of the status quo if the standards are used as measurement devices. Mathematics instructors must be familiar with several standards while also trying to cater to inter-disciplinary learning. Mathematics educators are considering the effects of assessment standards of proposed content and to explore alternative means of assessment such as rubrics and portfolios. Although national standards and a national curriculum are different entities, their relationship to one another also alerts mathematics educators to possible tensions that accompany cumulative change influenced by the implementation of a national curriculum and the adherence to national standards.

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FEMALE PARTICIPATION AT MATH CONTESTS: EFFECTS OF SCHOOL SIZE AND THE USE OF TECHNOLOGY

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While observing the Illinois State Math Contest, I noticed an unequal distribution of male students and female students at the Team Calculator Event and the Team Rely Event. At the Team Calculator Event, tables of 5 students representing a school used calculators to solve mathematical problems. Student from schools with large enrollments (AA Division) sat on one side of the room. Students from schools with small enrollments (A Division) sat on the other side of the room. At the Team Relay Event, rows of students representing a school sat facing the same direction. A problem set was given to the first student in each row. After a student completed the assigned problem, the student passed the set of problems to the next student on their team. A team was done after each student on the team had completed their problem correctly. Results of my tally of the number of students participating in the two events are presented in Table 1.

More female students who participated in the two events were from Division A schools than from Division AA schools. A Chi-square test for homogeneity for the number of males and the number of females participating in the two events from Division A Schools and from Division AA Schools was significant (45.6 > 6.6 Chi-square df = 1 alpha = .01).

Table 1
Participation at the State Mathematics Contest

	AA calculator	A calculator	AA rely	A rely
number of males	149	66	130	62
number of females	15	34	58	78

More female students participated in the Team Relay Event than the Team Calculator Event. A Chi-square test for homogeneity for the number of female students and the number of male students participating in the Team Calculator Event and the Team Rely Event was significant (42.4 > 6.6 Chi-square df = 1 alpha = .01).

MATH CAMP: AN INNOVATIVE EDUCATIONAL PROGRAM

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Many high school students continue to enter mathematics courses feeling as if they are under-prepared and have little ability to be successful. In addition, many high school students spend part of their summer months attending sports camps on university campuses. These camps promise to teach the basics of the sport and to sharpen the skills of all participants. Math Camp was designed to help meet the mathematical needs of all levels of high school students within a format modeled after a typical basketball camp. The objectives of the camp were:

- to give students a head start in math right before school starts
- to discuss the affective components of learning and thus help students realize that they can do math (internal control) if they know how to play the game
- to critique the thinking of students through discussion of hands-on work at the chalkboard

Seventy-five students attended the First Annual ASU Math Camp which met in the evenings during the first week of August, 1997. The format of the camp included three instructors (each in a classroom), 9 heterogeneously mixed teams each with a coach, and three teams per classroom. Three topics per night, drill, practice, team competitions, and individual hot-shot competitions comprised the first 2 hours and 45 minutes of each evening with the last 15 minutes going to snacks, socializing, games, and announcements of the daily winners and cumulative team scores.

Evaluations from participants, coaches, instructors, parents, and teachers (during the following school year) were every positive. Students left Math Camp with a personal strategy for success to begin high school. They felt more confident in knowing how to "play the game" that we call math.

SYMBOLISM AND SEMIOSIS IN GRADUATE STUDENTS' ETHNOMATHEMATICS PROJECTS

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In an attempt to understand processes involved in connecting students' cultural activities and formal academic mathematics at all levels, semiotics (study of semiosis) has provided a fruitful research framework. Through semiosis (activity of signs), and in particular chaining of signifiers, cultural practices in a "systematization of relationships" which was to Ada Lovelace the essence of mathematics (Noss, 1997). Examples are given of chaining of signifiers in diagrams which illustrate graduate students' construction of mathematics from practices which are authentic to their own cultures in some way. Chosen from a bank of 119 projects, illustrations of chaining of signifiers in mathematics of cultural practices include mountain, bike mathematics (USA), dominoes (Cuba), Hanukkah candles (Israel), and the Jamaican national flag.

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TEACHER BELIEFS
RESEARCH REPORTS

THE IMPACT OF A SECONDARY PRESERVICE TEACHER'S BELIEFS ABOUT MATHEMATICS ON HER TEACHING PRACTICE

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This paper describes one preservice secondary teacher's beliefs about mathematics and discusses how these beliefs were related to her teaching practice. The participant was interviewed and observed throughout the final academic year of her undergraduate preservice program (methods course and student teaching). Although she communicated beliefs emphasizing the importance of cooperative exploration by students to understand connections among mathematical concepts, some of her more narrow views about the importance of mathematical procedures inhibited her ability to successfully implement exploratory, student-centered learning activities during her student teaching.

In our effort to describe how preservice secondary teachers' beliefs about the nature of mathematics are related to their teaching, we build upon ideas discussed previously (Benken & Wilson, 1996) and report the results of a second case. Attempting to better understand preservice teachers' beliefs about the nature of mathematics has become an important area of study for mathematics teacher education (Cooney, 1994; Thompson, 1992). Teachers' understanding of the nature of mathematics strongly influences their views of mathematics teaching and can play a significant role in shaping teachers' patterns of instructional behavior (Ernest, 1989, 1991; Lerman, 1990; Thompson, 1992; Lloyd & Wilson, 1998).

The relationship between teachers' beliefs and teaching practices is extremely complicated. Although existing research suggests that these factors are interdependent (Wood, Cobb & Yackel, 1991), more research is necessary to better understand the nature of their interactions (Thompson, 1992). Often, there is a disparity between espoused beliefs and what a teacher actually does in the classroom (Ernest, 1989, 1991; Thompson, 1992). More research that examines preservice teachers' beliefs beyond the methods course into their practice during student teaching would be helpful to better understand this relationship (Pajares, 1993).

To aid in our interpretations, we used three categories described by Ernest (1989) to characterize individual's views of the nature of mathematics: *Problem-Solving, Platonist and Instrumentalist.* Ernest (1989, 1991) and Lerman (1990) have theorized that a teacher's philosophy of mathematics forms a basis for her mental models of the teaching and learning of mathematics. Other research corroborates this claim (Smith, 1996; Thompson, 1992).

According to the Problem-Solving view, mathematics is seen as a continually growing field of human creation that develops through conjectures, the generation of patterns, proofs and questioning. The results of mathematics remain open to revision. Teachers with Problem-Solving views are likely to think of themselves as facilitators who allow students opportunities to explore, generate and solve problems, and construct their own understandings. Problem-Solving teachers approach teaching in ways that stimulate classroom discourse, encourage student investigation and cooperative learning, and create an environment necessary for students to make connections within mathematics and to the world around them.

The Platonist view suggests that mathematics is "a static but unified body of certain knowledge," which is discovered and not created (Ernest, 1989, p. 250). This view involves a global understanding of mathematics as a "consistent, connected and objective structure." A teacher with Platonist views of mathematics is likely to see her primary role as explainer, and as one who should guide students to the solution or her intended goal. In general, a Platonist teacher emphasizes conceptual understanding with unified knowledge, and sees the students as "receptors of knowledge" (Ernest, 1989, p. 251).

The Instrumentalist maintains that mathematics is a fixed set of rules, facts and procedures for computing numerical and symbolic expressions to find answers. The Instrumentalist view is likely to be associated with a "telling" model of teaching, where teachers emphasize mastery of skills, rules, and procedures and usually strictly follow a text or scheme. A teacher's central role is to provide clear, step-by-step demonstrations of procedures, respond to students' questions, and provide opportunity for students to practice procedures (Smith, 1996). Solving problems is "a matter of recalling and applying the procedure appropriate for the given problem type" (Smith, 1996, p. 391).

Design

An ethnographic case study design was employed (Stake, 1995). Data were collected between September 1995 and June 1996. Twenty-four secondary mathematics methods students responded to a written survey at the beginning of the course, and we selected 2 preservice teachers for more in-depth study. This paper reports the experiences of one of these teachers (Leslie). Four semi-structured audio-taped interviews (approximately one hour in length), conducted during the methods course and student teaching, allowed Leslie to elaborate survey responses and communicate her views about mathematics, the methods course, and practice teaching experiences. Her bi-weekly tutoring experiences (part of the methods course) were observed. Photocopies of selected written assignments completed in the methods course were also analyzed. During student teaching, informal interviews (length varied from 20 minutes to one hour) followed seven observations. Finally, Leslie was observed during 12 student teaching seminars. Field notes of observations and informal interviews were taken and later analyzed.

Results

At the beginning of the methods course, Leslie communicated that mathematics was sequentially ordered, and that doing mathematics involved applying steps in order and mastering types of problems. For example, on the initial survey, she chose *cooking with a recipe*, from a list of eight similes, as the one that best describes what learning mathematics is like. In elaborating this response, she explained, "When I cook, I sometimes don't always have the exact tools and ingredients, so I improvise. However, I still do the basic steps in order, building from the bottom-up." Leslie's explanation suggests that she believed mathematics to be ordered, with each component building from the previous. To her, doing mathematics involved applying steps in a specific order. Such an emphasis on steps is most consistent with an Instrumentalist view of mathematics.

However, in some cases she articulated views that were more consistent with a Platonist view of mathematics. For example, Leslie saw mathematics as a connected, and somewhat creative, reasoning process. During our first interview, Leslie's elaboration of her survey response about cooking suggests she understood and valued connections within mathematics. She said,

> There's obviously some steps that you can't skip, or just kind of breeze through or you'll miss a lot. Like when you take the first course where you have to do a lot of proofs, if you don't really learn how to do the proofs, then you'll really have difficulty later on. Or, if you're doing trigonometry, and you don't get the relationships down between cosine and sine, then you'll have difficulty later on.

Leslie saw the steps (or various parts) of mathematics as being connected and important for the understanding of future topics. In her view, the ordering within mathematics was necessary for the understanding of and relationships among important mathematical ideas.

During the methods course, Leslie continued to struggle with her fundamental beliefs. She focused on the connectedness and understanding of mathematical ideas, the importance of and creativity involved in problem solving, and the value of applications. During the second month of the methods course, on an electronic shared assignment that required participants to comment on the changes recommended in the *Standards* (NCTM, 1989), Leslie articulated her view of doing mathematics as a logical thought process. She wrote:

> For me, it wasn't learning the rules of mathematics that made everything make sense now in calculus, differential equations, analysis, etc. It was, pardon the colloquialism, 'training my brain' to process the information and transform it, and above all, being able to logically reason that a given solution makes sense.

Although Leslie alluded to the rules of mathematics, she emphasized that a useful and important part of the foundation of mathematics is the ability to reason through and think about problems and connections between problems and solutions.

During student teaching, Leslie's comments confirmed that she viewed mathematics as a connected discipline. In many of her lessons, she stressed connections, both within mathematics and to the real world. She frequently claimed during discussions that she wanted to help her students understand these connections and see the usefulness of mathematics. For example, on one occasion, Leslie passionately commented:

Getting them to take things to higher level and not think about them on a really basic level was difficult. They still want an easy answer, they want a formula, they want to know what numbers to push on the calculator. I'm more concerned about getting them to understand what it means.

Here Leslie stresses that meaning is more important than formulas or answers. Leslie followed the above comment with an example using rational numbers. Although the properties themselves were important to learn, she said that she considered the understanding that would be gained by trying to explore and derive them to be even more important. These statements, along with others made at the end of the year, suggest that during Leslie's student teaching, her views expanded to include an even more connected, conceptual and useful view of mathematics.

During her student teaching, several patterns emerged in Leslie's practice that suggest she struggled with how to balance these desires and growing recognition of connections with her more Instrumentalist views of mathematics. For example, her teaching suggested she wanted to be certain that her students first understood the basics, or foundation, before she felt comfortable letting them explore and investigate. To Leslie, if she did not make certain that her students understood the basics, then they would have difficulty understanding future mathematical concepts, which are built upon the foundation. During a lesson she led on February 21, 1996, Leslie chose to review only components of a student project that she considered to be fundamental to the activity. In this review she chose not to discuss students' observations and written reflections of their findings.

During our final interview in April, Leslie reflected on several of her lessons that involved group activities and made the following comment:

They seem to enjoy doing them because they like to work together and they like to do something different, but I'm not always sure how well it helped them to learn what they are doing....[For example] in that general math class we were talking about area and I had them measuring the area of certain surfaces in the room....and they were having trouble with area being a two-dimensional measurement.

As this example illustrates, Leslie tried to develop activities that would help her students to understand the concepts she valued through cooperative learning and exploration. Although she did not want to fall into the book-lecture format, and would often search for other ways to "get at the same material without just telling," she reflected that often she made sure she went over certain material, in addition to allowing the students time to work together. Leslie struggled to balance student exploration with teacher dissemination of basic, fundamental concepts because she valued both.

On April 29, 1997, Leslie taught a lesson on subtracting integers. She used an interactive lecture format to demonstrate multiple ways to approach the concept. At the beginning of the lesson, before discussing examples that used number lines and manipulatives, Leslie told her class about rules for subtracting integers. She stated, "There are basically three ways to think about subtraction of integers. The first is 'SMATO,' or subtraction means adding the opposite." A student then asked if she could always "do that." Leslie replied, "If subtracting, yes. It always works." During our discussion before this lesson, Leslie indicated that she would be presenting three different ways of looking at subtraction. During her lesson she chose to show them the rule, or "straight forward algebra," way of looking at subtraction first, without added explanation or interpretation, then she introduced examples. This instructional pattern of introducing rules followed by examples, which Leslie repeated throughout the semester, illustrates how her view of mathematics as a connected set of fundamental procedures, played itself out during her student teaching.

Discussion

Can teachers implement practices consistent with reform while maintaining some core Instrumentalist views? Although Leslie communicated beliefs about mathematics during the methods course and student teaching that can be categorized as Platonist (e.g., emphasis on connections), her more Instrumentalist views appeared to inhibit her ability to fully implement some of her innovative ideas. From the beginning of the study, Leslie's belief that mathematics is learned sequentially and built upon a foundation, influenced her decision to make certain that she provided her students with what she perceived to be the basics before allowing them to explore and investigate problems. Leslie claimed that using cooperative group activities was a good way for students to learn concepts and generate algorithms, and she even occasionally attempted to implement them. However, when her students reacted negatively to the activities, she became frustrated and questioned the validity of cooperative, student-centered learning approaches.

Leslie's teaching practices were no doubt influenced by her own experiences as a mathematics learner and the traditional teaching practices of her cooperating teacher, who served as her mentor and model. Nevertheless, we believe that Leslie's main obstacle to implementing the teaching practices she envisioned were her own conceptions of mathematics and mathematics teaching. We are confident, however, that Leslie's conceptions of mathematics, teaching, and student learning will continue to develop, enabling her to implement the strategies she desires so her students can attain the understandings she values. Like many teachers who attempt to implement reform ideas, Leslie seems to have "one foot in each of two conflicting paradigms" (Goldsmith & Schifter, 1997, p. 28). Leslie's experiences can help teacher educators and others interested in implementing teaching ideas consistent with reform recommendations understand why the process of change is so difficult for teachers.

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IMPLEMENTING MATHEMATICS REFORM: A LOOK AT FOUR VETERAN MATHEMATICS TEACHERS

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This paper investigates four veteran mathematics teachers' beliefs about mathematics and the resulting differences in their implementation of mathematics reform. We analyzed a variety of data, spanning a four year time period, to document these teachers' beliefs, classroom practices, and leadership roles. Of particular interest is the finding that the two teachers whose actions most reflected current calls for reform described their change as "in transit" and personalized it as "my change" while those whose actions reflected a superficial implementation of reform talked in third person about the need for others to change and their role in helping them.

Mathematics reform as described in this paper is based on the NCTM Professional Standards for Teaching Mathematics (1991) and other related reform documents, where a reform classroom has several distinctive observable characteristics. For example, students are engaged in doing meaningful mathematics, often working collaboratively in small groups with the teacher facilitating students' investigations or discussions. It would not be uncommon to see students using technology to enhance their learning, presenting their findings to their peers, or studying real-world applications. Most importantly, the focus is on the students', instead of the teachers', mathematical thinking. Superficial reform involves many of the characteristics identified above, but lacks the fundamental emphasis on students doing the mathematical thinking.

Thompson (1992), in her synthesis of the research on teachers' beliefs, claimed that there is strong evidence to suggest a relationship between teachers' beliefs and their teaching in the classroom. Furthermore, there is evidence to suggest that teachers can, based on their belief structure, hold two seemingly dichotomous beliefs without experiencing conflict (Cooney, et al., 1998). This, in turn, can result in classroom practices which appear to be inconsistent with teachers' beliefs.

Kaplan (1991) provides additional insight into this apparent inconsistency. She argues that *surface* beliefs, which underlie *superficial practice structures,* can be distinct from *deep* beliefs which underlie *pervasive behaviors in the classroom*. This distinction provides a possible explanation for the fact that teachers who express similar views implement reform in different ways. In this paper, we investigate the beliefs and practices of four such teachers.

Methods

Participants. This study focuses on a subset of four participants from a cohort of thirty secondary school mathematics teachers who expressed a commitment to improving the mathematics teaching in their schools by participating in a three-year (1994-1997) teacher-leader program funded by the National Science Foundation (Grant No. ESI-9353513). The primary focus of the program was to prepare secondary school mathematics teachers to become participants in, and leaders of, reform by exposing them to contemporary mathematics content as well as current curriculum and pedagogical issues.

Each of the four teachers, two males and two females, was at least 47 years old and had been teaching for at least 23 years. Veteran teachers were chosen for two key reasons: 1) their commitment to reform belied the traditional stereotype of teachers not willing or wanting to change at an advanced point in their careers; and 2) they were more likely to have a firmly established view of mathematics and mathematics teaching. These particular four veteran teachers were chosen because they represented two very different approaches to implementing reform.

Data Collection. Data used in this study were collected during the three-year program and the initial follow-up year from a wide variety of sources including: an essay application to the program describing experience, professional development, and reasons for wishing to enter the program; reflective writing assignments related to classroom practice; annual extensive reflective final projects; impromptu and planned journal entries; a pre-test and post-test project beliefs survey; and classroom observations followed by interviews. The authors observed and videotaped at least one class period of each participant during annual visits. Immediately following each observation, the participant was interviewed for 30-55 minutes about his or her reflections on the lesson's content, pedagogy, and discourse. The interview questions built upon the participants' previous comments and were of an open-ended nature to encourage further reflection on the teaching that had just taken place. The participants' final projects included personal reflections of their progress during the year, emphasizing both successes and areas for further improvement.

Analysis. A grounded theory approach was used to analyze the data (Strauss & Corbin, 1990). Categories were allowed to emerge based on the data, instead of using the data to support pre-existing hypotheses. The first author read the transcribed data, then coded and analyzed it with the assistance of a qualitative data analysis computer program. Examples of the nodes that emerged include: Pedagogy/Resistance, Pedagogy, Mentoring Philosophy/Personal, Beliefs/Content, Beliefs/Learning and Self-Confidence/Self-Reflection. At various stages of the coding and analysis the authors met and discussed the developing categories.

During the coding process it became evident that these teachers were reflecting many of the characteristics described in Ernest's model of views of mathematics (1989). Ernest identified three views of mathematics commonly held by teachers: Instrumentalist, Platonist, and Problem Solving. An Instrumentalist would be one who sees mathematics as a useful collection of facts, rules, and skills. The Platonist views mathematics as a static unified body of knowledge which is uncovered, not created. Someone who holds the Problem-Solving view sees mathematics as a dynamic body of knowledge that is open for revision and can be created by the learner. In the following, we use Ernest's model as a reference point to investigate shifts in the teachers beliefs. To substantiate our findings, we asked participants during the 1997-98 follow-up interview to select the card that best reflected their beliefs from three cards, each of which contained a description of one of the views identified by Ernest (Edwards, 1996).

Evidence

For the purpose of this study the four veteran teachers are referred to using the pseudonyms Ken, Betty, Judy, and Fred. The following are brief descriptions of the four teachers that include data used to determine the view each veteran teacher held toward mathematics.

Ken is the department chair of a small-town middle-class homogeneous high school. He is extremely active in curricular changes (for all subjects) and sits on many policy making committees. He uses a non-investigatory, traditional-looking text and is pleased with the results of his students' performance on standardized tests. Ken was motivated to enter the program by the anticipation of a new state-wide proficiency test. He stated in his 1994 Application "As 1998 approaches I anticipate our high school curriculum can be modified to effectively prepare our students for success on this important test." In a reflective assignment for December 1996 he stated "In my opinion the understanding of mathematics requires students to demonstrate competency in certain procedural skills." Although in his October 1997 Interview he claimed to have changed his view of mathematics and identified himself with the Problem-Solving view, he remained externally motivated, and his classroom continued to be an environment of limited discourse that showed few signs of deep reform.

Betty teaches in a medium-sized rural town and has played an active role in policy making in the state concerning mathematics education. During her school's last curriculum adoption process, Betty chose a traditional college calculus text and an integrated Geometry text that is traditional in all other aspects. In her 1994 Application under "Benefits of Participation" she claimed, "This program would certainly help us to help make the necessary changes that are reflected both nationally by the NCTM Standards and the statewide Core Curriculum and ... make sure the majority of the ... students [are] assured of obtaining their state endorsed diploma." In her reflective assignment for October 1995 she likened mathematics to an orchestra by saying, "The teacher is in charge of the orchestration, the student hears and feels the music and responds by making conjectures, exploring, etc., and the tools are the sheets of music that the orchestra leader and the musicians read to make beautiful music." During her February 1996 Interview she stated, "Math is not static. It changes all the time." This leads one to believe that her view of mathematics was changing, however her classes were conducted in a manner similar to previous years where the classroom was quiet with students frantically taking notes. Betty readily agreed with the reform movement and aligned herself with the Problem-Solving view, however her changes seemed to be limited to the superficial ones that could be witnessed when passing by the classroom, such as groups of desks instead of rows and the presence of technology.

Ken and Betty "talked the talk," but the changes in their classrooms were superficial. Both displayed self-confidence, participated in leadership roles, conducted their classes with authoritarianism, and emanated a feeling of "control." Even though they verbally encouraged their students to speak out, the students did not appear to voluntarily respond.

During the 1997-98 Interview all four of the teachers studied in this paper chose the card which states: "Mathematics is a continuously expanding field of human inquiry that is not a finished product, but open to revision. Mathematics is seen as created, not discovered." (Ernest's Problem-Solving view). Although none of the four chose the Instrumentalist view as their professed belief, Betty and Ken stated in their interviews that they were most concerned with procedural knowledge and spent much of their class-time on facts, rules, and skills. This, with other data, suggests that they profess to believe the Problem-Solving view, but in practice typify the Instrumentalist view.

Judy teaches a fairly homogeneous group of students using a National Science Foundation (NSF)-funded reform curriculum in a large rural town. Judy's beliefs are best exemplified by comments made during her September 1995 Interview when she reiterated a conversation that had taken place with a colleague, in which she said, "We can do something so that you can at least get started on using it in the classroom to do more than just number crunching. To do it in a discovery type way." In the same interview, she said she was liking her curriculum "... even more because it immerses the kids in the discovery mathematics that I would like to see happening." Her motivation for changing her classroom was alluded to during her October 1997 Interview, when she stated that about 10 years ago she noticed that her students did not seem to be understanding the material when she was lecturing and they weren't retaining information. She seemed to be compelled to change by her personal desire for students to understand mathematics. Judy appeared to not only verbally align herself with the problem solving approach to teaching, but also allowed students to investigate and construct much of their mathematical knowledge in the classroom.

Fred, teaching in a medium-sized urban school district, chose with his colleagues to implement a NSF-funded reform curriculum during the course of the program. In addition, Fred taught a calculus course using a traditional college text. He entered the program with the intent to improve his teaching and learn what was new in mathematics. In his December 1995 reflective assignment, he shared, "This is the very reason I became involved in this program so I could learn how to implement investigations into my teaching." During an October 1997 observation of his calculus classroom, the discussion was driven primarily by the students. Fred was the moderator and would interrupt their discussion only to probe the students when they were not precise or when the conversation began to wane. In the follow-up interview Fred indicated that this day was typical of his current teaching practice. Observations of his classroom at the beginning of the project as well as his early reflections, indicated that this was a significant change from his earlier practice.

Fred seemed to have made the greatest change in both his beliefs about teaching mathematics and his practice. We noticed that he had taken ownership of his and his students' learning and became more intrinsically motivated. He became more reflective and aware of his own thoughts. He reflected on what transpired in the lesson, what he was thinking about while the lesson took place, and where he should go from there. These are qualities that Judy exhibited from the beginning of the study. Both Fred and Judy remained humble, making comments such as "I don't have all of the answers, but I won't give up" [Judy, Reflective Assignment Dec. 1995].

Results and Conclusions

The differences between these two pairs of teachers seem to lie in their motivation for changing their teaching and their perspective on their change. Fred and Judy were motivated more by their students' understanding of the mathematics than their performance on proficiency exams or other external measures of achievement. Fred and Judy's motivation seemed to stem from their beliefs that mathematics is created and that students must take an active role in their own learning. Thus as their understanding of reform grew, their classrooms were altered to allow students to actively participate in investigations and discussions, instead of passively receiving information. In addition, when they reflected upon their change Fred and Judy used the phrases "my change" or "I changed." Fred and Judy's humility and willingness to reflect on their own actions seemed to allow them to truly reflect on the impact their teaching had on their students.

In contrast, Ken and Betty were motivated by their students' test results and proficiency exam objectives. They felt they had changed and implied they had "arrived" and were now looking to help others change. During their reflections, they related their experiences in third-person, generally using phrases like "the teacher" to talk about classroom practices. Ken and Betty's satisfaction with their teaching and external measures of success impeded their ability to look objectively at their classroom practice and seemed to result in superficial changes.

This study supports Kaplan's (1991) hypothesis that deep beliefs are indicative of pervasive classroom behaviors while surface beliefs are connected to superficial practice structures. All four veteran teachers in this study had similar surface beliefs and superficial practice structures, but their deep beliefs and corresponding pervasive classroom behaviors fell into two categories: those reflecting current calls for reform and those reflecting superficial implementation of reform.

It is important to note that all four teachers in this study have made significant contributions to the reform effort, although not all have achieved a classroom practice reflective of deep reform. The teachers' different self-perceptions in relationship to reform provided the greatest impetus or impediment to the change process. Those who saw the reforms as "their" change as opposed to that of the NCTM or some other outside source and understood reform as a process instead of a destination had pervasive classroom behaviors more reflective of current calls for reform. This suggests that the constructivist theory upon which current reforms are based also applies to teacher learning. Moving beyond surface level implementation of reform will require engaging teachers in a long-term process that they claim as their own.

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REFLECTIONS ABOUT LISTENING AND DIALOGUE AS EVIDENCE OF TEACHER CHANGE

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This study examined one teacher's change across the first two years (1995-97) of implementation of cognitively guided instruction (CGI). Interviews were supplemented by written responses to a variety of questions. This teacher began the project as a "teller." In the first year she was a "passive observer and listener." In the second year, she became an "active participant within a community of learners." The jump from "teller" to "active participant" may be supported by intermediate stages during which teachers come to understand various characteristics of "learning communities" while remaining essentially removed from such communities.

This study documented teacher change across the first two years of implementation of cognitively guided instruction (CGI). The evidence came from one teacher's reflections on (a) dialogue among children in her classroom, (b) what she thought was important in children's dialogue, (c) her role in the classroom dialogue, (d) the substance of her conversations with peers about how to implement CGI, and (e) the substance of workshops about CGI. The analysis was part of the evaluation of a five-year teacher enhancement project (NSF Grant ESI-09450518) which focuses on disseminating CGI throughout North Carolina.

CGI is an approach to teaching mathematics in which knowledge of children's thinking is central to instructional decision making. Teachers use research-based knowledge about children's mathematical thinking to help them assess specifics about individual students' thinking and adjust instruction to match students' performance (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). The tenets of CGI fit well within Fosnot's (1996) framework of principles of learning derived from constructivism, briefly summarized below:

> Learning is not the result of development; learning *is* development.
> Disequilibrium facilitates learning.
> Reflective abstraction is the driving force of learning.
> Dialogue within a community engenders further thinking.
> Learning proceeds toward the development of structures. (Fosnot, 1996, 29-30)

The principle that might be most visible in CGI classrooms is the one about "dialogue within a community." Because students in CGI classrooms solve problems and share solutions, there is considerable conversation about mathematics. Tracking teachers' views about these conversations, as well as their conversations with peers about implementing CGI, might reveal important elements of teachers' change; namely, what they decide is important to hear and the importance of listening as a factor in their own growth.

Listening is an important part of dialogue and is a critical part of the process for understanding children's thinking. Within implementation of CGI, teachers have special responsibilities for listening. Early on, teachers tend to listen to children's explanations with the expectation that they (the teachers) are supposed to understand children's thinking simply on the basis of what the children say; in this paper we refer to this type of listening as "passive listening." In contrast, teachers who are especially effective at implementing CGI seem to understand that making sense of children's thinking is a joint responsibility; both the teacher and the children must contribute to generating "shared knowledge" about that thinking. These teachers often engage children in conversation about solution strategies so that both the teacher and the children come to understand how what is being said reflects real thinking; we refer to this type of listening as "active listening." A teacher's development of listening skills is consistent with allowing students to develop personal meaning. Too, listening takes time, so when a teacher chooses to take time to listen, children have time to think and develop meaning.

Similarly, as teachers interact with each other about implementing CGI, they might listen in more passive or less passive ways. They might simply allow each other to describe what is going on in their classrooms, or they might probe each other to develop shared meaning for the significance of classroom events. As teachers begin to understand the importance of having children develop shared knowledge about mathematics, they might also begin to understand the importance of developing shared knowledge with colleagues about the teaching of mathematics.

Method

Project. The 5-year CGI project (calendar years 1995-1999) is helping five teams (originally, 2 teacher educators and 6 classroom teachers on each team) learn to use CGI as a basis of mathematics instruction. In the first two years of the project, four workshops were held: May 1995 (3 days), July 1995 (10 days), June 1996 (8 days), and July 1997 (7 days). Between workshops, the teachers implemented CGI in mathematics instruction, each team met about once a month to continue learning about CGI and to discuss their progress, and each teacher was visited about once a month during mathematics instruction by one of the team's teacher educator members. Project staff visited each teacher during mathematics instruction once each semester to provide general support and encouragement.

Instrumentation. The primary source of data was interviews, lasting from 30-50 minutes each, conducted with one subject, Mrs. S, during each of the three summer workshops. The interviews were tape recorded and transcribed. Interviews were loosely structured around questions written by the authors; follow-up probes varied across the interviews, depending on Mrs. S's particular responses. Mrs. S's responses to written instruments were also examined: Transcript Analysis and Seven Questions (Bowman, Bright, & Vacc, 1997) and several "reflections" to particular questions posed by project staff.

Subject. At the beginning of the project, Mrs. S was a first-grade teacher with 20 years of teaching experience. Her principal was very supportive of having teachers learn to use CGI.

Analysis. Data were grouped into three periods. "Pre-CGI" designates Mrs. S's descriptions of her teaching at the start of the project, "early CGI" designates her descriptions of her teaching between May 1995 and June 1996, and "later CGI" designates her descriptions of her teaching between June 1996 and July 1997. (The project is on-going, so additional data will be gathered during Mrs. S's third and fourth year of implementation of CGI.)

Results and Discussion

Dialogue Among Children

Pre-CGI views. Mrs. S explained that establishing rapport with her students had always been important to her. Her comments suggested, however, that she had not asked students to share much about their solutions to mathematics problems. She explained that prior to CGI she "stood in front of the class and told them this was the way to do it" (Interview, July 13, 1995).

> *I've always encouraged my students to talk to me about things. I have always been aware of how close I was to my students; I never wanted to be the teacher that they were afraid to talk to or afraid to tell me anything.... [B]ecause I had that relationship with them anyway, maybe that's why they were free to share their solutions to problems that very first day [of trying out CGI] (Interview, July 13, 1995)*

In contrast, in response to the questions, "What do you look for as you watch children solve mathematics problems?" and "Why are those things important?", she mentioned "students working together" and "students talking through problem (to self or partner)" (Seven Questions, Spring 1995). She did not specify what the important substance of such "talking" might be.

Early CGI views. Classroom dialogue appeared to have been mainly between students and Mrs. S rather than among students. She seemed to treat each child as a separate "entity" for her to analyze and appeared not to expect that students would understand each other's thinking.

> *They knew that they could tell me anything about how they had solved this particular problem and I was not going to say, "No, that's the wrong answer." So, they felt free to say what they needed to say. (Interview, July 13, 1995)*

Mrs. S allowed children to share their solutions, but the children essentially took over the role of telling their classmates about solutions. In some sense, they took over her earlier role of standing in front of the class and telling the children about solutions.

Mrs. S seemed to begin thinking differently about her role, and to some extent the children's roles, as listeners. However, Mrs. S was not clear about what she was listening for, and she did not acknowledge that interaction with children is necessary in order for a teacher to understand clearly what children are thinking.

> *I believe that primary-age students need to ...*
> *— learn to explain to others their thinking as they solve problems*
> *— learn to listen to and respect other's thinking and solutions*
> *I ... should provide situations in which the above can happen. I also must observe their problem-solving and listen to their thinking to know what direction to move in next. (Seven Questions, Spring 1996)*

Mrs. S learned that children's initial attempts both at solving problems and at explaining those solutions were sometimes not reflective of children's full understanding. She learned to allow children the opportunity to complete an explanation without intervening and imposing her interpretation on the children's explanations, but there is little evidence that shared understanding was one of her goals. Rather, sharing by children almost appeared to have been "sharing for the sake of sharing." Mrs. S seemed to be a passive learner in early implementation.

Later CGI views. Mrs. S seemed to begin to expect children to take a more active role during instruction and to focus more on the learning of mathematics by allowing time for students to explain and negotiate meanings. When students made mistakes, time was allowed for them to learn from their mistakes. That is, Mrs. S seemed to be a more active listener.

> *My conversation with them is completely different now. It is almost like we are equal. And I am still in charge but I can learn from you just like I want you to learn things from me. And I talk to them differently than how I used to. (Interview, July 28, 1997)*

Mrs. S seemed to have put herself back into the classroom as an active participant in developing shared meaning and in learning from mistakes.

Not surprisingly, Mrs. S also developed deeper understanding of what was important to listen for as children explained their solutions., and she acknowledged the importance of children learning from each other.

> *So what if I give them the wrong problem type. It's not going to hurt that child.... I can make mistakes just like the children can, and I am learning because I am finding my own path. (Interview, July 28, 1997)*

Dialogue Among Peers

Early CGI views. Mrs. S provided no information about her interactions with peers in her pre-CGI teaching. Early in implementing CGI, Mrs. S indicated that she and the other CGI teacher (also first-grade) in her building shared problems to pose to children and ways that children responded to those problems. Her comments do not indicate whether they reflected on their conversation as a means of developing shared knowledge of CGI. Indeed, her language suggests that they tried to give ideas to each other rather than developing shared knowledge.

> *As we were writing the problems ... my partner and I began to talk about how we could start with this problem ... and then ... what would be a good problem to do next.... We've shared problems ... and helped with ... [classroom management] solutions for each other.... I guess we fed off each others' enthusiasm ... sharing what's going on — giving each other ideas. (Interview, July 13, 1995)*

Mrs. S seemed to have similar types of interactions with other teachers on her team and in the project. There was little sense of "deep" conversation to develop shared knowledge. Too, the importance of the visits by project staff to Mrs. S's class seemed to be in what the project staff "gave" to her rather than in developing shared knowledge.

Later CGI views. Although the notion of teachers' sharing (i.e., telling) information about their CGI implementation continued to be important to Mrs. S, she began to acknowledge the role of discussion as well as questions and answers during adult dialogue.

> *I think that the discussions we have had ...this year were more helpful than a lot of the [other] things.... We have a lot of successes and failures and questions and answers when we [the team] get together. (Interview, July 28, 1997)*

Conclusions and Implications

Mrs. S's discussion of listening and dialogue in her classroom suggests that there may be "phases" through which teachers pass as they are implementing CGI. Especially critical may be the phase of "passive learner and objective observer" that seemed to characterize Mrs. S's teaching during the first year of implementation. The jump from "teacher as teller" to "teacher as active participant in a community of learners" may be too great for many teachers to make directly. Rather, there are intermediate behaviors and beliefs that may have to serve as a bridge.

Mrs. S's discussion of listening and dialogue with peers suggests that it may be difficult for teachers to get beyond the level of "You tell me what's happening in your classroom and I'll tell you what is happening in my classroom." Although teachers see the need for having children explain and justify their solutions to mathematics problems, teachers do not so readily acknowledge the need for teachers to explain and justify to each other their solutions to pedagogical problems. Perhaps teachers do not see classroom challenges as problems, but rather only as events, so description rather than analysis is all that is needed.

It is not unreasonable to expect that implementation of any "reform mathematics teaching" will not happen without similar kinds of "bridges" between current practice and the reform practice. These "bridge phases" are clearly not the goal of reform, but they may be appropriate intermediate steps. Helping teachers recognize that they are passing through the bridge phases might help relieve some of the tensions that they feel about not immediately implementing reform mathematics teaching. Understanding these bridges also may help researchers and reformers refine appropriate evaluation techniques for determining the success of inservice programs

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SHIFTING BELIEFS: PRESERVICE TEACHERS' REFLECTIONS ON ASSESSING STUDENTS' MATHEMATICAL IDEAS

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The purpose of this study was to investigate the use of student assessment interviews as a means of shifting preservice teachers' beliefs about mathematics assessment. A total of 37 preservice elementary and secondary teachers participated in a written reflections assignment, follow-up surveys, and semi-structured interviews. Results indicate that after preservice teachers conducted assessment interviews, they gained a greater understanding of the diversity and depth of students' mathematical thinking and problem solving strategies, and they saw the interview technique as a feasible part of their mathematics assessment options.

Introduction

In response to the national interest in alternative forms of mathematics assessment, the educational community is calling for new approaches to assessment that evaluate students' conceptual understandings (Resnick & Resnick, 1992; Wiggins, 1989). The purpose of this study was to investigate the use of student assessment interviews as a way of encouraging preservice teachers to examine their beliefs about how students communicate mathematical ideas, and to examine the effectiveness of this technique in shifting preservice teachers' beliefs about mathematics assessment. Research (Owens, 1993) reports that teachers often teach and assess students the way they were taught and assessed. To break this cycle, preservice teachers need to be engaged in experiences that encourage them to reflect on their growing knowledge of how students think mathematically.

In light of these current reform trends, this study examined the following questions: (1)What are preservice teachers' beliefs about students' mathematical thinking, (2)What are preservice teachers' beliefs about methods of alternative assessment in mathematics, and (3)What is the impact of the student assessment interview technique on preservice teachers' beliefs about alternative assessment strategies?

Theoretical Perspectives

The American educational system has embraced the notion of standardized testing. Research has shown that testing shapes instruction and that "test-defined instruction has the effect of driving out good teaching" (McNeil, 1998). Because assessment is such an integral part of instruction (Cooney, Badger, & Wilson, 1993), it is imperative that assessment strategies reflect the current vision of mathematics (NCTM, 1995) that allows educators to conceptualize the full range of students' mathematical thinking.

Recent reform efforts in mathematics education (NCTM, 1995) suggest a broader base of strategies for assessing students' mathematical thinking. Yet many preservice teachers' beliefs about mathematics reflect a linear model of what it means to know and do mathematics. Some believe that mathematical activity consists primarily of identifying a correct answer to a well-defined problem, following a fixed set of rules and procedures, and recalling and applying learned algorithms to solve a problem (Borasi, 1990). How teachers think about mathematical learning is how they assess mathematical learning. As long as preservice teachers view mathematics as a set of rules and procedures to be memorized, moves toward the use of alternative assessments will be seen as irrelevant to their instructional methods.

The use of assessment interviews as a part of preservice teachers' experiences is a commonly utilized technique that allows them to make inferences about students' mathematical knowledge and reflect on the role of assessment in instruction. This paper examines the effectiveness of this technique for shifting preservice teachers beliefs about mathematics assessment strategies.

Methodology

This study occurred during one full academic year, August 1997 to May 1998. Participants were preservice teachers enrolled in elementary or secondary mathematics methods courses during the fall semester 1997, and in full-time student teaching internship placements during the spring semester 1998. A total of 37 preservice teachers conducted student assessment interviews and completed a written reflections assignment and a follow-up survey. A subset of 10 preservice teachers, 5 elementary and 5 secondary, were selected randomly to be interviewed.

The study used three sources of data: a written reflections assignment completed by preservice teachers immediately following the student assessment interview (fall 1997), a follow-up survey of the interview process completed at the end of the semester (fall 1997), and semi-structured interviews with a subset of the participants during their student teaching internship (spring 1998).

The written reflections assignment required preservice teachers to: (a) research an appropriate mathematics topic; (b) review information on assessment and questioning; (c) develop appropriate tasks; and (d) evaluate the effectiveness of the assessment interview strategy (Huinker, 1993; Stenmark, 1991). Written reflections assignments were used to identify preservice teachers beliefs about students' mathematical thinking and about the interview technique as an assessment tool.

Follow-up surveys were used at the close of the semester to gather evidence of preservice teachers' beliefs and opinions about the assessment interview as an alternative form of assessment.

An interview protocol was developed by using constant comparative analysis of the written reflections assignments and follow-up surveys (Strauss, 1987). In the spring of 1998, semi-structured, 45 minute audio-taped interviews were conducted with 10 preservice teachers. Interviews were fully transcribed and used to identify beliefs about how students learn mathematics and shifts in beliefs about assessment strategies.

These data sources were analyzed using analytic induction, a method of analysis that involves scanning the data for themes and relationships, developing hypotheses, and modifying them on the basis of the data (LeCompte, Preissle, & Tesch, 1993). Initial data analysis consisted of coding the interview transcripts for issues related to understanding students' mathematical thinking and evidence of shifts in beliefs about assessment practices.

Results and Discussion

Themes that cut across the three data sources included: observations that the assessment interview technique was different from anything preservice teachers had experienced as students, reports that the assessment interview gave preservice teachers greater understanding of students thinking, and indications of shifting notions of mathematics assessment practices.

Understanding Student Thinking. Surveys indicated that although almost 100% of the elementary preservice teachers were familiar with only paper-and-pencil tests, they believed that this method of assessment did not show a student's full range of mathematical thinking. Their surveys indicated the following insights: (a) students have more complex mathematical ideas than they are able to express in whole-group settings, (b) students need many varied and continuous opportunities to demonstrate what they know and understand, and (c) students are unique in their constructions of mathematical knowledge. One student wrote,

> *Students minds work differently so they need to be assessed differently. It is important for each child to have a real chance to demonstrate their knowledge. What they do on paper may not reflect what they do in other situations or what they really know and understand.*

Written reflections assignments indicated that elementary preservice teachers observed students using manipulatives and drawings as tools for understanding. A common theme was the importance of allowing students to see and manipulate concrete objects during both the instruction and assessment process. As students talked aloud during interviews, preservice teachers gained insight into the depth and variety of student thinking, and they felt they were truly able to understand the strategies students employed. One preservice teacher stated,

> She [the student] amazed me in her verbal thought processes. She understood exactly how to tell me what she was thinking. And that amazed me the most. And sometimes I wouldn't understand but she would show me and say, 'Look, this is exactly how I am doing this. This is how I got this answer.' I think I learned that her thought processes were a lot more detailed than I ever imagined.

A majority of the preservice teachers wrote similar comments,

> *In doing this interview, I learned that each student's mathematical strategies are different. Students learn differently and teachers need to assess each student in the best way for that individual student in order for greater understanding.*

Shifts in Beliefs and Practice. Elementary preservice teachers reported that they would use the assessment interview or a modified version of it, citing that it would be worthwhile in making instructional decisions and reporting specific progress to parents. Others reported it to be a highly favorable technique and expressed interest in using other forms of authentic assessments, explaining, "paper-and-pencil tests just aren't adequate forms of assessment for young children." A few of the preservice teachers commented that although the interview was useful, it may be difficult to find time to conduct in a classroom of 25 elementary children.

Interviews with elementary preservice teachers indicated that they were trying to adapt the interview technique as an assessment strategy in their placement experience. Some of them interviewed small groups of three or four students, while others conducted a short 5 to 10 minute interview with a child during seatwork or center time. Many reported that they had never thought about using an oral mathematics assessment until they conducted the interview. One elementary preservice teacher said, "You can see that they understand a concept as they talk about it," and another remarked, "I would have never thought to give an oral assessment for math ever. If anything, you've changed one person!"

All of the elementary preservice teachers interviewed reported that the interview technique was an assessment strategy they would use in some form in their own classrooms. They believed different types of assessments given with greater frequency would give them a more complete picture of students' mathematical knowledge. Their comments indicated that they began experiencing shifts in beliefs about assessment during the project. Two elementary preservice teachers reported,

> Before this [interview] I thought I could tell exactly what was going on in their heads just by looking at their work. Now I know it has so much more to do than just that, because their thought pro-

> cesses are a lot different than what they put down on paper for a test.
>
> I've definitely realized that I cannot do paper-pencil tests to understand exactly what the child is thinking. It needs to be more performance assessment.

Another preservice teacher summarized her experience as she defined assessment,

> I learned so much about one student in that time. And I think that if I could do that with each child, I think the children would learn more in math throughout the year. And I would know exactly what they needed help with. That is what an assessment should be.

Understanding Student Thinking. The secondary preservice teachers were challenged to make conjectures about students' mathematical thinking. On their surveys they reported that students had very weak problem-solving skills and struggled with solving non-routine problems. One preservice teacher said that the two students he interviewed "did not know how to organize the information they were given. They did not know how to get started attacking the problems." Students' inability to problem-solve was echoed by other preservice teachers. They reported that students seemed to think of mathematics as a concrete, cut and dry discipline. Moreover, they view mathematics as a set of rules and formulas, tend to follow steps they have memorized, and see only one way to work or solve problems. One preservice teacher noted,

> I don't think many students do think mathematically. They tend to compartmentalize knowledge. Most of the students I encountered wanted to memorize facts rather than take information, put it together, and make an assertion. That is what mathematical thinking is to me. (Reflection assignment)

During interviews, preservice teachers were pressed further to come to grips with how the students they interviewed think mathematically. They seemed to hone in on making conjectures about how students learn mathematics, reporting that students tend to learn mathematics by visualization. Although they complained that students seemed to be poor problem-solvers, they reported that when faced with the unfamiliar, the students drew pictures to help them solve problems. They believed this indicated that students were visual learners and understanding mathematics concepts was contingent upon visualizing them.

Shifts in Beliefs and Practice. During interviews, preservice teachers reported that although they were assessed in high school by only paper-and-pencil techniques, they now think differently about mathematics assessment. They reported that the assessment interview differed from the way they were assessed as students because it allowed them to "probe." Probing meant that they had more flexibility to ask students "why" and it allowed them to better understand students' thought processes. The preservice teachers believed that this "probing" would be lost through traditional paper-and-pencil tests. They further stated that the assessment interview helped them assess whether or not students understood particular concepts, and it pressed students to explain their thinking and problem solving strategies. This notion of "explaining" could not be easily demonstrated on paper-and-pencil tests. One preservice teacher stated,

> Students would gain a fuller understanding of mathematics if they were required to explain why they solve problems using certain methods. This would require them to understand the methods as well as when to apply them. Student thinking must be taken beyond simply finding an answer to being able to defend the way they found it.

Preservice teachers' ideas about the usefulness of this strategy seemed to be couched in the notion of "teachers understanding students' understanding." One preservice teacher said,

> This [assessment interview] helped me realize that as a teacher, you need to understand where your kids are coming from, how they are thinking, how they are trying to solve problems, and maybe just taking a test, grading it, and just sending it back is just not going to give that level of detail of understanding. It's better to listen to them and to watch them than just to grade a test and then hand it back.

The preservice teachers reported that they would use interviews in their own teaching. However, they struggled with the notion of "grading" and reported that it would be challenging for them to assign a numeric value to students' responses during interviews, suggesting that the technique was not easily quantifiable. Although they said they thought of mathematics assessment differently because of the assessment interview, they reported that it would be hard to implement in a typical mathematics classroom of 25 to 30 students. They indicated that they would use the interview technique to identifying particularly "needy" students who were struggling in mathematics to gain a better understanding of their thinking and to help them become successful.

Conclusions and Implications

Preservice teachers' comments indicate that the assessment interview technique does encourage them to question beliefs about mathematics assessment. The elementary preservice teachers viewed the interview as one of the many varied and continuous forms of assessment they would use as classroom teachers. Secondary preservice teachers saw the assessment interview as a mathematical tool for use, particularly, with "needy" students struggling in mathematics. Most of the preservice teachers reported that interviewing students gave them greater insight into the depth of understanding students attained in mathematical topics.

The implications of these results are that preservice teachers' beliefs about mathematics assessment need to be changed before they enter the classroom. Experiences that significantly challenge their beliefs about assessment while they are undergraduates have the potential to encourage them to utilize meaningful mathematics assessment techniques as classroom teachers. Although these results are encouraging, the process of implementation may hinder their efforts to use alternative assessments in their first years as classroom teachers. Follow-up on these teachers as they begin their careers will provide helpful information on the extent to which these assessment techniques have lasting effects.

UNDERSTANDING HOW PROSPECTIVE SECONDARY TEACHERS AVOID ACCOMMODATING THEIR EXISTING BELIEF SYSTEMS

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The results presented in this paper identify some mechanisms involved in the production of a well-known phenomenon, namely that many prospective teachers tend to assimilate new experiences into their traditional belief systems rather than substantively change their existing conceptions. A prospective secondary mathematics teacher was able to remove the potentially perturbatory effects of class activities designed to challenge his procedural view of mathematics through the compensation moves of seeing meaning in his procedures and proceduralizing the new meaning that he developed in class.

Background and Purpose

There is a growing awareness of the role that teachers' beliefs play in the implementation of reforms (Gregg, 1995; Smith, 1996). Consequently, interventions have been designed to induce changes in beliefs concerning mathematics and its teaching and learning. While researchers have reported some success with efforts to change the beliefs of prospective teachers (Cooney, Shealy, & Arvold, 1998), these same researchers and others (cited in Thompson, 1992) note that many prospective teachers appear to assimilate new experiences and ideas to fit their original belief systems rather than accommodate their existing schemas. Thompson (1992), in her review of the literature on teachers' beliefs, recognized the need for further research, stating that previous studies "have not provided the detailed analyses necessary to shed light on the question of why it is so difficult for teachers to accommodate their schemas and internalize new ideas (p. 140)."

The purpose of this study is to carefully examine the moves that prospective secondary mathematics teachers (PSTs) make to avoid restructuring their existing belief systems while they are enrolled in a mathematics course designed specifically to induce such changes. However, the case study presented in this paper is not simply a story of resistance to change. The subject engages with the class content and forms ideas that are coherent to him and that appear to be influenced by the course content. It is our goal to account for the subject's emerging ideas.

Theoretical Perspective and Instructional Approach

The resolution of perturbations plays a central role in constructivist approaches to learning and in recent research on changing teachers' beliefs (Cooney et al., 1998; Piaget, 1975; Shaw & Jakubowski, 1991). We organized instructional experiences for this study around the notion of presenting challenges to PSTs' traditional belief systems (as summarized in Smith, 1996), in an effort to help PSTs develop beliefs that are more consistent with recommended changes in mathematics education (e.g., as described in Clarke, 1997). This contrasts with teacher education approaches that present reform ideas without first showing the limitations of a traditional approach.

We used mathematical *meaning* as an organizing construct to create potentially perturbatory experiences. Many research studies indicate that students who can accurately perform mathematical procedures are often unable to demonstrate an understanding of underlying concepts (e.g., Hiebert & Carpenter, 1992, or NAEP results reported in Lindquist, 1989). By helping students see gaps in their own knowledge and by having students react to NAEP reports, we hoped to show PSTs the limitations of a belief in mathematics as a set of procedures. For example, we asked students to demonstrate the meaning of division of fractions by drawing a picture and by creating a real world problem, in an effort to expose common gaps in PSTs' understanding (as previously identified by Ball, 1990)[1]. We hoped that this experience and others like it would create conflict between the PSTs' beliefs about mathematics and their view of themselves as mathematical experts.

Methods

Eight PSTs from a class of 26 volunteered to participate in the study. This paper presents the findings for one of the subjects, who we will call Antonio. Since the class was taught by one the authors, the interviews were conducted by a graduate student, and the data were not analyzed until after grades were submitted. Each subject was interviewed six times throughout the semester, in videotaped sessions outside of class, each lasting about 30 minutes. The interviews were designed to help students feel comfortable voicing ideas that they might not express in class (e.g., for fear that the ideas sounded "too traditional"). Interview tasks relied heavily on "case-like" situations that encouraged subjects to project themselves into a vision of their future classrooms—a requisite condition for change in teachers' beliefs according to Shaw and Jakubowski (1991). Documents generated by students for class, including homework, were also collected. The qualitative analyses of the data followed interpretive techniques, including the search for disconfirming evidence (Strauss, 1987). Additionally, we adopted Simon and Tzur's (in press) methodology of "explaining the teacher's perspective from the researchers' perspective" in an attempt to capture the coherence and sensibility of the PSTs' emerging ideas.

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[1] We are using *meaning* here to indicate a quantitative understanding of the quotitive interpretation of division of whole numbers as extended to fractions.

Results and Discussion

Beliefs and subject matter knowledge prior to the start of class instruction. Antonio entered the course with fairly traditional beliefs about mathematics and teaching, as demonstrated in his responses on the first homework assignment (collected before instruction began). For example, he outlined a step-by-step demonstration of the invert and multiply (IM) rule as his method for teaching division of fractions. However, he exhibited some concern for understanding. He was the only student in the class who attempted to explain why the IM rule worked. (His argument involved showing that $\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \text{ x } \frac{2}{1}$ since $\frac{3}{4} \div \frac{1}{2} = \frac{\frac{3}{4}}{\frac{1}{2}} \text{ x } \frac{\frac{2}{1}}{\frac{2}{1}} = \frac{3}{4} \text{ x } \frac{2}{1}$) Furthermore, Antonio writes, " I hope my students will realize that conceptual understanding is as important as getting a solution of a given problem through a series of steps" (Homework 1). Despite the fact that he appears to value understanding, Antonio's performance on a quiz during the first day of instruction provides evidence that his own understanding of mathematics was limited. He was unable to draw a picture to illustra $3 \div \frac{1}{2}$ and to generate a real world problem that could be solved by finding $\frac{3}{4} \div \frac{1}{8}$. Thus, we thought that if we introduced Antonio to the meaning of division of fractions, he would value this for his own understanding of mathematics and his teaching of mathematics.

Move #1: Meaning can be found in procedures. Although Antonio was unable to draw a picture of $3 \div \frac{1}{2}$ on the first quiz, his response on the second homework assignment indicated that he had come to realize that $3 \div \frac{1}{2}$ means how many one-halves are in three wholes. We concluded that we had introduced him to the meaning of division of fractions. Thus, we were surprised when Antonio was given a list of a dozen instructional goals for division of fractions in the first interview, and he did not select the item addressing this meaning (i.e., "know how to interpret a division equation, e.g., $3 \div \frac{1}{2}$ means how many one-halves are in three wholes"), even when he was told to imagine that he had as much time to teach division of fractions as he felt was necessary.

Antonio's comments later in the interview indicate that he had not accidentally overlooked the quotitive interpretation of division of fractions. He felt that traditional instruction had been wrongly discounted in class, and that the class was overlooking the meaning embedded in the procedures themselves. For example, he said, "I think . . . we don't concentrate enough on how much, on the conceptual aspect of just the algebraic steps" (Interview 1). We wondered what meaning Antonio saw in the procedures. His description in the second interview of how he would teach division of fractions provided some clues.

Like I said, I would teach the invert and multiply [rule]. Boom, here's the answer. Then I would go back and say, OK, now let's look at the answer. Now we can see that the answer multiplied by this fraction gives you the initial fraction. So you can see when you multiply these together you get this answer. But if you divide this fraction by this fraction, you get this answer. So you get some kind of, you get a relationship.

Antonio repeatedly offers the relationship between division and multiplication as the reason why the IM rule "works"—it works because he can check the answer using the procedure for multiplication of fractions and because he can use multiplication along with the multiplicative identity of one to produce a justification for the IM rule. It appears that Antonio has constructed some conceptual knowledge involving the inverse relationship between multiplication and division (even though this understanding may be best described as a logically consistent set of interrelated procedures and may lack some important connections). The fact that Antonio sees meaning in the procedures allows him to discount the instructor's claims that traditional instruction lacks meaning.

Move #2: Procedures can be found in the meaning. It is clear from the first interview that initially Antonio was unable to extend the quotitive interpretation of division of whole numbers to difficult fraction problems. For example, when asked how he would show $\frac{1}{4} + \frac{2}{3}$ Antonio was clearly confused and used diagrams inappropriately (Interview 1). By the second interview, however, Antonio appears to have developed proficiency with solving division problems using diagrams. Nevertheless, it was in this interview that we began to see that his view of the meaning of division of fractions differed significantly from ours. In particular, it seemed that Antonio came to view what we saw as the meaning of division of fractions as just another procedure for obtaining answers to division of fractions problems (i.e., through drawing pictures based on the quotitive interpretation for division of fractions). When asked whether or not he thought students should be taught the IM rule, Antonio indicated that he conceived of both approaches as rules.

> I think both rules ought to be taught. Some people concentrate on just one. Other, other teachers just concentrate on the conceptual, uh, approach...But if you actually ask them to, on the test, do this one *by conceptual* without the invert and multiply, and do this one with the invert and multiply, then you would be getting both ideas

> across. So I think she should have at least shown it, taught it, because why not? If it's another approach, why not teach it? I mean, isn't that what school's about? Teaching different approaches? (Interview 2)

His strong orientation toward finding answers keeps him from seeing the power of the quotitive meaning of division as a foundation for reasoning and as a basis for an explanation of the IM rule that transcends giving rules to justify rules. Instead, Antonio seems to assimilate the quotitive meaning for division of fractions as a new procedure (see Figure 1). His understanding appears to consist of three separate parts. Antonio repeatedly demonstrates his ability to use pictures and real world objects to establish the meaning of the dividend and the divisor. Additionally, he thinks that interpreting the quotient is important. For example, in Interview 6, Antonio explains that if students find an answer of 3/4, then he wants them to understand what 3/4 means, e.g., " Is it 3/4 of 1, or is it 3/4 of 2, and if it's of one, one what? One dollar? One basket of apples?" However, when Antonio uses the IM rule or his "conceptual way" to find quotients, the numbers are no longer adequately connected to the meanings he has developed for them.

Does Antonio Change? Despite the fact that Antonio seems to have removed the perturbatory effect of initial class experiences, he does appear to be in conflict in Interview 1, perhaps because "Dr. Lobato thinks teaching conceptually is the best approach....and I know that she's probably right," i.e. he feels pressure to incorporate "this conceptual stuff" into his views in some way. This struggle is highlighted in his attempts to fit the "conceptual way" of teaching division of fractions into his vision of his own teaching. In Interviews 1 and 2, he resolved this conflict by keeping his initial vision relatively intact, i.e., by presenting the IM rule first, and then teaching the "conceptual way" at the end as another (and somewhat redundant) strategy for solving division of fractions problems. However, in the final interview,

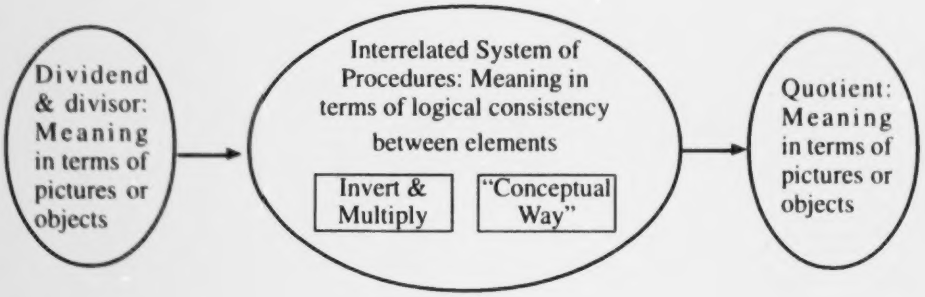
Dividend & divisor: Meaning in terms of pictures or objects
Interrelated System of Procedures: Meaning in terms of logical consistency between elements
Invert & Multiply
"Conceptual Way"
Quotient: Meaning in terms of pictures or objects

Figure 1.

Antonio strongly insists that the conceptual approach should be taught first. He contends that pictures (something that was not part of his initial vision for teaching division of fractions) are now *necessary* for understanding the IM rule, "because you won't see why this [the IM rule] works unless you tie it into what I've done here [referring to pictures]." His insistence that pictures are an important part of finding meaning in the IM rule seems to suggest that his values and beliefs about what constitutes meaning in mathematics are broadening. Thus, it is possible that conceptual understanding is starting to become constituted as something other than a part of the interrelated system of multiplication and division procedures.

Concluding Remarks

This study provides an initial step toward understanding the specific moves that PSTs make to avoid substantive reorganization of their belief systems. We expected that a mathematics major would be disturbed by the fact that he was unable to demonstrate the meaning of division of fractions, and that he would question his procedural view of mathematics as a result of this experience. However, Antonio did not view his initial lack of understanding as a threat to his view of mathematics because he saw meaning in his procedures. Furthermore, once he understood how to draw pictures, he assimilated this into his current structures and saw it as another way with to produce answers. While his belief system remained fairly stable throughout the semester, he appeared to make some small steps toward change, steps that appeared to be motivated by more general perturbations in class caused by the instructor's ongoing messages regarding the importance of conceptual understanding.

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SELF-ESTIMATING TEACHERS' VIEWS ON MATHEMATICS TEACHING - MODIFYING DIONNE'S APPROACH

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The following report has originated in a joint, extensive investigation with E. Pehkonen (Helsinki) on investigating teachers' conceptions on teaching mathematics. However, in the following presentation we restrict to focus on one aspect of the research, namely different *self-estimations* of the teachers on their views on mathematics teaching and their representations. It was Dionne (84) who proposed some numerical self-estimation of personal views on mathematics by distributing 30 points to three components. This rough adhoc-approach, however, vividly visualizes differences in the personal constructs on mathematics. Here, we present some methodological modifications leading to a more detailed description of underlying teachers' views. Our test subjects amounted to a total of 6 experienced German teachers. The research was conducted during the spring and summer of 1994.

The literature contains controversial and abundant contributions concerning teachers' beliefs as well as the process by which teachers change in regard to their professionalism. Marginal attention is dedicated, however, to the corresponding methodological questions concerning the research of these beliefs (Grigutsch 1994).

(see http://www.uniduisburg.de/FB11/PROJECTS/MAVI.html)

It was Dionne (1984) who proposed some numerical self-estimation on personal views on mathematics by distributing 30 points to three components. In this research note, we present some methodological modifications leading to a more detailed description of underlying teachers' views. Originally, our test subjects amounted to a total of 13 experienced German teachers, five of whom teach in the *Gymnasium*, two in the *Realschule*, one in the *Hauptschule*, and five in the *Gesamtschule*. Here, we will limit ourselves to the researching of six people.

1. Theoretical Frame - What are Beliefs?

The concept of belief has many definitions in the literature (e.g. Thompson 1992, Pehkonen 1994, 1995). Here, we understand *beliefs* as one's stable subjective knowledge (which also includes his feelings) of a certain object or concern to which undisputable ground may not always be found in objective considerations.

As stated elsewhere, an individual's beliefs form a structure: We will call this structure of beliefs, or belief system, his/her *view of mathematics*.

This wide spectrum of beliefs (and conceptions) contains, among others, four main components: (1) beliefs about mathematics, (2) beliefs about oneself as a user of mathematics, (3) beliefs about teaching mathematics, and (4) beliefs about learning mathematics. These main groups of beliefs can be, in turn, split into smaller pieces. However, generally it is not easy to differentiate between the beliefs on mathematics and teaching resp. learning of mathematics. This is particularly true if there is no 'world outside of school mathematics' for the test subject. Along these lines, the term 'mathematical world view' which originated from Schoenfeld (1985), was recently used by Törner & Grigutsch (1994), and elaborated in a newer paper by Grigutsch; Raatz & Törner (1998).

1.1. Different Views of Mathematics

For his research, Dionne (1984) used the following three perspectives of mathematics: (A) Mathematics is seen as a set of skills (*traditional* perspective); (B) Mathematics is seen as logic and rigor (*formalist* perspective); (C) Mathematics is seen as a constructive process (*constructivist* perspective). In his book, Ernest (1991b) describes analogously three views of mathematics: *instrumentist, platonist* and *problem solving*. These correspond more or less to the three perspectives Dionne (1984) mentioned above. Furthermore, the same three-component model was rediscovered ten years later by Törner & Grigutsch (1994) calling (A) the *toolbox aspect*, (B) the *system aspect* and (C) the *process aspect*. Recent research shows that there are at least four basic components in an individual's view of mathematics: the fourth component seems to be application (Grigutsch, Raatz & Törner 1998).

1.2. The German Educational System

For sake of shortness of this report it is not possible to provide the reader with the framework of the underlying German education system. The reader is refered to Robitaille (1997).

2. The Basic Ideas of this Research

The teachers we interviewed in the study came from each type of school present in the German educational system. This was cause to take the simpliest possible approach as a plane of projection of their views. As a result, we asked the participating teachers in one of the inquiries to share with us their self-assessment with regard to both the actual and the intended teaching methods of mathematics and to employ the catagories of Dionne. It is obvious that the tendency between their vision and the reality of their accomplishments contain pertinent information. With this statement the raised beliefs of teaching mathematics can be understood in regard to the type of data as three dimensional vectors through which the entries as weights stand for the different dimensions in which teaching mathematics is presented. Thus, the question arises how to visualize these vectors.

In this paper we shall introduce a new pictorial method of self-estimation to serve as a supplement to the numerical Dionne-data. Although these two methods, presenting data in a tabular and representing them graphically, are mathematically equivalent, there are not redundant in a *methodological* sense. With respect to the considerations above, our main research question reads: *How well do information from these two different methodological sources investigating teachers' views of mathematics fit together?* where we will mainly restrict only to the informations derived from the self-assessments. It will be reported in (Pehkonen & Törner 98) how valid information will be received with the method of Dionne compared also with the rich data sources through the questionnaire and interview method.

3. The Realization of the Research

Since in our earlier attempts with the same methods there was some confusion with respect to the view on mathematics under consideration, we let the teacher explicitly *distinguish* between his/her *real* and *ideal* view of mathematics. So, not only the self-estimation with respect to the categories mentioned above were of interest for us, but also we paid much attention the pattern of transition in the self-estimations with respect to their real and ideal view respectively.

We asked the teachers to share 30 points between their three real and ideal perspectives of mathematics written it in a tabular form. In addition, we decided to use another type of inquiry, namely a pictorial representation of the self-estimation. Dionne's categories are assumed to be represented by the vertices of an equilateral triangle. Any normalized estimation towards the three categories can be visualized by some point in the interior of triangle with the distances to the vertices as a measure of closeness. Thus, the teachers were asked to mark their positions of the scores within an equilateral triangle.

4. The Results

4.1. The Numerical Self-Estimation

In Table 4.1 we give the scores which teachers attribute to three components of the view of mathematics. At first glance the following is made obvious by the table: (4.1.1) None of the teachers chose an extreme position, neither in their real view nor in their ideal view. (4.1.2) Each teacher with the exception of K wanted to change, in the direction of process; e.g., H1 wanted to emphasize the process aspect more. (4.1.3) Each teacher regarded the process aspect as the most important factor. It was H1 who gave the process aspect the highest loading. (4.1.4) It is notable that the estimation of K and L, the teachers (having the same formal qualification!) not having met each other before. The same applies to their formal qualification. (4.1.5) Teachers D, J and H2 share the highest loadings with respect to the toolbox aspect. It should be noted that the interviews supported also these entries in Table 4.1. (4.1.6) Again, the mentioned three persons are exactly those teachers who are not quite satisfied with their own teaching in mathematics and would like to change their situation, however, through quite different ways.

Table 4.1
Scoring to the Self-Estimation

Teacher	Tool	System	Process	Tool	System	Process
	real			ideal		
D	**15**	5	10	5	5	**20**
H1	9	1	**20**	4	1	**25**
H2	**14**	8	8	6	**12**	**12**
J	**15**	3	12	10	5	**15**
K	8	10	**12**	10	8	**12**
L	9	9	**12**	6	9	**15**

Tool = math as a tool box, System = system aspect of mathematics, Process = process aspect of mathematics. The leading positions are marked in boldface.

On the basis of the interview (Pehkonen & Törner 98) we may classify D and H1 as the most innovative among these six persons. The tendencies in their ideal view of mathematics are the same, however there are small differences concerning the role of systems and structures in mathematics (D, System = 5; H1, System = 1). However, on the basis of the figures for the real classroom lesson the assessment of D is considerably more rational than H1. Perhaps this discrepency is explained by the fact that D, in contrast to H1 has passed through a full academic university course of study, so D's mathematical horizon can be regarded as broader.

It seems evident to us, that primarly the **weights** set by the teachers, not the exact numerical scoring, **indicate their understanding of mathematics teaching** which leads to a linear ordering of the components. Thus, we derive table 4.2.

(4.1.7) Apparently, with the exception of K, all teachers held the formalism aspect (system) in last place in their real teaching; however, H2 and L would like to change the order. This can be understood by the interviews: K was extremely in favor of the formalism aspect in the past. (4.1.8) It is notable that (only) K and L give the formalism aspect a second ranking in their real teaching. Probably, this fact can be explained through their

Table 4.2
The Ranking of the Components Derived from Table 4.5

Teacher	real	ideal
D	T > P > S	P > T = S
H1	P > T > S	P > T > S
H2	T > P = S	P = S > T
J	T > P > S	P > T > S
K	P > S > T	P > T > S
L	P > S = T	P > S > T

T = Tool, S = System and P = Process aspect

teaching career at *Gymnasium* affording a high mathematical qualification. (4.1.9) On the other hand, with respect to ideal teaching, the process aspect is ranked first among all of the teachers and, in case of H2, on the same level with the system aspect. H2 seems to be obligated to mathematics which he has been taught to be of structural importance at the university and, therefore, feels guilty since his actual situation makes it impossible to present this subject in an adequate manner according to his view.

4.2 Triangular approach

In Figure 4.3 we illustrate the marks within the equilateral triangle from the teachers' responses.

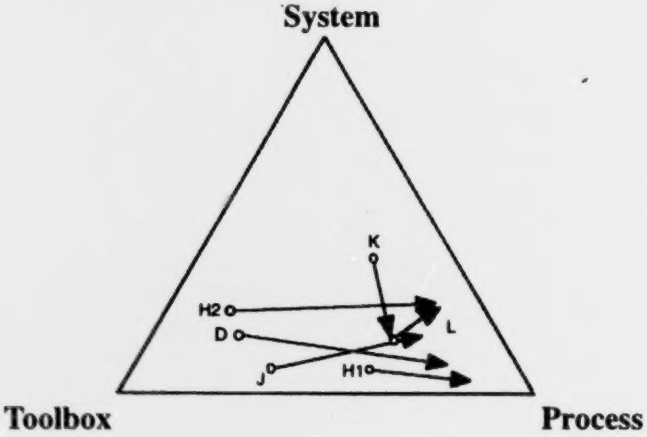

Figure 4.3. The self-estimation-data in graphical form as given by the teachers. Arrows (real to ideal) indicating tendencies are drawn by the authors

First, the *outer distribution* underlines previous observations (see 4.1.1, 4.1.2 and 4.1.3). Next: (4.2.1) It becomes obvious that there is more or less a common view on ideal mathematics teaching. The positions are closer than they are in the actual teaching of mathematics teachers of different school forms. (4.2.2) Teachers D, J and H2 are found along with their implemented lessons in the toolbox corner (4.1.5). These estimations can be supported by corresponding quotes (Pehkonen & Törner 98).

Secondly, the predominant *tendency* of the change of the teachers underlines the importance of the process aspect. In particular: (4.2.3) The two *Gymnasium* teachers K and L show some slight differences (in contrast to 4.1.4).

One should note the fact that L does not estimate the necessity of change in his own classes to be very high. As it became clear in the interview, possibly the daily disturbances taking place his classes lead only to marginal frustration, because for him (as is with K) mathematics exists outside of the classroom and is, as a result, a pure and philosophical discipline worthy of respect.

Thirdly, if one takes the *length* of the arrows as an indicator for the magnitude of change these metric informations are *not made transparent* by Table 4.1. Note: (4.2.4) Again, teachers D, J and H2 claim the largest change with respect to their teaching (see the length of the arrows). (4.2.5) Probably, the arguments of 4.2.2 may also explain why K and L are claiming only limited changes.

4.3. Comparison of Both Self-Estimations

The question arises which data one should hold to be of primary importance: the graphical or numerical data? Both positions could be supported by arguments; both information sources have their own message. It is evident that the numerical and graphical data do not correspond exactly. This fact should therefore not be overemphasized. It seems to us that the test subjects unconsciously intend to express different information through these mathematical equivalent representations. Thus, they serve as two independent data sources covering different aspects!

For example, H2 estimated his math view by Toolbox = 14, System = 8, Process = 8, thus the aspects of System and Process are playing an equivalent but low estimated role. And, in the ideal teaching his scores are Toolbox = 6, System = 12, Process = 12. This feature is not reflected in the graphical representation where the aspect of System seems to remain unchanged. However, the length of vector indicates his feeling that his real teaching differs greatly from ideal teaching.

Apparently there is some inconsistency in the numerical and graphical data of K. His estimations of real and ideal teaching show some interchanging of the roles of Toolbox and System which should be represented by a reflection of positions within the equilateral triangle. On the other hand, the arrow in the graphical mode calls for some change in particularly towards more Process aspect and less System.

Whereas it is easier to realize the tendencies and the direction of the changes, the table may also show some clues as to how the changes should take place. Note that all three, D (System = 5), H1 (System = 1), and L (System = 9) would not be likely to change the absolute value of the factor System; they only prefer an exchange between the Tool aspect in favor of the Process aspect. It must remain an open question whether or not it is an intentional exchange, or perhaps if it is merely just a strategy to treat the data which is to follow the Dionnian catagorization and to distribute 30 points twice among three entries.

5. Conclusions

The derivable conclusions are two-fold in nature. They reflect our considerations on the different methodological sources as well as there are informations on German teachers' view of mathematics. Here, we are restricted to a discussion of methodological aspects.

First, without having done interviews (see Pehkonen & Törner 98) it would have been difficult to give an extensive interpretation of the numerical or pictorial data. Thus, the Dionne-data is of limited importance if there are no additional information sources.

Furthermore, it is indisputable that Dionne´s three-pole must be seen only as a primitive model. Nevertheless, in spite of its simplicity it still possesses a high degree of clarification, especially when concerning a first approach to the problem. The detailed comparision of the self-estimations of the Dionnian components makes clear that numerical data is in need of commentary.

It is decisive that one highlights in such examinations, whether the data under question concerns the real teaching of mathematics or if they are valued as the ideal teaching of mathematics.

Furthermore, it seems that the use of pictorial self-estimations completes in some sense the 'rough' and primitive, yet illustrative, method of Dionne. The additional expenditure for the test subjects is of marginal magnitude. Although, it is a matter of processing redundant data, the noticeable inconsistancy should not be overly valued, because both sources of data make allowances for varying emphasis.

Metrical aspects especially play a strengthened role in the process through pictorial illustrations. The examinee can highlight his/her basic discrepencies, and, finally, a direction of change will become evident in relation to the three components, which are represented by the three corners.

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TEACHER BELIEFS
SHORT ORAL

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COPING WITH MATHEMATICS ANXIETY IN COLLEGE: A STATISTICAL ANALYSIS

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The following research study was conducted at a community college in order to examine the coping strategies of approximately 300 mathematics students. College students enrolled in either a remedial algebra course or a nonremedial precalculus course completed the Composite Math Anxiety Scale in order to provide a mathematics anxiety score. The students also rated a list of ten coping strategies with regard to frequency of use and helpfulness. Mathematics faculty and counselors rated the same strategies, but only in terms of helpfulness. A multivariate analysis of variance (MANOVA) was performed on the student data. The three independent variables were mathematics anxiety, gender, and course level. The dependent variables were the ten coping strategies, each of which was rated for frequency of use and helpfulness. Additionally, the mean scores of the coping strategies (in terms of the helpfulness factor) were rank ordered and compared among the students, mathematics faculty, and counseling faculty.

The MANOVA yielded three significant main effects for mathematics anxiety level, gender, and course level. Low mathematics anxiety students utilized and valued the majority of coping strategies more than did high anxiety students. Mathematics students, faculty, and counselors agreed in rating the helpfulness of most strategies. Completing homework assignments on time, informing your instructor if you don't understand the course material, setting aside extra study time before exams, and asking questions in class received the highest ratings. The results of this study were used to develop math anxiety awareness workshops for faculty.

TEACHER BELIEFS
POSTERS

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RETHINKING THE CONCEPT OF *GOOD* PROSPECTIVE MATHEMATICS TEACHERS

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Research in secondary mathematics teacher education suggests that preservice teachers' beliefs and orientations toward authority influence their learning to teach and their subsequent teaching (Cooney, Shealy, & Arvold, 1998). In a study extending this research emphasis, relationships among teachers' goals for teaching mathematics and their previous mathematics performance provide additional insights into explanations of their teaching practices. Theories of interactionism and constructivism framed the case studies of the goals, beliefs, and orientations of seven teachers during their twelve-month teacher education program. The findings suggested that we rethink our notions of attributes of a "good" mathematics teacher.

The poster presentation provides an opportunity for conference participants to reevaluate their perceived notions of the "good" prospective mathematics teacher.

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A CONCEPTUALLY ORIENTED TEACHER'S IMAGE OF TEACHING

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A conceptually oriented teacher tends to have „an image of a system of ideas and ways of thinking that she intends the students develop“ and „an image of how these ideas and ways of thinking can develop“ (Thompson, Philipp, Thompson, and Boyd, 1994, p. 86). The goal of this study was to better understand one conceptually oriented teacher's images of these systems of ideas and how they develop. The author thus analyzed videotapes and transcripts of one conceptually oriented instructor (Mark) as he taught a mathematics content course to prospective elementary school teachers.

In every classroom episode analyzed, Mark had a unifying idea in mathematics in mind that he wanted the students to understand. That understanding included the ability to connect the unifying idea to a) other mathematical concepts; b) mathematical procedures; and c) the ways that students made sense of the unifying idea, other mathematical concepts, and mathematical procedures. These ideas and the network that relates these ideas together represent the system of ideas Mark wanted his students to develop. Mark intended for his students to make sense of various systems of ideas. These centered around the unifying ideas of making sense of numbers, reasoning additively and multiplicatively, generalizing, making connections among mathematical ideas, and examining underlying structure.

Mark's image of how students come to develop a system of ideas (over the course of a lesson or series of lessons) compelled him to expect students to share their thinking, focus on aspects of a student's thinking that were connected to the unifying idea in the system, use diagrams and explanations to help make explicit both students' reasonings and the connections among each other's ways of reasoning, and make their thinking about their own and each other's thinking about the mathematics an object of conversation. Although Mark intended the students develop various systems of ideas throughout the semester, his fundamental image of *how* students develop a system of ideas remained constant.

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DEVELOPING A SCHEMA FOR METAPHORICAL ANALYSIS

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Metaphors have been used for investigating mathematics teachers' beliefs (Cooney, Shealy, & Arvold, 1998; Fleener & Fry, 1994) because of the insight they can provide. Tobin (1990), in his work with science teachers, found that changing teachers' metaphors for teaching and learning was a precursor to changing their classroom practice. This poster reports on the use of metaphors to assess the compatibility of teachers' conceptions of mathematics teaching with current reforms.

Twenty-two secondary school mathematics teachers, who had completed a three-year program designed to prepare them to become participants in and leaders of reform, were asked to identify from 11 occupations which were most like/unlike a secondary school mathematics teacher. The first two authors individually coded the teachers' elaboration for their choices and reached consensus on the resulting categories. The categories that emerged were: Learning, Instruction, Environment, Assessment, and Extraneous Duties. Each of these categories was further divided into one or two dichotomies representing reform versus non-reform. For example, under Learning, statements were categorized as to whether they represented the student or the teacher being responsible for doing the mathematical thinking.

Although the teachers chose a variety of metaphors, the vast majority of their responses were tallied in the reform column. This suggests that these teachers' conceptions of mathematics teaching at the end of their program were compatible with current reforms. The categorical schema developed as part of this research delineates obstacles to reform and areas of contradiction that exist within teachers' beliefs systems. This provides information for the kind of focused professional development that is required to move reform forward.

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CHANGES IN TEACHER BELIEFS AS A RESULT OF WEB-BASED PROFESSIONAL DEVELOPMENT

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The present study examines changes in beliefs for two participants in a semester-long distance inservice program using a Web site called INSTRUCT (Implementing the NCTM School Teaching Recommendations Using Collaborative Telecommunications) located at http://instruct.cms.uncwil.edu. The model will be presented as an initial attempt to represent and thereby better understand the complex dynamics of Web-based professional development. The model takes into account the possibility of a shorter cycle of training, implementation and evaluation of new practices, thus allowing for changes in teacher beliefs to inform future staff development efforts. Additionally, an attempt has been made to display the effect that each of INSTRUCT's options, such as online chats, has on the overall professional development cycle.

Participants in the INSTRUCT program reviewed and implemented material from the six NCTM Standards for Teaching Mathematics, taking two weeks to read through, discuss, implement, and reflect on each of six standards. Teacher 1 in the study began the semester viewing herself in a traditional role, that of dispenser of knowledge. As the semester went on, she gradually became convinced that she could become more of a facilitator, allowing students to discover much of the mathematics for themselves. The central belief for Teacher 2 involved her need for control in the classroom. As a result of the semester experience, she became convinced that student decision-making can play an important role in learning mathematics.

ELEMENTARY MATHEMATICS TEACHERS' STATED, INTENDED, AND ENACTED BELIEFS

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It has frequently been assumed that teachers' beliefs about content and about learning and teaching have a direct impact on the teachers' practice. Although some research has shown a relationship between beliefs and practice, it does not appear to be a direct one. Pajares (1992) suggests that researchers attempting to understand teachers' beliefs should attend to their stated beliefs, their intended beliefs, and their enacted beliefs.

The project discussed here was designed to investigate seven elementary teachers developing practice. Data on teachers stated beliefs were collected in interviews and through questionnaires; data on their intended beliefs were collected during discussions in six day-long meetings that focused on the mathematical thinking of children in the teachers' classes; and data on their enacted beliefs were collected in the form of videotapes the teachers made in their own classrooms.

The teachers' stated beliefs sometimes appeared to be in conflict with the beliefs the researchers inferred from their practice. However, the intended beliefs the teachers articulated as they discussed their practice with their colleagues proved an important link between stated and enacted beliefs. The discussions revealed meanings given by the teachers to the terminology of their stated beliefs and clarified the beliefs inferred from their practice.[1] As a result, it became clear that their stated and enacted beliefs were not as disparate as might have been thought. This study indicates the importance of informal interactions among a community of teachers and researchers in understanding the teachers' beliefs and the impact of those beliefs on their teaching.

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[1] We are using *meaning* here to indicate a quantitative understanding of the quotitive interpretation of division of whole numbers as extended to fractions.

TEACHER EDUCATION
RESEARCH REPORTS

LEARNING TO TEACH MATHEMATICS USING MANIPULATIVES: A STUDY OF PRESERVICE ELEMENTARY SCHOOL TEACHERS

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Current thinking about how children learn mathematics suggests that mathematics instruction in elementary schools be based on manipulative materials. Prospective elementary school teachers rarely experience this type of teaching in school or in the "regular" mathematics courses they take in college. The mathematics "methods" course, therefore, must not only expose teacher candidates to this teaching methodology but also ensure that the students acquire some expertise in the process. This study examined the experiences of a group of preservice elementary teachers as they learned how to learn and teach mathematics with manipulatives.

> "Modeling mathematical ideas through the use of representations (*concrete* , visual, graphical, symbolic) is central to the teaching of mathematics." (my emphasis) *Professional Standards for Teaching Mathematics* (National Council of teachers of Mathematics, 1991, p. 151).

This statement reflects the importance currently attached to the use of representations in teaching school mathematics. Concrete representations in particular, are considered very important at the elementary school level because children tend to understand abstract mathematical ideas better when they interact with physical embodiments of these ideas (Bruner, 1964; Dienes, 1960; Piaget, 1952). Some studies of mathematics teaching, however, found that few elementary school teachers incorporate work with manipulatives into their mathematics lessons. (Fey, 1979; Scott, 1983).

Some teachers avoid using manipulatives with children because they do not know how to plan, teach, or manage mathematics lessons based on such materials (Hollingsworth, 1990; Trueblood, 1986). Other teachers fear that children will become dependent on manipulatives, and will, as a result, not "master basic computational algorithms and related concepts " (Senger, 1998; Trueblood, 1986). Having acquired much of their own mathematical knowledge only through transmission, and symbolic representations, teachers often are not confident that they can help children build mathematical knowledge in any other way. So, many just teach mathematics as they had been taught. Clearly, the type of mathematics instruction envisioned by the *Professional Standards* will not materialize as long as teachers' pedagogical knowledge remains inadequate, and their beliefs, in conflict with the spirit of reform.

This paper will report on a teaching experiment designed to teach prospective teachers how to use manipulatives to learn and teach mathematics. This analysis will focus on fractions where "the use of manipulatives is crucial in developing students' understanding of fraction ideas" (Bezuk & Cramer, 1989, p.158).

Hypotheses

It was hypothesized that by participating in this experiment, preservice teachers would be able to:

1. Use manipulatives to (a) model fraction concepts, (b) explain fraction algorithms, and (c) solve fraction problems,
2. Plan and teach activity-based lessons to develop fraction concepts or skills, and
3. Develop a positive disposition towards mathematics teaching and learning involving manipulatives.

The study also provides information about three pertinent questions:

1. What is the attitude of preservice elementary school teachers towards using manipulatives to learn or teach fraction concepts?
2. What difficulties do teacher trainees face as they use manipulatives to learn or explain fraction concepts?
3. What kinds of errors do teacher trainees make when working with manipulatives and how can we explain these errors?

Theoretical Background For The Study

For this study, the researcher adopted the view that mathematical learning is primarily a constructive process based on the learner's experience and activity (Cobb et al, 1993) Proponents of this view believe that "learning occurs when individuals reflect on their activity, including sensory motor and conceptual activity, and reorganize their interpretive framework" (Cobb, et al, 1993, p. 21). From this perspective, mathematics instruction should engage students in meaningful, goal-oriented tasks that foster collaboration, active exploration, problem solving, discourse and sense-making.

Method

The approach used in this study complemented elements of the "teaching experiment" methodology (Rachlin, et al., 1987), with careful observation, and reflection as espoused by Schon (1983). As applied in this study, Schon's notion of "reflection-in-action" helped the researcher uncover students' cognition as he observed the students' interactions with their peers and instructional materials in the classroom. It also informed his analysis of data obtained from interviews, video recordings, and students' written work.

Procedure

The subjects of this study were twenty-one elementary education majors enrolled in a mathematics methods course. The sample included one male student. Although all the students had passed the professional examination required for licensure as elementary school teachers, fifteen were diagnosed as having serious difficulty with rational number concepts and algorithms.

Classroom Activities

The students sat in groups of four or five around hexagonal tables in a classroom equipped with such manipulatives as Cuisenaire Rods, Egg Cartons, Fraction Bars, Fraction Plates, Pattern Blocks and Texas Instruments' fraction calculators. These materials were used in instructional tasks like the following:

Open explorations. In these activities, small groups of students were asked to explore the instructional potential of a given manipulative. The students' findings were in turn used to generate new questions that initiated further explorations or led to the refinement or extension of earlier findings. Here is an example.

As students came into my classroom, I went around and placed a dozen home-made fraction bars on each table. The bars represented fractions from halves to twelfths; each was 12" long and made of colored, matting paper. Since we were just beginning the unit on fractions, I asked the students how they might use the bars with a fourth grade class.

After interacting with these bars for a short time, volunteers responded by using the bars to illustrate unit fractions, draw line segments, and measure lengths. Next the students were asked to search for possible relationships among the fractions represented on the bars This, and subsequent leading questions resulted in the students' (re)discovery of:

order relations such as : $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \ . \ . \ . \ > \frac{1}{11} > \frac{1}{12}$, and
equivalence relations like : $\frac{1}{2} \spadesuit \frac{2}{4} \spadesuit \frac{3}{6}$, and $\frac{1}{3} \spadesuit \frac{2}{6} \spadesuit \frac{3}{9} \spadesuit \frac{\triangle}{12}$.

They classified many fractions shown on the bars into three groups depending on how close the fractions are to zero, one half, or one. As a result, they were able to estimate sums and differences of fractions. Finally, in efforts to determine what fractions, if any, there are between 1/2 and 1/3, the students experienced, in a concrete way, the density property of rational numbers.

Goal-oriented, guided investigations. The difference between these and the open explorations is that the questions, in this case, were designed to generate discoveries about specific objectives. Here is an example designed to help the students make sense of division of fractions.

This investigation was preceded by a review of the partitive and measurement interpretations of division of whole numbers. After this, the students were asked to cover the yellow hexagonal block in as many ways as they could using only blocks of the same color. The students, already familiar with pattern blocks, quickly displayed coverings by two red trapezoids, three blue rhombi, and six green triangles. "If the yellow hexagon represents one, what fractions do the red, blue and the green blocks represent, and why?", I asked. Again the answers 1/2, 1/3, and 1/6 together with explanations, came very quickly. I then wrote the problem $\frac{1}{2}-\frac{1}{6}$ on the chalk board and asked: "Can anyone use pattern blocks to explain what this problem means, and then solve it?"

There was no answer for a while despite the flurry of activities and hushed discussions in the small groups. Eventually, one student offered the opinion that pattern blocks could be used to show fractions, but not to solve problems like the given one. She offered this explanation: "This is one half" moving a red trapezoid aside. "To divide by 1/6 you have to flip this green triangle." She flipped the triangular block. Appearing puzzled, she said. "When you flip 1/6, you get 6/1; but this block won't give you that!" "I know how to do this problem, the answer is 3, but the blocks confuse me. May I do the problem without the blocks?"

It took some time and many hints and questions for some of the students to understand the problem. Their fixation with the "invert and multiply" algorithm and failure to connect the given problem with measurement division of whole numbers apparently blocked their efforts to solve the problem.

In a third type of activity, students in groups of two or three, took turns to model, teach or explain fraction concepts and processes to each other. Afterwards, they exchanged constructive critiques of each other's performance.

Students were also required on occasions, to explain in writing, how they would do certain computation exercises, and then draw sketches of an appropriate manipulative to illustrate steps in their solution process.

Twice during the semester, each group of four or five students planned and team taught an activity based lesson on an assigned fraction topic. These lessons not only involved manipulatives, some included connections to home economics and science.

One of the summative tasks students did in this course was to use specified as well as self-selected manipulatives to model given concepts, illustrate relationships, or do a computation exercises. Here are two sample problems: (a) Use paper folding to develop the idea of equivalent fractions;

and (b) Explain what $\frac{1}{4} \div 3$ means, and then use a suitable manipulative to do the problem. Each student was video-taped while doing this task individually in my presence.

Data

The data for this study were gathered from every day activities in the class. They include: (a) students' written work; (b) videotape of the summative task described above; (c) my notes and reflections on observed classroom events, such as the students' discoveries , comments, task commitment, and presentations; and(d) the notes I took as each student did required tasks.

Results

The data provided sufficient evidence to support the first two hypotheses of the study. As for the third, the result appears mixed. On one hand, it seemed that using manipulatives enabled most of the students to make sense of multiplication and division of fractions for the first time. Some spoke enthusiastically about using manipulatives in their own classrooms. In fact, for some students, the ability to apply measurement as well as partitive interpretations to fractions transferred to the division of decimals. On the other, students with already high computation skills, sometimes had a bad attitude about using manipulatives. Convinced that they already knew the material, some of these students often acted like they considered using manipulatives a waste of their time.. They often made comments like "I already know this stuff," "Using blocks just makes simple things difficult," "This method is confusing." "Isn't it more straight forward to tell children to invert and multiply than to go through all this?" It was apparent that to succeed in the course, some merely tried to imitate what they saw me do in class, or memorized the step-by-step procedures in their textbooks.

To further shed light on students' learning, student errors and misconceptions were collected and analyzed. These data came from my observation notes and from the students' written work. The data included semantic, algorithmic, conceptual and representational errors. There were instances of students interpreting a problem like $1/2 \div 1/6$ as "one half divided into six equal parts." Representational errors occurred when students applied shifting frames of reference in dealing with fractions in a problem. Consider, for example, a student's representation of the fractions to be added in this problem $\frac{1}{4}+\frac{1}{6}$. Using pattern blocks, the student represented 1/4 by a red trapezoid, and 1/6 by a green triangle. Combining the two blocks , she came up with 4/6 for an answer. To determine which block would represent 1/4 , she used two hexagons as her unit, but to select the block for 1/6, she used as her unit only one hexagon!

It is sometimes said that algorithms drive out thought. The data for this study found some evidence for this claim. Consider a student's attempt to solve $\frac{1}{2} \div \frac{1}{6}$ with pattern blocks. Using one hexagon as her unit, this student represented the dividend by a red trapezoid. But before proceeding to represent the divisor, she inverted it and obtained 6. Having inverted the divisor, she proceeded to "multiply" the trapezoid by 6 resulting in six trapezoids. She rearranged these to form three hexagons which represent the number 3. This, she said, shows that $\frac{1}{2} \div \frac{1}{6} = 3$. After having a whole class discussion about the student's solution, many began to see the importance of using manipulatives in teaching children mathematics.

In summary, the data for this study show how manipulatives can help a teacher uncover children's misunderstandings about mathematics concepts. The results also show the kinds of misconceptions that may also lurk undetected behind correct paper-and-pencil computations. It appears that fluency with paper-and-pencil increases a pre-service teacher's resistance to change. Hence, any attempt to make teachers adopt new teaching strategies must also find ways to soften their resistance to change.

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PROSPECTIVE ELEMENTARY TEACHERS' CONSTRUCTIONS OF UNDERSTANDING ABOUT MULTIPLICATION WITH RATIONAL NUMBERS

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The purpose of this study was to research the constructions of understanding of prospective elementary teachers about multiplication with rational numbers. One participant's learning progression is described and schematically represented. The need for a centrally–organizing framework for thinking about multiplication, and the need for development of new concepts (in increasingly difficult number situations), and the need for physical quantitative experiences involving (or modeled by) multiplication with non–whole numbers was observed.

Purposes

The purpose of this study is to research the constructions of understanding of prospective elementary teachers about multiplication with whole and rational numbers[1]. An earlier study (Azim, 1995a) identified common dimensions of understanding about multiplication with whole and rational numbers among prospective elementary teachers that were constructed during the study. The data in the earlier study suggested that the construction–of–understanding processes of the prospective teachers were individualized despite the shared course experiences and student interactions that occurred during the course/study. Their developmental progressions in understanding about rational number multiplication were observed to be very different both in sequence and in methods of constructing understanding.

The research purposes for the current study were, therefore, to: (1) describe the individual learning (or "construction–of–understanding") progression of each prospective elementary teacher in the study as he or she develops understanding about rational number multiplication, and (2) identify similarities and/or differences (patterns) in the learning progressions of these individual prospective elementary teachers.

Perspectives & Theoretical Framework

The theoretical framework within which this research is conducted is the theory that multiplication — first learned about in the context of whole numbers — must be re–conceptualized in the rational number domain, and that this re–conceptualization is neither a simple nor a quick process (Greer, 1994). The development of understanding about multiplication is not a linear process, but, a multi–dimensional process of developing understanding of interconnected concepts and procedures in the "Multiplicative Conceptual Field" (MCF). (Vergnaud, 1988, p. 141.) The view of learning ascribed to by the author of this research is the constructivist view (e.g., Davis, Maher, & Noddings, 1990), and provides the second dimension of the theoretical framework within which this study is conducted.

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[1] In this article, a contrast is drawn between multiplying with *whole numbers* and *rational numbers*. The term rational numbers is used loosely to represent non–whole rational numbers for the purposes of this article.

Methods

Qualitative research methodologies were used (Howe & Eisenhart, 1990). These methods included collecting written inventory data, coursework, and semi–structured individual interview data that provided perspectives on the understandings about multiplication of the study's participants. A data analysis process of interpreting, analyzing, and schematically representing each participant's learning progression was conducted and followed by an analysis of observed similarities in and differences between these learning progressions.

Data Sources/Evidence

The participants in the study were 20 prospective elementary teachers enrolled, 1997–1998, in two sections (taught by the researcher) of their university's required elementary mathematics methods course. The data contributed by these volunteers included: (a) a written inventory about rational number multiplication (including questions about the meaning of multiplication and about putting multiplication expressions into real world story problem contexts); (b) two one–hour audio–taped interviews, one early in the quarter and one at the conclusion of the quarter, about participants' reasoning and understanding about multiplication with rational numbers (and during which interview guides were used); and (c) coursework that gave evidence of participants' understanding about rational number multiplication.

Results/Conclusions

Purpose 1: In answer to the first research question, one example of a learning progression of one prospective elementary teacher, Mark (a pseudonym) is provided.[2] Mark reasoned through his sense of number relationships and through his sense of operations. But he ran into contradictions between the two senses (represented by the "clash" symbol in the schematic representation of his learning progression found in Figure 1). He described his interpretation of 1 1/2 x 1/3 as follows: "I looked at it, thinking if you have 1 1/2 of something, times it by 1/3, I would be getting 1/2 because I could visually look at — I thought of it in my head, you know. Here's my pizza. Here's 1 1/2 of a pizza, and I have 3 buddies. Each buddy gets 1/2. But what are you doing? See, that is dividing." The conversation continued, with Mark trying to use the fact that he had observed the number relationship between 1/3, 3 *halves,* and the total quantity 1 1/2, and yet feeling wrong to have constructed a story about *dividing* — rather than *multiplying* — the 1 1/2 pizza among 3 friends. Without his sense of number relationships and/or operation (multiplication) sense to guide, Mark became very frustrated. He was unable to give meaning to this (and other) fraction multiplication expressions with which we were working.

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[2] A longer version of this article including more in–depth learning progressions is available from the author.

Mark's strength *was* his sense of numbers and his sense of operations. When I asked Mark, therefore, to restate his pizza story problem not using the number 3 (which leads to asking a dividing question) but using the number 1/3 (which he was given), he was able to match the structure of the expression with a story problem: "You would be saying I have 1 1/2 pieces of pizza — of which 1/3 has pepperoni. How much would have pepperoni on it?" (The answer would be 1/2 of a piece.)

Mark was a student who *wanted* to get his bearings. This rephrasing of his story was the start of his re–conceptualizing multiplication with rational numbers (less than 1 — multiplication where the result is less than the original amount). In order to help him get his bearings, I then asked him to explain what 1 1/2 x 2 means. "You get 3. Well, that would be like you have a dollar and a half, timesing it by — timesing it twice. Maybe you're putting it in a copy machine and you get another dollar and a half. How

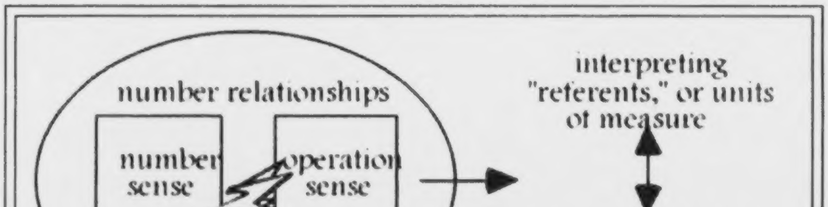
number relationships
number sense
operation sense
multiplication patterns & meanings
interpreting "referents," or units of measure
multiplication as a(n) (invariant) "scaling up or down" process

Figure 1. *Schematic Representation of Mark's Learning Progression.*

much do you have total? Well, you have 3." He led himself immediately to question the meaning of 1 1/2 x 1/3 in this context: "Um, but you have a dollar and a half here, and it goes to the copy machine. It comes out —cut up in 1/3. How many 1/3's do you have in 1 1/2? You would have 1/2. It doesn't make logic." He had reverted to division reasoning: how many 1/3's are in 1 1/2? I returned to the *multiplication* structure in the copy machine analogy by asking him (in regards to multiplying by 2): (Interviewer:) "How many copies of the $1 1/2 did it make there?" (Mark:) "2." (I:) "How many copies of your $1 1/2 does it make here? (multiplying by 1/3)" (Mark:) "Well, it would only make 1/3." (I:) "Okay, and what's 1/3 of a copy of $1 1/2?" (Mark:) "1/3 of a copy of $1 1/2 — would be 1/2." (I:) "1/2 of what?" (Mark:) "1/2 of a dollar." (I:) "What happens when you multiply a number by 1/3?" (Mark:) "Uh — it breaks it up into 3's. It divides it up in 3's."

Mark used the copy machine analogy, that he *knew* was a correct interpretation for multiplication with whole numbers (e.g., 2) to realize that multiplying by 1/3 would *divide* the 1 1/2 dollars into 3 parts, or by 3. He, then, wanted a deeper understanding of *why* this would be. We continued. I asked Mark to choose a number. He chose the number 9. I asked him to think about what happens to the number 9 when you multiply it by 3, 2, 1, and 0. I wrote these problems down, and he filled in the answers. I then asked him what it would seem the answer to 1/2 x 9 would or should be, based on this pattern. He answered, "You're cutting it in 1/2. It's half of in between those two [0 and 1]. So if it was 1/3, you're cutting it into 1/3 of that."

Using his number and operation sense, Mark had looked at a pattern of multiplication equations and, from it, discerned what the product of a fraction (multiplier) and a whole number (9) would be — a reduction of the whole number (9) to *that fraction (1/3) of* its size.

Mark needed to develop his understanding further in the interview, however. He was still very uneasy and seemed to need some closure that he had not yet experienced. He returned to the issue of constructing story problems for multiplication: "I get so confused. It's so frustrating because you know the answer [but can't construct a story to match it]." We started to work on a new story problem. (Interviewer:) "You go out in the backyard and you get a board 1 1/2 yards long, and you are saying that you don't need the whole board. What do you need?" (Mark:) "You don't need the whole board. You need 1/3 of the board. If you have 1 1/2 yards of board, and you only need 1/3 of that board, you would only use 1/2 of the board." (I:) "Clarify that. What is the 1/2?" (Mark) The material which it takes to make the birdhouse." (I:) "1/2 of what?" (Mark:) "1/2 of a yard. 1/2 of a yard — I see. Here's your board, here's your material. Well, I only need to use this much. How much are you using for each one of these? Well, I'm using 1/2 for each of these. Now that you think about it, that's completely logical. That would have made sense, and I probably would have thought of that, but you get so turned around."

The issue of what "referent," or "unit of measure" belongs to what fraction seemed to have added to Mark's frustration. (This was a common issue among the study participants.) Mark reached resolution that day. He resolved his conflict regarding the numbers (dividing) and the issue of what the 1/2 represented in that one final version of a story problem for 1 1/2 x 1/3.

Mark constructed meaning for multiplication with rational numbers during the interview quoted above that took place during the second week of the study. He also constructed some understanding of the role of "referents" (or "units of measure") in interpreting fraction story problems. When Mark came in for his second interview, at the conclusion of the course, he explained that he had created a method of visualizing rational number multiplication that he was now using as a framework for interpreting problems and situations. Mark explained to me, during the second interview, that he had developed a rectangular model for interpreting fraction multiplication. He would draw the "whole" that was being multiplied (such as the 1 1/2 pizzas) using rectangles (e.g., 1 and 1/2 rectangles) and then would show how much each rectangle (or rectangular part) was being reduced, when multiplied by a number less than 1, or enlarged, when multiplied by a number greater than 1. He worked several examples, giving rough sketches of how the problems would work. He used his model accurately (visually) for each problem (but did not use it to determine the number answers).

Mark demonstrated during this interview that he understood what was quantitatively occurring when two rational numbers were multiplied. He had re–conceptualized multiplication in the realm of rational numbers and had a framework for interpreting the meaning of new problems or situations. The numbers in the problems and the actions in the illustrations made good sense (to him). He had developed the beginnings, too, of interpreting "referents" accurately in interpreting rational number multiplication situations.

Mark's journey was not unlike the journeys of others in the study. They, too, experienced difficulties because of their individual background knowledge, ideas, or contradictions that arose from their combination of previous learning and new questions being raised. His constructions, like theirs, were in early stages of development, needing the tests of experience to reinforce them more securely through more experience (and success) with simple expressions and experience with increasingly more difficult expressions.

Purpose 2: Patterns or seemingly important observations that emerged through studying and schematically representing the learning progressions of all of the participants in the study are (very briefly) described below.

1. *The Need for a Centrally–Organizing Framework*

   Participants who demonstrated the most flexible and sturdy (able to be probed without weakening) understanding about multiplication were participants who had developed at least one well–understood centrally–organizing framework for interpreting multiplication with rational (and whole) numbers.

2. *The Need for Growth within a Framework* (or Dimension of Understanding)

   Participants who could interpret rational number multiplication meaningfully with some multiplication expressions were very often unable to do this with similar but more complex expressions. This suggests that opportunity for growth with concepts is important in learning about rational number multiplication and is supported by the conceptual field theory (Vergnaud, 1988) that as new dimensions of understanding are developed, growth within the dimensions and the development of connections between the dimensions are important parts of the construction of the whole conceptual field.

3. *The Need for Quantitative Experiences with Rational Number Multiplication*

   In general, participants who had measured, drawn, or thought about situations or number problems requiring that they take a fraction, whole number, or mixed number multiple of another quantity prior to the course — or who devoted a good quantity of time to doing this during the course — moved more steadily and quickly through the construction process of developing understanding of multiplication with rational numbers than participants who had not.

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NARRATIVE AS A TOOL FOR FACILITATING PRESERVICE MATHEMATICS TEACHER DEVELOPMENT

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This paper reports on a study that investigated preservice mathematics teacher development with a focus on self-understanding as a basis for professional growth. The goal of the study was to determine the effect of using narrative as a tool for facilitating self-reflection of the nature of the preservice teachers' thinking in the teaching of mathematics. Twenty-two preservice high school mathematics teachers were studied using a humanistic research approach to interpret the effect of the narrative approach on their thinking. The study was conducted during a secondary mathematics methods course. Data consisted of the participants' writing and group discussions associated with the narrative activities used in the course. The outcome revealed that narrative provided an effective way for helping the preservice teachers to broaden their views of mathematics and its teaching and learning.

Introduction

Current reform movement in mathematics education (NCTM, 1989; 1991) poses a significant challenge to both practitioners and teacher educators. The former are expected to teach mathematics in a way they were not taught and the latter to provide meaningful and effective intervention to help these teachers to achieve this. Thus, a significant component of the reform movement is a shift in how both inservice and preservice mathematics teachers are educated. This paper reports on a study that investigated preservice high school mathematics teacher development with a focus on self-understanding as a basis for professional growth. The goal of the study was to determine the effect of using narrative, or narrative knowing, as a tool for facilitating self-reflection of the nature of the preservice teachers' thinking in the teaching of mathematics.

Theoretical Perspective

The study was framed in the context of teacher learning based on reflection on action, thinking, and past experiences (Dewey, 1963; Schon, 1983, 1987). Reflection has been promoted as a way in which teachers construct and re-construct the meanings/beliefs and knowledge that guide their actions in the classroom (Schon, 1983). Thus, the importance of reflection in teacher education is usually linked to the relationship between teachers' beliefs about content, teaching and learning and their classroom behaviors. More generally, reflection is linked to creating awareness of the beliefs or conceptual systems framing teachers' thinking and actions. Reflection has, therefore, gained significant acceptance as a basis of teacher education (Bennett, 1996; Grimmitt & Erickson, 1988; Halton & Smith, 1995). Its importance to teacher preparation programs is linked to findings that preservice teachers have well developed personal and practical theories regarding teaching acquired during many years of being a student (Brookhart & Freeman, 1992).These theories have been shown to influence the way the preservice teachers approach teacher education and what they learn from it (Calderhead & Robson, 1991). In general, preservice teachers tend to rely on their personal experiences as learners in constructing meaning for classroom events (Britzman, 1991). Thus, a growing number of researchers argue that we can strengthen the impact of teacher education programs by focusing on prospective teachers' initial beliefs and background knowledge about teaching (e.g., Bullough, 1994). It is within this context that reflection becomes a necessary process to enable preservice teachers to become aware of their personal theories in order to facilitate their professional growth beyond such theories.

The importance of reflection in mathematics education is evident in the constructivist approach to teacher development (Simon & Schifter, 1991; Wood et al, 1991). It is also evident in the literature on preservice teacher education (Cooney, 1994). However, particular modes of reflection are often treated, not as the primary foci, but as implicit factors of an intervention program in studies involving mathematics teacher education. In this study, a particular mode of reflection, narrative, was investigated as a basis for facilitating reflection by preservice teachers on their beliefs and actions regarding mathematics teaching. The use of narrative in this study is based on the view that narrative is a way in which one makes sense of one's world (Bruner, 1986; Polkinghorne, 1988). In this view, narrative is a cognitive scheme (Polkinghorne, 1988), a way of knowing (Bruner, 1986). Thus, reflection in a narrative context involves the use of story, not merely as an object to be analyzed in a fragmented way, but more importantly, as a humanistic process of making sense of reality. It is a process of conscious storying and restorying of past, present and future events one has experienced or intends to experience. Narrative has been used in recent years in research on teaching and in teacher development (Carter, 1993), but not specifically in relation to mathematics teaching.

Methodology

Methodology involved working with 22 preservice high school teachers during their practicum year. The study was conducted during a secondary mathematics methods course taught by the researcher. The course ran for 10 weeks in the fall, followed by 4 weeks of practice teaching in the schools, then resumed for 7 weeks in the winter term. Narrative was interwoven throughout the course so that the participants wrote stories before and after formal theory on mathematics teaching (e.g., NCTM standards) was covered and before and after the 4-week practice teaching. The before-theory stories focused on experiences that likely influence the development of preservice teachers' beliefs and knowledge about mathematics teaching, e.g., experiences with school and instruction, and experiences with formal knowledge. Post-theory stories involved rewriting the pre-theory stories and creating new stories of imagined mathematics lessons to reflect the theory. Post-teaching stories focused on actual teaching experiences. The following is an outline of a portion of the narrative activities:

Before theory and practice teaching: Participants wrote stories of personal experiences as students of a good and a bad mathematics lesson. They rewrote the story of the bad lesson to reflect how they would teach it and rewrote the good lesson using a different teaching approach.

After theory and before practice teaching: Participants compared their stories to theory then wrote a new story of them teaching a mathematics lesson.

After theory and close to practice teaching: Participants wrote a story of them teaching a mathematics lesson based on a topic they expected to teach during practice teaching.

After 4 weeks of practice teaching: Participants compared their actual teaching to their stories.

The stories were accompanied with participants' small- and large-group sharing and discussions and journal writing on their reflections. Discussions and reflections focused on participants' thinking and actions in relation to the nature of mathematics, perspectives of teaching mathematics, and perspectives of learning mathematics. Some journal topics were selected by participants while others were prescribed (e.g., requiring that they compare and critique the teaching perspective reflected in their pre-theory stories in relation to the perspectives presented in the theory). For each story, instructions like the following were given to the participants:

Write a story of your experience of a mathematics class. The story should describe a complete mathematics lesson from beginning to end and provide as much details as possible on the following: What the teacher/instructor did and said. What you did, said, thought and felt. How the mathematics content was dealt with or presented. What other students did and said during the lesson. The lesson should involve the presentation of a mathematics concept for the first time. The story must be written in first person. Do not analyze anyone or anything in the story. Just describe the situation as it actually happened.

Data and Analysis: Given the focus on story, a humanistic perspective of research involving analysis of narratives (Polkinghorne, 1995) seemed to be most relevant for this study. Data for this narrative research approach include written documents (personal journals ..., autobiographies, biographies) and oral statements ... involving storied narratives [Polkinghorne, 1995, p. 12]. Thus, data for the study was all of the participants' writings associated with the narrative activities. The writings consisted of all of their stories and their journals of their thinking in reflecting on the stories.

There was also one journal on their thinking and experiences of the nature of mathematics and its teaching and learning that they wrote prior to any of the narrative activities and one at the end of the course in which they reflected on the impact of the narrative activities on their learning about themselves in teaching mathematics. Their group reflections in 2 classes before and 2 classes after practice teaching were audio-taped and transcribed. In general, the writings and discussions were considered to be an indication of the nature of the participants' reflections, learnings, and understandings resulting from the narrative activities. The analysis- of-narrative method involves looking for various kinds of responses, actions, and understandings that appear across the storied data in order to describe the themes that identify particular occurrences within the data (Polkinghorne, 1995). In this study, themes regarding the effect of the narrative approach in facilitating self-understanding was identified by thoroughly scrutinizing the data. Themes from pre-and post-theory stories were compared and triangulated with the transcripts of group reflections and the reflective journals.

Results

The narrative experience seemed to allow the preservice teachers to gain insights into their thinking and actions, regarding mathematics and its teaching and learning, that were very revealing to them. There were many surprises for them, particularly in relation to conflicts between their thinking (e.g., what they valued or wanted to be) and their actions in the classroom. One noted,

... It was interesting to go back and read the story from the fall. The first thought that came to mind was how incredibly naive I was (am?) about the realities of teaching. ...In my proposed lesson on conics, I think I have assumed too much in terms of how self-guided students can be when "discovering" new concepts.... [Rose, after practice teaching]

Thus, the effectiveness of the approach peaked for them when comparing actual teaching stories to pre-teaching stories. Such comparisons generally focused on conflicts between the self the preservice teachers wanted to be and the self in action in a way that informed them of what to consider to restory the two in a more compatible way. The dominant theme that emerged in this comparative context was the preservice teachers' struggle with classroom discourse. For example,

... When I read my story, I could see many of the faults I have and many changes I would make. .... I was surprised at ... how much I'd fallen into the traditional role of standing pinned to the blackboard defending myself with a constant volley of low level questions. This was disappointing even at the time but I did feel that as long as I was conscious of my faults I had a chance to improve. [Amy, after practice teaching]

...In the story I saw myself having answers and questions all the time. It seemed very easy to ask questions, but really it is not. .. There were times when I asked a question and the students came to one answer. I tried to keep them rolling, but I did not know how. There were other times where I realized I could have asked a question to get students involved and thinking, but I lectured instead.... [Tara, after practice teaching]

The participants were able to recognize that their deep beliefs (e.g., a traditional view of the teacher's role) and not surface beliefs (e.g., a constructivist view of the teacher's role) were the stronger influences on how they acted, how their students responded, and how they themselves felt. With this self-understanding, they were able to confront some of their taken-for-granted conceptions (e.g., teaching as telling) and wanted to rethink them in alternative ways. Thus, they became more open to consider things they previously assumed they understood and, consequently, broadened their understanding of what mathematics teaching entailed in the context of the reform movement.

In addition to self-awareness, the narrative approach allowed the participants to focus on mathematics pedagogy in a humanistic context (Bruner, 1986) instead of a paradigmatic context (Bruner, 1986) where the former deals with context sensitive and particular explications and the latter on context-free and universal explications. In working with the narratives, the preservice teachers' focus was more on a holistic and contextual way of thinking about teaching mathematics as opposed to merely isolated procedures or techniques. For example, they looked at teaching, learning, and content in a related way when considering discourse in the context of lived experiences. As one reflected,

... I also need to better anticipate students' questions and think through responses. As well, I need to think about good questions that I should be asking about the concepts being taught, so that I avoid asking trivial questions and ask those which will lead students to clarify and extend their thinking.[Rose, after practice teaching]

However, the humanistic context was initially a source of varying levels of anxiety for the participants (males in particular) in terms of seeing the relevance of writing or self-reflection in a course that dealt with teaching mathematics. As one explained,

To be quite honest, reflection of this nature is not something that I find myself compelled to do naturally. On the other hand I feel it is important for me to say also that looking back on my earlier stories, reflecting on my practicum experience, and trying to resolve the motives between the two as well as comparing the fictional with the factual forced me to think critically about what I am honestly trying to do and the best ways of achieving this end.

Conclusion

The potential of narrative as a vehicle for raising preservice teachers' awareness of their thinking and broadening their view of mathematics teaching seems to be significant. Thus, the storying approach seems to be a worthwhile avenue to pursue as a way to enhance mathematics teacher education programs. It could likely develop the kind of flexibility in preservice teachers' thinking that will allow them to realize reform recommendations in the teaching of mathematics.

TEACHER CHANGE DURING AN URBAN SYSTEMIC INITIATIVE

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The vision of school mathematics contained in the NCTM *Standards* documents requires fundamental changes in mathematics teaching practice. In recent years, NSF has funded a number of Systemic Initiatives aimed at unifying and making more effective what the agency perceived as haphazard and unstructured approaches to reform. However, early reports from several sources are pessimistic about the effect of these Systemic Initiatives. The authors report some of their experiences and findings while working as external evaluators for an Urban Systemic Initiative which was guided by a *Constructivist Vision for Teaching, Learning, and Staff Development.* Analysis of data collected for the evaluation (observations of staff development, observations of classrooms, surveys of teachers, students, and administrators, focus groups of teachers and administrators, and teacher interviews) indicates that mathematics teachers in the district are well aware of the constructivist underpinnings of the USI, hold emerging understandings of constructivism, and are beginning to implement changes in their teaching practices.

The National Council of Teachers of Mathematics (NCTM) has provided a major impetus for reform in school mathematics through its development and wide dissemination of standards documents (NCTM, 1989, 1991). Indeed, the education departments in no less than 41 states have used the *Curriculum and Evaluation Standards for School Mathematics* as a model for revising their state guidelines in mathematics, while the *Professional Standards for Teaching Mathematics* suggests new models for teaching, learning, and staff development. Clearly, bringing to life the vision of school mathematics which arises from these *Standards* documents requires fundamental changes in what occurs in most classrooms.

In recent years, the National Science Foundation (NSF) has attempted to stimulate reform in school science and mathematics education through systemic initiative grants. First at the state level and later at the level of individual urban districts, NSF instituted State and Urban Systemic Initiatives (SSIs and USIs) in an effort to bring order and focus to what the agency perceived as a haphazard and unstructured approach to professional development and reform efforts. By focusing change initiatives on a large system, it was thought, past problems in effecting real change in classrooms might be avoided.

However, Cuban (1987) holds little hope for the success of state-wide reform initiatives, calling the notion that state agencies can mandate change in classrooms "defective at its core" (p. 23). Moreover, Fullan (1982, 1991), who has carefully studied the process of teacher change over many years, has noted that change is a highly personal experience and a difficult, lengthy process. Sowell and Zambo (1997) also suggest that large-scale reform efforts frequently fail to take into account the interaction between teachers' beliefs and the proposed reforms. There is support for the importance of teachers' beliefs to the process of change in practice in Thompson's review (1992) of the literature on mathematics teachers' beliefs, which posits a dialectic relationship between beliefs and practices.

These viewpoints appear to mirror what has actually occurred. Sowell and Zambo (1997) note that studies of state-wide mathematics reform efforts in California and Kentucky have apparently produced no evidence of wide-spread change in classroom practice (see, for example, Ball, 1990). Sowell and Zambo also report that the effort to change mathematics teachers' practices in the State of Arizona "has produced little reform in classrooms" (p. 354).

Theoretical Framework

From a socioconstructivist perspective, the process by which mathematics teachers change their practice of teaching is similar to the constructive process by which students learn mathematics. Fostering change in teaching practice requires changing teachers' beliefs about the learning-teaching process and about the nature of mathematics itself. Just as students who are learning mathematics must experience a perturbation of their existing ideas, and then reconstruct ever more powerful ideas, so also must mathematics teachers who are attempting to change engage in a similar process (Nelson, 1997; Schifter, 1996; Schifter & Fosnot, 1993; Simon & Schifter, 1991).

In light of the foregoing, those who would design systemic reform initiatives in mathematics education should recognize that change must occur in individuals before it can occur in systems. They should understand that change is a difficult process that takes time and support, and that real change in mathematics teaching practice is fundamentally interwoven with individual teachers' beliefs about learning and teaching mathematics, and about the nature of mathematics itself. Consequently, they should design professional development activities for teachers that are long-term, that provide contexts within which teachers can examine the proposed reforms in light of their current beliefs and practices, that aim to help teachers examine and reflect upon their current beliefs, and practices, and that help teachers develop a vision of what the proposed reforms might look like in their own classrooms.

Methodology

Perhaps a fundamental problem with the concept of a State Systemic Initiative is that the unit of analysis is simply too large and complex. The authors have been members of the external evaluation team during the first four years of an Urban Systemic Initiative (USI) in a large city in the midwest. At the outset of the USI grant, the school district commissioned the writing of a constructivist vision statement (Stein, Edwards, Roberts, Norman, Sales, & Alec, 1994) which served as the theoretical underpinning for the entire project. Nine principles which support a socioconstructivist orientation to teaching form the basis of this vision statement:

- In order to understand, students must actively construct their own meanings;
- Learning depends upon students' prior understandings;
- What, and how much, is learned depends upon the context in which learning occurs;
- What is learned depends upon the shared understandings that students negotiate with the teacher and with each other;
- Effective teaching involves engaging students at their present levels and helping them to move to higher levels;
- Specific teaching methods can foster students' active constructions of knowledge;
- Learning how to learn is an important part of the teaching-learning process;
- Continuous assessment can facilitate learning; and
- Teachers should become aware of the constructive nature of their own learning.

Early in the first year of the USI grant, the school district identified benchmarks which determined "levels of change" for individual schools:

- Tier 1, or the level of implementation,
- Tier 2, or the level of preparation for implementation, and
- Tier 3, or the level of becoming aware, ready, and committed to implement.

Over the course of the first four years of the USI grant, appropriate staff development activities were planned and conducted for teachers from schools at each level of change.

During annual formative USI evaluations, the evaluation team sought evidence of change from a number of sources. Data were drawn from:

- observations of a sample of the staff development activities;
- survey questionnaires administered to students, teachers, and administrators;
- focus groups conducted with teachers and administrators; and
- case studies of individual schools.

Members of the evaluation team observed 12 to 16 staff development sessions each year, often participating in the activities right along with the teachers so as to better gauge the teachers' understandings of the activities. These evaluators kept fieldnotes of each session and wrote summary reports of their observations.

Separate but parallel survey questionnaires appropriate for use with students, teachers, or administrators were developed by a panel of experts which included members of the evaluation team and mathematics and science education specialists from the district. The questions addressed in the focus groups were similarly developed, and the focus group discussions were audiotaped.

The case studies typically involved examining the school's "action plan," observing teachers' work in the classroom, interviewing individual teachers and administrators using an interview guide developed by the evaluation team, and examining samples of student work. Some of the case studies were conducted by a single evaluator; others were conducted by a team of evaluators consisting of one science and one mathematics education specialist.

The staff development data were analyzed to identify patterns suggesting the extent to which staff development activities modeled the vision of teaching and learning presented in the constructivist vision statement, as well as the extent to which teacher-participants were aware of, and able to make sense of, constructivist orientations to teaching. The survey data were analyzed to identify items for which there was agreement across all three groups that were sampled, and which tended to confirm or disconfirm that the goals of the USI were being achieved. The data collected from the focus groups were similarly analyzed, and agreement between the focus group data and the survey data was sought. Finally, the case study data was analyzed for patterns suggesting the extent to which mathematics instruction in the case study schools mirrored the vision of mathematics teaching and learning articulated in the constructivist vision statement. Throughout all of the data analysis, there was a strong emphasis on the identification of multiple sources of information which support interpretations of the data.

Results

The analysis of these data provides evidence that many teachers who have undergone staff development activities provided by the USI are well aware of the constructivist underpinning of the proposed reforms and have developed a working understanding of those underlying principles. The data analysis also indicates that significant and worthwhile changes are occurring, albeit somewhat slowly, in many classrooms throughout this school district.

Most of the early activities of the USI focused on providing staff development which supported a constructivist perspective of mathematics teaching and learning. All mathematics teachers in the district were afforded multiple staff development opportunities during each year of the USI.

Evaluator fieldnotes from the staff development observations provide evidence that information on constructivism was incorporated into all of the programs, the importance of reflection and writing was stressed, teachers were active learners themselves as they participated in "hands-on" activities, cooperative and collaborative learning was fostered, and an emphasis was placed on changing programs and strategies to fit students' needs. During staff development activities, the teachers observed frequently used the term "constructivism," indicated that they had read *A Constructivist Vision for Teaching, Learning, and Staff Development,* and demonstrated an emerging understanding of this perspective. This interpretation is also supported by data from the teacher surveys, focus groups, and teacher interviews conducted during the case studies. These data leave little doubt that many of the mathematics teachers in this school district are now embracing a constructivist orientation to teaching.

Analysis of the survey data, as well as classroom observations, indicate the success this USI has achieved in bringing to life a vision of constructivism in mathematics classrooms. Analysis of teacher surveys indicates that most of those responding have implemented at least one strategy drawn from their staff development activities, and this contention is supported by the analysis of student surveys. Moreover, classroom observations made in Tier 1 classrooms suggest that most of those mathematics teachers who were observed are implementing strategies and approaches that were presented in the staff development sessions. For example, during one year of the evaluation, over 90% of the teachers surveyed reported using group activities "weekly or more often," 86% of the students surveyed indicated that they worked in small groups "sometimes" or "almost everyday," and, in 65% of the classroom observations, it was reported that cooperative learning was used. It is also worth noting that nearly two-thirds of the teachers who responded to the survey say that they desire additional staff development.

Discussion

An awareness and understanding of the underlying philosophy which is driving the reform, coupled with observed changes in classroom practices, is perhaps the most striking finding in the evaluation of this USI. This may well be evidence that these teachers have examined and reflected upon the proposed reforms, come to their own understandings of the reforms, reconciled the proposed changes with their belief structures, and are beginning to implement change in their classrooms.

On the other hand, because data collection began at the end of the first year of the USI, it was not possible to collect baseline data. Without such a baseline, it is difficult to assess change in teachers' practices over the course of the USI grant. However, our interpretation of the data does indicate that a shift toward a constructivist orientation to teaching has occurred at some point in time for many of the teachers in this district.

While the four-year life of this USI grant may seem to be a relatively long period of time, it is noteworthy that four years is just nearing the five to seven years that Fullan (1991) has suggested are necessary for real and lasting change to occur. A number of changes in mathematics teaching have been documented in this district. It remains to be seen how lasting these changes will be.

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CASES AS CONTEXTS FOR TEACHER LEARNING[1]

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"Knowledge in teaching...is particularistic and situational, intimately connected to the contexts and events of teaching. Consequently the most effective way to represent knowledge in teaching is through cases that capture both the routines and the problematic, unique situations that call for reflection, analysis and continuing inquiry" (Sykes & Byrd, 1992, p. 513). However, those who argue that cases are powerful learning tools also caution about the potential difficulties of case use. Despite the cautions, we actually know very little about the parameters and conditions under which cases are most useful. Our goal is to design a process for documenting teacher learning in the context of cases and to use those methods to study the effectiveness of two particular case types. Specifically, we examine the use of cases that use the development of children's mathematical thinking as their core. One case type is more typical and includes classroom contexts where teachers make use of children's thinking. The second case type uses teachers own student work as the basis of the case.

Introduction

The results of prior case-based instruction studies point to wonderful successes in using cases with teachers (Barnett, 1991), mixed results when looking at mentor and novice teachers (Levin, 1995), and cautions about the ways in which cases may or may not be effective (Williams, 1992). Some case work enables us to understand the importance of thinking about the use of cases (Adler, 1996; Tillman, 1995). Others raise issue, both within and outside of the education arena, about the degree to which cases lend themselves to the abstraction of concepts and principles (Williams, 1992). Beyond these individual studies we also are beginning to build comprehensive ways of thinking about the issues surrounding the use of cases within teacher education (see Doyle, 1990; Grossman, 1992; Merseth,

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1996; Shulman, 1992; Sykes & Byrd, 1992) Thus, combining these ideas we have information that helps us think about how to construct cases, work with the teachers, and understand what needs to be studied. However, we have little research information about the parameters of effectiveness of cases with teachers.

Our goal in engaging teachers with case based examples has been to provide teachers opportunities to (1) begin to understand the principled ideas underlying the development of children's mathematical thinking (2) create an understanding of how to make use of children's mathematical thinking in their classrooms and (3) help teachers see that their classrooms can be sites for their ongoing learning. We want to help teachers see themselves as learners, whereby their changing knowledge, beliefs, and practice become self-sustaining and generative (Franke, Carpenter, Fennema, Ansell, & Behrend, in press; Schifter, 1997; Sherin, 1997). Our central question has been whether the case-based approach allows teachers to make connections between the development of children's mathematical thinking and their classroom practices so they can continue to grow. We have designed a process for documenting teacher learning from cases. We examine teacher learning in two case types, both with the development of children's mathematical thinking at their core: 1) classroom cases and 2) student work cases. We were most concerned with examining the differences in the use of these two types of cases as they pertain to teachers' ongoing learning.

Method

For a year we worked with one school of kindergarten through fifth grade teachers. The 30 teachers from this urban school, serving predominately low income, native Spanish speaking children, engaged in monthly workgroup meetings. During the monthly workgroup meetings, half of the teachers discussed a classroom case: an example of a teacher using children's thinking. These cases were drawn from other professional development work with teachers. The other half of the teachers discussed cases of student work that they brought from their own classrooms. We documented the teachers' beliefs, knowledge and classroom practices throughout the year.

Procedures

Kennedy Elementary School is a year around school, with four tracks. At any one time three tracks of students and teachers are in attendance. Each track hosted a teacher workgroup. Within each track between 4 and 12 kindergarten through fifth grade teachers participated in the monthly meetings. Two of the tracks were assigned to the classroom cases while the other two tracks were assigned to the student work cases. The teachers within each workgroup varied in their beliefs and knowledge related to the teaching and learning of mathematics and in their teaching of mathematics. The school as a whole, however, has a reputation of being skill oriented in mathematics. Each month the teachers met for one and a half hours. The teachers began each meeting by bringing in student responses to a problem that all participating teachers posed to their students. All teachers filled out an initial reflection sheet that asked them to either talk about the classroom case or their own student work. We discussed the classroom case or the student work for approximately an hour. The teachers then spent the last 10 minutes summarizing verbally what they had learned during the conversation.

The Cases

All of the cases addressed the critical aspects of the development of children's mathematical thinking in the domains of addition/subtraction, multiplication/division and place value (see Carpenter, Fennema and Franke (1996)). Classroom cases were created from examples of real teaching episodes: teachers working in classrooms with children, where children's thinking was central. Not all of the cases presented exemplary pedagogy; however, each raised issues related to the integration of children's thinking and classroom practice for discussion. The cases were not designed to leave the teacher with a critical question to answer or a given problem to solve; rather, the cases allowed teachers to see classroom settings where children's thinking was the focus. The problematic nature of the classroom cases came from the teacher matching what he or she read with their own thinking and then discussing consistencies and inconsistencies. The cases we developed are more consistent with Doyle's (1990) knowledge and understanding category - where teaching knowledge is imbedded in the case - although knowledge is also brought to the case. The cases instantiate theoretical knowledge about teaching. Our cases also served to engage the teachers in problem solving and decision-making related to these theoretical, research based ideas.

The student work case examples drawn from the teachers themselves are work samples and descriptions of students' thinking. Each month the teachers posed the same problem to their students. They brought at least three examples of student work on the problem to the workgroup meeting. The teachers were not directed about which student work to bring.

Data Collection

We collected four types of data from the teachers in order to understand the impact of these cases. First, we interviewed and observed the teachers before the first workgroup meeting in order to document their initial beliefs and knowledge about the teaching and learning of mathematics and the degree to which they saw themselves as learners. We conducted a final interview with the teachers that elicited the teachers' views about the teaching and learning of mathematics, specifically in relation to the development of children's mathematical thinking, their views about their classroom practice and how they felt the cases influenced their thinking and practice. Second, we collected data related to the teachers' thoughts about the cases before group discussions. Then we audiotaped and took field notes during the case discussions. Third, we tracked the ways in which the teachers adapted the problem we gave them to pose to their students. We also kept track of the student work that they elicited. Fourth, we observed in the teachers' classrooms at least once in between the workgroup meetings to see what, if any, impact the case discussions were having on the teachers' classroom practice.

Results

Our results at this point are preliminary, however, we have learned a great deal through our work with the teachers this year and through our final set of interviews and observations of the teachers. The most striking result at this point is that at the end of a year of workgroup meetings, all of the classroom case teachers decided they would rather not engage with these cases any longer, while all of the student work case teachers have chosen to continue with the student work cases. Most notably, the classroom case teachers reported wanting to focus on their student work. The teachers request to focus on their student work comes, not from conversations with their colleagues, but from posing a monthly problem to their students. These teachers posed the same problem each month to their students as the student work case teachers. However, rather than discussing the student work explicitly these teachers discussed a classroom case that used the same problem structure. The teachers did not see these classroom cases as relevant to their students in their classrooms.

We have seen that asking all of the participating teachers to pose the same problem to their students has been powerful, especially if the student work is discussed. The teachers had an opportunity to talk across grade level about what it would mean for the students to solve problems in different ways, the teachers had an opportunity to see what mathematical ideas might develop when students go on to the next grade and what students may bring to their classrooms from their previous grade. Having a problem to pose their students also immediately engaged the teachers in thinking about the relationship between their children's mathematical thinking and their classroom practice. Posing the problem led teachers to engage in practical inquiry. The teachers would pose the problem with one set of numbers and then adjust the numbers and pose it again to see what the students would do. Engaging in practical inquiry at this level did not occur for all of the student work case teachers, however, all but a few of the teachers became intrigued with their children's mathematical thinking.

The student work case teachers spoke about how the professional development we engaged them in was different from other professional development. The main distinction the teachers pointed out was that our professional development focused on their own students in their own classrooms over the course of a year. So from the teachers' perspective the professional development was not a one shot deal where they had to figure out how to use, but rather our professional development provided an opportunity for them to think about how to change their teaching of mathematics in an ongoing and substantial way. Although the student work case teachers were enthusiastic about their learning, there were differences in what the teachers learned about the development of children's thinking, their teaching of mathematics, and their own student's mathematical thinking. We hypothesize at this point that the student work workgroups provided more opportunities for engaging teachers in self sustaining generative change, but it is a bit early to tell.

Discussion

Engaging with the two case types over the past year has provided us with an enriching learning environment of our own. We set out to contrast the two case types in a systematic and rigorous way. The details of what we have learned are still to be determined and reported. However, we do now feel that the teachers in the classroom case group would have benefited from engaging with student work cases. We think that these cases, especially initially, would have provided a mechanism for the teachers to see the discussion as relevant to their own students in their classrooms. We think these student work cases would have engaged them more readily in practical inquiry. However, we also believe that the student work case group may be at a point where classroom cases would push their thinking and provide a forum for a different type of conversation. We will continue to investigate this as we work with the teachers next year. We have found that we need to carefully think about and study when to use what types of cases with teachers and that the use of cases in and of themselves are not the answer to the professional development needs of teachers.

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PRESERVICE ELEMENTARY TEACHERS LEARNING TO INTEGRATE MATHEMATICS WITH OTHER SUBJECTS THROUGH INTERDISCIPLINARY INSTRUCTION

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The purpose of this study was to have preservice elementary teachers learn about different ways to integrate mathematics with other subjects through interdisciplinary experiences in methods courses, and to explore their ideas related to designing and implementing their own interdisciplinary instruction. Specifically, this paper will focus on describing the ways these preservice teachers designed interdisciplinary instruction involving mathematics and their ideas about integrating mathematics with other subjects. Implications for teacher education programs are provided.

Teachers are being encouraged to use interdisciplinary instruction, especially at the elementary school level. This implies that future elementary school teachers need to be prepared for designing, implementing, and assessing the results of interdisciplinary instruction. In particular, preservice elementary teachers need to know how to integrate mathematics with other subject areas such as language arts, social studies, music, art, and science, especially if they will be expected to use such instruction in the future. Knowing how to integrate mathematics instruction with other subjects involves experiencing interdisciplinary instruction, and practice with designing and implementing such instruction.

Theoretical Framework

Interdisciplinary instruction implies instruction that involves more than one discipline. Typically, when thinking of integrating mathematics with other subjects, one thinks of natural connections with science. Recently, suggestions for connecting mathematics with literature have been quite popular (Burns, 1992; Thiessen & Matthias, 1992; Whitin & Wilde, 1995). Using literature in mathematics class also opens the door to connections with other subjects such as social studies (e.g., Smith, 1995). Music and art also provide many opportunities for mathematical connections. Preservice teachers need to be aware of and familiar with examples of combining mathematics with many different subject areas to be able to design and implement interdisciplinary instruction with future students.

Interdisciplinary instruction may occur in several different ways (Jacobs, 1989; McKinnon, Nolan, Openshaw & Solar, 1991). For example, McKinnon et al. (1991) identify three ways that interdisciplinary instruction occurs: (1) the teaching of one subject in the context of another, (2) the linking of subjects by addressing a common theme, and (3) the application of knowledge and skills from a variety of subjects to study selected issues and problems. Specifically, Welchman-Tischler (1992) describes several ways to link mathematics and literature such as providing a context, posing an interesting problem, and developing a mathematics concept or skill. Schiro (1997) distinguishes between using children's literature as a springboard into mathematics activities and integrating mathematics and literature through mathematical literary criticism and editing. The latter method of integrating mathematics with literature seems to be aligned with a definition of integration by Schmidt et al. (1985) as "the *purposeful* intertwining of subject matters to achieve multiple goals" (p. 307). Preservice teachers ought to be aware of the different ways that mathematics could be linked with or intertwined with other subject matter. Not only is it important for these preservice teachers to read and hear about successful interdisciplinary instruction in classrooms, but it is at least equally, if not more, important for them to experience the kinds of interdisciplinary instruction that teacher educators model and hope are eventually implemented by these preservice teachers in future teaching experiences. This paper will describe the ways that preservice teachers who experienced interdisciplinary instruction in their methods courses designed interdisciplinary instruction involving mathematics and their ideas about integrating mathematics with other subjects. Implications for teacher education programs are also included

Method

Forty-six preservice teachers were enrolled in two sections of an elementary mathematics methods course in the semester just prior to student teaching. They ranged in age from 20 to over 40 years old, and ten of the preservice teachers were male. These preservice teachers were also enrolled in the following elementary education methods courses: reading, music, special education, and either science or art. The three university faculty who taught the mathematics, reading, and music methods courses collaborated with each other to provide team-taught interdisciplinary instruction and common assignments across their courses. Common assignments included designing an interdisciplinary unit, and designing at least three lesson plans that each integrated two or more subjects. The mathematics and reading methods faculty emphasized mathematics and literature connections in a variety of ways throughout the semester. Therefore, the preservice teachers were required to write at least one lesson plan linking mathematics and literature in some way. In addition, the preservice teachers were encouraged to try out this lesson with elementary school students during their field placement.

Qualitative methods were an appropriate vehicle to explore the views of preservice elementary teachers regarding interdisciplinary instruction. Sources of data included preservice teachers' written reflections on class discussions and activities, copies of preservice teachers' responses to common course assignments, videotapes of team taught interdisciplinary instruction class sessions, and an open-response questionnaire administered at the end of the semester. The data were examined for emerging patterns and themes regarding students' views of interdisciplinary instruction, and their specific ideas about how to integrate mathematics with other subjects.

Results

The preservice teachers thought about interdisciplinary instruction in at least two distinct ways. The preservice teachers participated in interdisciplinary instructional activities, modeled by the faculty, involving mathematics, reading, writing, and music around a common theme. Then nine groups of 5-6 preservice teachers each were asked to brainstorm and make a web of instructional activities for a thematic interdisciplinary unit. Five of the groups constructed webs for an interdisciplinary unit by listing each individual subject (e.g., mathematics, reading, music) and then listing specific activities for each subject (e.g., calculate the time needed to travel from earth to another planet for mathematics, read a book about space travel for reading, make up a song about traveling in space for music) that related to a central theme (e.g., Traveling in Space).

The other four groups thought of theme related activities that combined several subjects. For example, one group chose Halloween as a theme and described one activity that involved carving pumpkins. From this one activity, the group generated ideas about integrating art (carving the pumpkin), social studies and reading (learning about the history of pumpkin carving), mathematics (measuring and weighing the pumpkins before and after carving), science and writing (plant the pumpkin seeds and keep a journal), and music (make instruments using pumpkin seeds).

These examples suggest that some preservice teachers preferred to think of the different subject areas first and then brainstorm theme related activities for each subject, which may indicate their thinking about interdisciplinary instruction as "parallel disciplines" (Jacobs, 1989), or linking subjects by addressing a common theme (McKinnon et al., 1991). Other preservice teachers preferred to think of theme related activities first and then identify the subject areas involved in each activity. This strategy for designing an interdisciplinary unit might reflect thinking about interdisciplinary instruction as teaching one subject in the context of another (McKinnon et al., 1991) or combining several subject areas for a more holistic approach. These two ways of thinking about designing interdisciplinary instruction were also reflected in the webs generated by individual preservice teachers for their interdisciplinary unit common course assignment. More than half of the preservice teachers appeared to generate ideas for their interdisciplinary units by listing different subject areas first, and then listing instructional activities related to the unit theme for each subject area.

After a team-taught class session on developing interdisciplinary units, one preservice teacher commented, "I learned how you can integrate almost every content area imaginable into [a lesson plan], including some areas I didn't think that could be integrated, such as social studies and math." Other preservice teachers commented about their new awareness of how "easy" they thought it was to generate ideas for instructional activities around a theme. Several preservice teachers noted the benefits of brainstorming as a group to maximize the number of ideas for instructional activities connecting different content areas.

Since the mathematics and reading methods courses emphasized connections between mathematics and literature, it was not surprising that many comments from students related to connecting these two subjects. All of the lesson plans written by the preservice teachers that focused on linking mathematics with literature used the following format (with minor variations): the teacher reads a story to the class and then the elementary students pose, write, and solve problems based on the content or context of the story. The objectives of these lessons emphasized communicating mathematical ideas clearly through writing problems and explaining solutions (orally or in writing). This is consistent with a majority of preservice teachers who made comments similar to "[writing] is a powerful tool through which understanding and creativity can be fostered" and "including writing into the learning of mathematics can really help pull things together when integrating math with other subjects." Designing a lesson where elementary students would write and/or solve story problems based on the context of a book related to the unit theme was a "quick and easy" way to incorporate mathematics with other subjects from their point of view.

In general, a dominant theme throughout the students' reflections on their experiences of the interdisciplinary class sessions was the preservice teachers' appreciation of the modeling of interdisciplinary instruction by the faculty. Comments that reflected a majority of the responses included: "I found today's experience to be quite valuable. Seeing theory put into practice helps clear up any misconceptions," and "[a]s I continue to witness integration, I become much more confident and educated in exactly *how* to integrate. The more exposure I have to the process, the more confident I become as a teacher … learning is so much more meaningful when teachers model and 'practice what they teach.'"

Conclusion and Implications

Preservice teachers think about interdisciplinary instruction in different ways. Many struggle with designing instruction that connects mathematics with other subjects, even after experiencing several interdisciplinary activities. Most of the preservice teachers in this study were able to eventually connect mathematics with other subjects through successful links with literature. This study provides further support for the model of teacher education where learners of "new" instructional techniques need to experience these methods first as students. Modeling appropriate teaching practices is necessary, although insufficient, for the process of learning to teach. While several preservice teachers were able to actually try out interdisciplinary lessons with elementary students, more opportunities are needed for preservice teachers to try out, modify, and refine such lessons. Teaching experiences involving interdisciplinary instruction and the opportunity to reflect on those teaching experiences are also important components of teacher education. The results of this study provide valuable insight into how preservice teachers initially think about and develop interdisciplinary instruction. The next step is to follow these preservice teachers into their student teaching experience to see how their ideas about interdisciplinary instruction are maintained, changed, or modified based on their new teaching experiences.

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USING CONCEPTUAL DESIGN TOOLS TO FOSTER LEARNING IN THE GAME DESIGN ENVIRONMENT

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We propose game design as a learning environment for teachers to build on and challenge their existing understandings of mathematics and engage in relevant and meaningful learning. Sixteen preservice teachers organized in four teams participated in game design sessions. We analyzed the various games of the teachers to understand how they conceptualized the task of creating virtual game learning environments for others and to understand how conceptual design tools can provide a common platform to develop meaningful fraction contexts. We found that when we provided teachers with conceptual design tools such as game screens and design directives that facilitated an integration of content and game context, the games and the teachers' thinking increased in their sophistication. In the discussion, we elaborate on how the conceptual design tools improved the instructional design process.

Introduction

Traditionally, instructional design activities are created by those outside the classroom, like curriculum designers and content specialists. Here, we suggest that teachers, along with their students, become instructional designers themselves and instructional design becomes the forum for problematizing (Hiebert et. al, 1996). Problematizing sets the parameters for a learning environment where teachers and students engage in mathematical inquiry that leads to the creation of ongoing learning opportunities.

In considering a context for instructional design, educational games as such do not come naturally to mind. Games, especially video games, have become a significant part of children's culture in the late 20th century (Provenzo, 1991). A considerable number of research studies point out that teachers need to link their instructional practice to culturally relevant materials and activities (Lave, 1996; Resnick, 1987). Many teachers have used educational games for their motivational benefits. We build on these motivating and culturally relevant aspects of games but with a distinct difference by having teachers construct their own educational games—and to reexamine their mathematical understanding in the process.

Previous research studies point out the learning benefits of engaging students and teachers in instructional design (Baylor, 1997; Harel, 1991). But this work also indicates that while students definitely benefit from the process of instructional design, the quality of products created did not equal or represent in most cases the quality of the instructional process. Students within these instructional design projects often followed well-trained routines in learning fractions by representing decontextualized problems with a focus on understanding fraction operations rather than concepts. Our current study attempts to address the imbalance—"good process but weak product"—found in traditional instructional design for learning. As teachers place themselves in the role of the teacher and the learner, we expect them to examine their instructional principles, reflect on how they incorporate both children's mathematical thinking and the mathematical content, and consider how they can provide explanatory representations and activities (Carpenter, Fennema & Franke, 1996). In this process, teachers construct mathematical representations, strategies, and contexts that become available for examination by others in their design of educational games (Lampert, 1989; Streefland, 1991).

Methods

Data Collection

Sixteen pre-service teachers from a teacher certification program that were organized into four teams participated in the game design sessions. The teachers participated in three consecutive sessions lasting an hour each. Each team was asked to design a video game to teach fourth grade students about equivalent fractions. After twenty minutes, each team presented their final game designs. Each presentation was followed by a class discussion in which the preservice teachers evaluated each other's game designs and raised questions about the game designs and their potential effectiveness.

In the second and third game design sessions, the teachers designed games after they were introduced to the conceptual design tools. "Conceptual design tools" were design directives given to teachers after their presentation of initial game designs. One conceptual design tool came from the researcher asking the game designers to "create a game without asking questions" as many of the initial games simply consisted of posing questions or testing students. Another conceptual design tool shared was a blank page of empty computer screen frames, two to a page, on which the teachers could draw and annotate their game designs. A third conceptual design tool focused teachers on the issue of dynamic representations by sharing with them an example from the students' designs where the students created a dynamic representation to teach fractions. Finally, teachers were asked "to use fair sharing" as a teaching context for fractions.

Analysis

All three of the teachers' game design sessions were video taped. In addition, we followed one team of game designers throughout all of the design sessions by recording all of their group interactions and discussions. We transcribed all of the videotapes and annotated the actual game designs using the teachers' own words from their descriptions of their games. While it was obvious that within the short time frame of one design session the teachers would not be able to generate many full-fledged games, it became apparent that the game ideas proposed by the teachers were of different specificity. Using the transcription of the group presentations and the ensuing class discussions, the twelve games created by the teachers were evaluated using the coding scheme described in Table 1. Each game idea developed by the students and the teachers was analyzed according to the following categories: (1) degree of fraction integration into the game's narrative or theme, (2) type(s) of fraction representation used in the game, and (3) degree of consideration displayed by the designers in accommodating the users' thinking processes. Each game for each session was coded separately by two coders.

Table 1
Game Design Coding Scheme

Codes	Operational Definition
Integration	
extrinsic	Game idea and fraction content are separated
intrinsic	Game idea is merged with fraction content
constructivist	Game facilitates players' making of their own fraction content
Representation	
none	Player does not see or have access to any fractions
representation	Player has access to fraction symbols
symbolic	Player has access to a pictorial fraction representation but cannot manipulate
fixed pictorial	Player has access to multiple fixed representations where the pictures differ
fixed multiple	Player has access to multiple representations that can be manipulated
free multiple	
Consideration of User's Thinking	
none	Issues of children's thinking contexts are not apparent in the game
problems	Problems are designed to afford opportunities for children's thinking
	Design includes thought to different strategy possi bilities as well as problems
strategies	

Results

Overall patterns showed that after introduction of the conceptual design tools, the teachers shifted from extrinsic game designs, where fraction content and game idea were separate, to intrinsic game designs, where idea and content were integrated. In a few cases, the game designs even allowed the player to create and, thus, naturally blend content with game idea. Similarly, after introduction of the conceptual design tools, there were shifts in the representations employed in the game designs. The representations changed from either no productive representations or only symbolic representations to the use of pictorial representations, and in some cases to multiple representations where the player could choose his representation from some given alternatives. In consideration of the user, designing a more sophisticated game required the designer to consider alternatives for a player's thinking and in some way tailor the game to the player's thinking. Not all of the teachers were successful at reaching this level of sophistication, yet this level was attained by some of them. In their game designs, the teachers also drew on their knowledge of the development of children's thinking. The games designed by the teachers included design elements that took into account what research tells us about the different ways to vary problems and the range of solution paths students may use in addressing those problems. The teachers designed games that to varying degrees accounted for the players' development and learning.

Discussion

Clearly, the preservice teachers learned as they engaged in game design. Our analyses suggest that the learning occurred not only because the teachers engaged in game design but as a result of the use of the conceptual design tools. The use of conceptual design tools allowed us to engage the designer in thinking about the process while still maintaining sight of the product. Often within instructional design we see a shift away from the focus exclusively on the product to a focus on process. However, what occurs then is that the product becomes lost, and what becomes produced is disappointing even though the process "worked." Essentially we asked the designer to try and think concurrently about the process and the product: they thought about designing a game for someone else to play. But the task itself, designing the game, was not enough to insure that the designer remained focused on both. This occurred as a function of the conceptual design tools. The story screens provided a concrete way for the designers to work through their design and served as a reminder that the sequence of the game was critical for the player. Designing a game without asking a question focused the designers on the relationship between their design and what the player would be doing and learning.

The conceptual design tools pushed the designers to think about both process and product in three specific ways. First, the conceptual toolbox included tools that enabled the teachers to create externalizable and share-able representations (game design screens). Second, the toolbox included tools that encouraged a focus on learning and the learner (Lave, 1996). Rather than thinking about how to design a game that teaches, the designers turned it around to 'How can I design a game in which the student will have the opportunity to learn?' Here then, new design space was created; the designer created room for the player to negotiate the design space using their existing knowledge and skills. And third, the tools led the teachers to think about principles rather than to focus on the procedures. The question posed to the preservice teachers about designing a game 'without asking a question' pressed teachers to come to think about how to engage students in understanding rather than practicing. As they focused on the principled ideas, the designer looked beyond routines to create opportunities for learning big ideas. The conceptual tools taken together problematized the situation for the designers, pressed them to problematize situations for the player and thus connected a rich process with a dynamic product.

Game design in fractions afforded opportunities for teachers to think about contexts that were mathematically powerful and to engage in ongoing reflection about the teaching and learning of mathematics. Game design provided a situation that naturally combined issues of practice and theory, and reflection on those relationships and game design provided opportunities for discussion, reflection, and collaboration within a meaningful context. Further, the parallels we have seen in the game design of the preservice teachers and students (in our prior work, Kafai, et. al, in press) demonstrates how game design can become a powerful learning context for teachers and students in classrooms together.

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REDEFINING "THE OBJECT" OF ASSESSMENT IN CLINICAL INTERVIEWING

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This paper argues that clinical interviews may actually reveal more about teacher-interviewers' conceptions than about children's knowledge. Three levels of teachers' object relations with mathematics are described to explain how elementary teachers' approaches to clinical interviewing are reflected in children's responses.

The purpose of this report is to share some evidence and a framework we have developed regarding the roles and perspectives elementary teachers take in relation to the use of clinical interviews for student assessments in mathematics. While we take the view that clinical interviews reveal a great deal about students' knowledge of mathematics, we argue that the efforts of teachers as interviewers may reveal more about teachers' conceptions and the learning context than about children's knowledge. This view is consistent with Derry and Potts' (1998) observation that the shaping of students' behavior is determined by the constructs tutors use to describe and categorize students in the context of the tutoring situation. Yet, we find that there is scant information in the literature regarding the significant pedagogical effects of teachers' thinking and affective displays toward mathematics as evidenced in the clinical interview.

Current reform approaches (NCTM, 1995) provide support for concentrated efforts by teachers to introduce non-test-based forms of assessment into their classrooms. The underlying constructivist tone of this thrust and its subsequent manifestations in state-level implementations emphasize attempts by teachers to understand how their students "are thinking" as a legitimate component of their classroom practices. As Davis (1996) notes, "...the listening teacher works with the contingencies of the particular classroom setting. It is founded on the realizations that no learning outcome can be prescribed, no active setting can be controlled. But neither must we forego attempts to influence (or fail to acknowledge our influence upon) what might come about." (p. 271). This report examines the clinical interview as an embodiment of the relationships described by Davis at the heart of non-test-based assessments of mathematical thinking.

Our presentation is based on several years of experience in introducing clinical assessment techniques to pre-service and advanced elementary mathematics teachers in a variety of contexts (Appelbaum & Kaplan, in press; Ginsburg, Kaplan, & Baroody, 1992) including the viewing and sharing of reactions to videotaped models of clinical interviews with children and engaging teachers in their own experiences in conducting short clinical interviews. For this report we highlight examples from prototypical cases of teachers' videotaped interviews of children, each of which illustrates a key perspective of our framework for understanding of the teacher's defining role in clinical interviewing. The framework we use applies object relations theory (Scharff, 1996) to describe the quality of the interviewer's attachment to mathematics and the impact of this attachment upon what is or is not revealed about the child's thinking.

We note that during the clinical interview, teachers' questions, responses, as well as their initial selection of an interview task, suggest at least three possible levels of object attachment to mathematics, each of which parallels a comparable level of engagement with mathematics that the children interviewed display. We have identified a first level of attachment in which teachers regard mathematics as a collection of facts and prescribed procedurally accurate answers that have nothing to do with one's sense of self. In our second level of attachment the teacher's focus on procedural correctness is supported by a more personally involving appreciation of logic and applications of mathematics. We postulate yet a third level in which teachers regard mathematics as part of the positive self and view the world quite naturally in mathematical terms expressed in a dialogue of mutually heightened mathematical thinking during the interview.

At the first level, mathematics is something of an alien content or body of knowledge to which the interviewer is minimally connected and which, in general, has nothing to do with definitions of self. Such teachers essentially engage with mathematics as a rote activity, the rules of which have been determined by others. Children they assess appear to be similarly oriented toward mathematics. During the clinical interview, teachers at this level can react in one of two ways, acceptance of only one type of response or acceptance of any type of response.

In the case of single response acceptance, we find that as interviewers these teachers may listen for accurate recall and statements of fact to questions they pose, questions that for them have only two kinds of answers - correct or incorrect. They do not go beyond the answer given, but greet each utterance with correction or praise followed by a predetermined new question not based on the child's response. For example, Mary interviewed Josh at the end of first grade to find out what he understood about place value and the writing of numbers. She began the interview by asking Josh to write down the number 5 followed by the number 7, both of which were easily executed by the child and praised by the interviewer. Mary then warned Josh that the numbers would be getting bigger and asked him to write 10 and 23. Again his response was accurate and praised. Finally Mary told Josh that she would give him a really big number and said "one hundred and twenty four." Josh responded by writing "10024" to which Mary reacted by saying, "Look at your number Josh. You have 100, good, but you wrote 100 and then 24. Try it again. The number one. Try that and then the number 24." Josh rewrote the number conventionally and Mary responded with praise then asked Josh to write a new number, "one hundred and fifteen." This time the child looked at his previous answer and correctly wrote "115." Mary praised him, saying, "Terrific, you know numbers" and then went on to a completely different task. At this point Mary probably believed that Josh understood place value rules for writing 3-digit numbers and Josh felt he has done a good job based on Mary's directed feedback. However, neither Josh nor Mary was clearer about the child's conception of place value and number writing than when they began, and, in fact, what the interview accomplished was to help them both maintain a distance from mathematics concepts and the logic behind them. This distancing from the object of mathematics results in a kind of interviewing technique that does not reveal much about the child's thinking, but reflects more about the interviewer. The teacher is like Putnam's (1987) tutor who responds to error with corrections that are intended to move the student back to the path of the interviewer's original script.

The same type of mathematically disassociated interviewer may alternatively approach the interviewing task with awe and an openness to accepting almost anything that the child suggests as indicative of interesting mathematical thinking. This type of teacher is unable to make reasonable judgments about the child's thinking because of his or her own distance from the subject matter. For example, we have the teacher-interviewer, Sherri, who asked a fourth grade student to help her figure out how many times she would need to fill an empty quart milk container with water in order to fill up a gallon container. Because the teacher herself actually did not know how to answer the question, she was open to accepting any type of answer that the student could offer. During the course of the interview as the child experimented with different ways of combining linear measurements of the two containers, the interviewer expressed repeated approval of the child's techniques although the efforts did not lead to any clear answer to the original question. In this case the interviewer's own absence of a connection to the task allowed her to appear to "discover" a variety of interesting observations about volume and dimensions of containers along with the child. Because none of these observations actually made much mathematical sense, however, the child appeared to be confused. Perhaps, though, because the interviewer was unable to respond with sharper more critical questions about the child's efforts, we might conclude that the child's potential for understanding the mathematics was never tapped.

At a second level of object attachment, some teachers view mathematics as a set of rules and prescribed procedures, but also see the logic of these components as well as the contexts in which they can be appropriately applied. For these teachers, mathematics is engaging, but their attachment is to its authority and predictability rather than to its more subtle, intellectually challenging, and even aesthetic characteristics. These procedurally engaged teachers tend to present interesting yet relatively routine problems as the context for a clinical assessment, noting how students follow or interpret rules and when they apply procedures. Take for example, the case of Dana who opened her interview by asking the fifth grade child to help her divide her class of 28 children into groups of four members each. When the child immediately responded that there would be 7 children in each group, Dana followed up with "How did you do that?" The child said, "Just multiply 4 times 7 or divide" to which Dana responded by asking her to write the problem down as division. Following the written work, Dana probed further, asking the child to tell what each of the numbers represented. After the child correctly identified what each number represented, Dana praised her and probed by asking the child to explain the arithmetic process to a hypothetical new child in class. When the child did this with ease and accuracy, Dana suggested that she explain what the division is "really doing." When the child referred to breaking things up and checking with multiplication, Dana praised her and asked her to create a situation in which "we might have to do something like this." The child quickly thought and said, "It's kind of like here. There were 48 people and you want six groups and how many people in the groups?" Dana's follow up here as before was to ask the child to write the equation describing the arithmetic, define the terms in the equation, and propose alternative arithmetic solutions (e.g., multiplication). In the end, Dana summed up the interview by saying, "OK, good. You still always come out with the same answer." In essence this interview focused on very detailed responses to procedural steps regarding the division algorithm and its associated meanings. Thus, the exploration and attachment in this case was confined to mathematical rules and their finite variations rather than to nuances and highly personalized conceptions of mathematical relationships that emerge in more playful and spontaneous interactions with mathematical objects.

At the third and rather more hypothetical level, teacher-interviewers who are in fact searching for such a playful manipulation of mathematical objects regard mathematics as part of the positive self and view the world quite naturally in mathematical terms. When this kind of interviewer interacts with a child in some mathematical context, a kind of synergy takes place in which both the interviewer and the child travel to a new dimension of knowing beyond that which either might engage alone. In this circumstance, the interviewer becomes part of the child's process and connection to the mathematics. Even when the child is not a spontaneously mathematically connected individual, this kind of interviewer may be able to break down barriers of procedural ritual or illogical random guessing so that in the process of the interview the child is able to see new mathematical meaning in the tasks at hand. To a large extent we see here the positive scaffolding effect described by Vygotsky (1986) and the development of operational thinking through relations of cooperation described by Piaget (DeVries, 1997) wherein the more advanced and experienced learner is able to act as a catalyst and support for the less experienced one. Such expressions of attachment to mathematics as evidenced in the clinical interview, however, have not been directly observed among our elementary teacher interviewers, but rather express an ideal to which we would like to see our teachers aspire. In fact, this third level appears to be much like Piaget's (1972) conception of formal operations in that it is not easy to detect, does not seem to appear universally, and may be limited to specific contexts of knowledge.

In general then, our observations have led to the formulation that clinical assessments are a function of the interaction between student and teacher in which the teacher's level of attachment to mathematics largely directs the assessment process and colors the responses made by the child. Although children do come to the assessment with their own emerging and established relationships with mathematics, it is evident to us that teachers filter children's attachments through their own unconscious investments in mathematics. These investments are defined by the extent to which teachers regard mathematics as essentially *egosyntonic* or *egodystonic* (i.e., as part or not as part of the self concept). In this view, teachers selectively evoke and listen to those elements of students' encounters with the world of mathematics that can be recognized and acknowledged as consistent with their own perceptions and feelings about the subject matter. Thus, in a sense, clinical interviewing as administered and interpreted by teachers is more reliably a statement about teachers' views and object relations with mathematics than an accurate and revealing statement about students' mathematical understandings.

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PROFESSIONAL DEVELOPMENT AND SYSTEMIC REFORM: SOME IMPORTANT CONNECTIONS

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This paper examines the impact of a two-week professional development course on the mathematical content knowledge of secondary mathematics teachers. During the workshop, teachers explored contextual problems using technological tools to deepen their understanding of mathematics. Preliminary results of this study showed that while teachers' content understandings tended to be somewhat superficial, the context within which teachers explored mathematics influenced the extent to which they engaged in the content at hand. Teachers' work in large group discussions tended to be more reflective and thoughtful while their written work or work in smaller groups tended to be more task-oriented.

Introduction

In this study, we set out to create and examine a professional development intervention which would contribute to the larger framework of systemic reform while building the mathematical content knowledge of the individual participants. In order to link our intervention to system-wide reform efforts in the context in which we chose to work, our research group negotiated and collaborated with various stakeholders in this setting to create an appropriate professional development intervention. Through this process, we designed a two-week professional development course for the entire mathematics faculty of an inner-city high school in Texas. This course had the following three goals:

- to expand secondary teachers' mathematical content knowledge by using college-level materials in a multi-media environment;
- to develop teachers' skills with technological tools, specifically computer applications, to support the teachers' developing mathematical constructions;
- to support the building of a teacher network among the workshop participants to help them sustain reform efforts in their classrooms.

By creating, implementing, and studying this course, we hoped to contribute to a better understanding of how professional development can contribute to systemic reform. In this paper, we will discuss results pertaining to the first of our goals: Increasing the mathematical content knowledge of secondary teachers (see Lachance, in preparation, for a complete discussion of the results of this study). The implications these results have for systemic reform will be discussed in the concluding section.

Method

Because different participants in this research had different perspectives on and experiences with this intervention, data collection and analysis took place within an interpretivist framework (Smith, 1993; Guba & Lincoln, 1989; Patton, 1990). As a paradigm for inquiry, interpretivism asserts that knowledge does not exist separate from the knower. In effect, interpretivism acknowledges that all knowledge is constructed and that such constructions are influenced by the prior beliefs, knowledge and experiences of the knower (Smith, 1993). As a framework for research, interpretivism expects multiple perspectives and encourages their solicitation and representation in research.

Given the interpretivist framework for this research, data was collected using qualitative methods. Each of the eighteen teacher participants in this course was interviewed in the spring of 1997, before the workshop began, and after they had returned to their classrooms in the fall of 1997. They also submitted a portfolio of their coursework including write-ups of the content problems they completed and written reflections on their learning during the course. All course sessions and problem solving activities were videotaped. A content analysis of these videotapes was conducted using a system of categorization (Patton, 1980).

Content Knowledge Defined

For certification and employment purposes, the content knowledge of teachers is often derived by the number of subject area college courses they have taken and/or their scores on standardized tests. These measures seem to imply a type of "reservoir" view of content knowledge. There is a fixed body of mathematical knowledge that teachers should know and store for use in teaching.

To us, such a view of content knowledge is problematic in many regards. First, it assumes that teachers can and will be able to store mentally all the mathematical content they will need in teaching. Given that there are multiple understandings of mathematical content, some of which have yet to come to light, such a task is not only impractical but most likely impossible.

Secondly, this view of knowledge does not take into account the ongoing evolution of understanding and knowledge creation. We know that learners of all kinds develop different and deeper understandings of subject matter when they work with others. In working with their students, teachers should almost constantly be revisiting and rethinking various understandings of the content they are presenting.

Finally, the type of mathematics instruction which reform prescribes requires teachers to have much more than a discrete set of mathematical understandings at their disposal. Instead, it requires teachers to have facility with a number of tools with which they can think about and then facilitate students' constructions of mathematical concepts. We thus see content knowledge as a "tool box" within which sit the following tools.

1. Basic Computational Facility and Content Understanding. Teachers need to be able to perform the procedures related to the content they teach with confidence and ease. If lack of use makes some content difficult to recall, teachers should be able to easily reconstruct this content.
2. Reflection. Teachers need to be able to reflect upon the content they are teaching or exploring to uncover its connections to other topics in mathematics and their curriculum. In their own development of mathematical understandings, as they set upon solution paths (alone or with their students), teachers will need to ask the same questions we ask students to ask themselves: Does this path make sense? Does my answer have meaning in the given context?
3. Flexibility. Teachers must be able to tap into a variety of representations for a given topic. They must help students understand the various approaches to different pieces of content. When presented with student thinking or unfamiliar content, teachers need to have the mental resources to explore the unfamiliar and critique knowledgeably the outcomes of such explorations.

Results

Given this definition, how do we view the content knowledge development of the teachers in this workshop? Interestingly enough, on different sets of data, teachers revealed different sets of strengths and weaknesses. On the portfolios, there was a great deal of technical or computational accuracy. However, in small group work and even in large group presentations, we saw significant gaps in teachers' working knowledge of content. For example, almost all the teachers chose to write-up a rather challenging workshop problem called "The Parking Garage" (Confrey & Maloney, in preparation). Using transformations on the prototype of the greatest integer function, most of the teachers accurately created mathematical models to predict the fees for parking, given the amount of time parked, under each of five proposed fee structures. Since many of these teachers had never worked with either transformations or the greatest integer function, their work on this problem was admirable.

However, in both small and large group discussions, several of the teachers had difficulty with some basic content procedures. In the following excerpt, a pair of teachers solving a problem involving a mountain lion attempting to prey upon some young antelope needed to convert 70 kilometers per hour to an equivalent number of meters per second.

Eduardo: 70x60 over 1000.

Phyllis: Why times 60?

Eduardo: To change the hours to..wait a minute - it's 3,600 for seconds. So it's 3.6 times 70.

Phyllis: Or just hit 36 over 10 times 70.

Eduardo: So 252 meters per second is how fast the young ones can go (Day 6, Tape 2).

The correct conversion is actually 70 x 1,000 to get the number of meters in 70 kilometers then divided by 3,600 (seconds in an hour) to get 19.44 meters per second.

While technical accuracy was stronger on the portfolio work, the level of reflection and flexibility tended to be low on teachers' written work. Teachers' answers to each problem's accompanying questions tended to be brief and superficial. In the Parking Garage Problem, teachers often did not explain or justify the models they created. Similarly, their work in small groups did not prompt many of these teachers to be more reflective or flexible. In the excerpt given above, not only did the teacher pair make a major mistake in converting kilometers per hour to meters per second, but they worked with the incorrect number for over thirty minutes. Even when this incorrect speed made it difficult to solve the problem, they never thought about whether it made sense that an animal could travel 252 meters in a second (a facilitator pointed this out).

However, in large group discussions, teachers seemed to be at their best in terms of content reflection and flexibility. They often built upon each others' contributions to uncover and tackle some of the hidden complexities of the mathematics under examination. An example of this can be seen in teachers' discussion of the Nautilus Problem (Confrey & Maloney, in preparation). For this problem, teachers were given nautilus shells and measuring tools and asked to take various measurements on the chambers in the shell. Working with the data they collected, teachers constructed mathematical models which would describe a particular aspect (length of septa wall, area, volume) of a chamber given the number of that chamber in the shell's sequence. When small groups finished their explorations, they presented their results to the group. In the excerpt below, Roy and Eduardo have just revealed that to predict the cross-sectional area of a given chamber, they used the function $y = 67(1.14)^{(x-1)}$ where y is the area and x is the chamber number.

Lead Facilitator: Why did you use 1.14 for your ratio?

Eduardo: It looked like midway - basically an average [of the ratios between each pair of data points].

Lead Facilitator: An interesting question - when you have a situation as with your ratios, is the arithmetic mean the best choice of means for your model? Or would you use a geometric mean?

Roy: I think I'd use an arithmetic mean. But I'd also look at extreme values and throw them out.

Lead Facilitator: ...I think I would argue I'd use a geometric mean. [Silence from teachers.]...The thing that we are modeling has multiplicative relationships... Because these relationships are based in this similar triangle context in that the shared sides of the chambers in the nautilus shell are modeled by similar triangles, I would argue we want to take a geometric mean, and not something that gives me a midpoint reading.

Roy: I don't know how to do a geometric mean.

Glenda: You'd take your two extremes and set them up in a proportion and find your ratio.

Lead Facilitator: You can also think of them as an exponential and take the root... It is just something to think about in terms of the mathematics. (Day 4, Tape 3).

Before this exploration, several teachers in this group were clearly unaware of the definition and application of geometric means. However, throughout the remainder of this discussion on the Nautilus problem, teachers kept revisiting this issue. They asked for further clarification from the facilitator, and when some groups that presented later in the day had reworked their models to make use of their new-found understanding of the geometric mean, the teacher audience challenged their presenting peers to explain their reasoning and procedures. By the end of the nautilus shell presentations, it was clear that most of these teachers had a deeper and clearer understanding of the concept of geometric mean.

Discussion

Why did weaknesses in reflection and flexibility seem to show in portfolios and small group work while these skills seemed to be strengths in larger group demonstrations and discussions? One explanation could come from teachers' history with mathematics. Because of their mathematical background, teachers would have had significant experience in doing mathematics problems. Even though the workshop problems were probably very different from those teachers had solved in their traditional mathematics courses, having to write up these problems for the facilitators to assess put these teachers back into "student mode." Thus, their concern may not have been to understand or reflect, but to produce for the evaluator. The technically accurate equations and the formal writing of problem write-ups are evidence of this.

The small group work also seemed to put these teachers in "student mode." Even though they were working with one or two other people, there still seemed to be some pressure to perform mathematically. Especially early in the workshop, the interaction between groups was less helping and more comparing: How far did you get? Did you figure this one out yet? This may not be evidence of competitiveness, but rather of an insecurity that they were "getting it right." Again, this may stem from past experiences in mathematics classrooms where students are often in competition with each other for grades that are assigned on the curve.

However, in the large group discussions, there was a different dynamic. First of all, everyone's ideas were made public. And it was not to a very critical public, but rather to a supportive one. Each group's presentation was appreciated for what it had to contribute to the discussion, and NOT for what it was lacking. In large group discussions, teachers appeared to be unexpectedly open about what they did and did not know. As a result, teachers seemed more free to reflect, connect, and engage with each other, the facilitators, and the mathematics.

Thus, context appears to be vital to the learning processes teachers engaged in during this workshop. When asked to engage mathematically in situations which were reminiscent of traditional instructional situations, these teachers appeared to concern themselves with the technical and surface level aspects of the content. They focused more on product, task completion, and "getting it right." However, when given the opportunity to participate in a lively discourse, to observe the work and thinking of others, and to question the conclusions of peers and facilitators, teachers were able to challenge themselves and deepen their understandings. Their goal in these discussions seemed to be to gain insight, satisfy interest, and increase their knowledge.

Conclusion

This result points to an important consideration when thinking about professional development and systemic reform. The types of classroom activities suggested by reform movements will make mathematics learning a much less predictable process. Teachers will have to be prepared to follow and then support students' thinking. Without a set of broad and flexible mathematical understandings, teachers will be unable to facilitate and develop student learning in reformed classrooms. Teacher content knowledge thus becomes a crucial foundation for reform of both classrooms and educational systems.

To improve teachers' content knowledge such that they have the content understandings, reflection skills, and flexibility to undertake mathematics instruction in reformed classrooms, professional development must work to reduce the influence of teachers' prior mathematical learning experiences. Teachers must be given the opportunity to work together on content problems and then share understandings and perspectives on these problems. The pressure to perform must be reduced and ample support for teacher learning from peers and facilitators must be provided. Most importantly, the emphasis must be on the process by which mathematical knowledge and understanding are created, not simply on the products of this process. By helping develop teacher content knowledge in this way, professional development courses can empower teachers to reform both their classrooms and the systems within which they work.

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AN ANALYSIS OF TWO NOVICE K–8 TEACHERS USING A MODEL OF TEACHING-IN-CONTEXT

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Two K–8 novice teachers, who, during their senior year in college, participated in a one-year intervention that focused on using students' thinking, differ one year after graduation in their pedagogical approaches to teaching a lesson on area and perimeter . We use, in part, Schoenfeld's model of teaching-in-context to better understand the learning-to-teach process as it relates to the role of students' thinking in these two teachers' lessons.

Two K–8 teachers, Mr. C. and Ms. W., had been involved in a one-year intervention project (Lubinski, Otto, Rich, & Jaberg, 1995) during their senior year at a midwestern university in which each had access to an extensive body of research on how children learn to understand mathematics. The project focus was on assisting novice teachers to build their pedagogical-content-knowledge base by considering students' thinking. Yet, our preliminary analysis of these two first-year teachers' end-of-the-year lesson on perimeter and area suggested that their teaching styles differ considerably with respect to their use of students' thinking. Ms. W. utilized students' thinking as a major pedagogical tool in her lesson, whereas Mr. C. did not. Furthermore, it is important to note that Mr. C. is a mathematics specialist and had taken several additional mathematics courses, including one on geometry, in which instructors modeled current reform-based pedagogy. Ms. W. had only taken the required two mathematics content courses. Both had taken the same methods course taught by one of the researchers who focused on using students' thinking for planning mathematics instruction. Their student teaching experience was also structured to include the overarching goal of using students' thinking when making instructional decisions. Thus, we have the question: What can we learn about the pedagogical approaches of these two novice teachers?

Purpose

It is well established in the literature that the interaction of teachers' knowledge and beliefs greatly influences the decisions they make during instruction (Thompson, 1992). Evidence exists that as teachers' pedagogical content knowledge develops, their beliefs and teaching practices may change, although as Franke, Fennema, and Carpenter's (1997) work indicates these changes in beliefs and practices do not necessarily occur and if they do they may not occur at the same time nor in any particular order. Furthermore, the role of experience in the process of teacher change is not yet well understood (Lubinski, et. al., 1997).

In order to accomplish our goal of analyzing the different pedagogical approaches of these two novice teachers, specifically their use of students' thinking, we focused on Schoenfeld's work "Toward a Theory of Teaching-in-Context" (Schoenfeld, in press). His model of studying how beliefs, goals, and knowledge base interact provided a framework with which to analyze our data. We used his model, in part, to better understand how our intervention influenced these novice teachers. More specifically, we asked, What can we learn about novice teachers' assessment and use of students' thinking when making and modifying their instructional decisions?

Data Sources

Our method involved gathering information to develop an image of Ms. W. and Mr. C.'s teaching style in order to provide insight as to why they made the decisions they did during one instructional session. For this paper we looked at data on Mr. C. and Ms. W. at the end of their first year of teaching fifth and fourth grade respectively. Our data included survey results from The Mathematics Beliefs Scales (Fennema, Carpenter, & Peterson, 1987) and a Survey on Teaching and Learning (Lubinski, Otto, & Rich, 1992); and a videotaped lesson on area and perimeter, a pre-lesson plan, post-lesson reflection, and a transcription of a stimulated-recall interview (SRI) for the videotaped lesson. It should be noted that similar data were also collected before and during the intervention and included journal reflections for the entire intervention year.

The Mathematics Beliefs Scales measures teachers' beliefs about learning and teaching. It consists of 48 items (covering four construct areas) each connected to a five-point Likert scale response. A high score for each construct indicates a professed belief that learning, teaching, or content decisions need to be based on the consideration of students' thinking and that teachers operate from a cognitively-based perspective. The Survey on Teaching and Learning consists of 12 pedagogical problems related to mathematics. Responses provide information about pedagogical content knowledge as well as beliefs and content and curricular knowledge.

Findings

After one year of teaching, Ms. W.'s Belief Survey scores on each of the four constructs were higher than Mr. C.'s, significantly so on the constructs of teaching and learning. On the learning construct, Mr. C.'s score was 3.8, while Ms. W. had 4.3, reflecting a greater consideration of students' thinking. On the teaching and presentation construct, Mr. C.'s score was 3.5 while Ms. W.'s was 4.4. Mr. C.'s belief scale score over all four constructs was 3.6 while Ms. W.'s was 4.4. Furthermore, Mr. C.'s scores on individual statements clustered around the middle of each item's Likert scale, while Ms. W.'s did not, reflecting definite preferences.

Statements reflecting beliefs also appeared in these teachers' responses on the Mathematics Survey responses for situations involving geometry and measurement and their SRI statements following the videotaped lessons on area and perimeter. After one year of teaching, Ms. W. commented on the use of student thinking in her responses on the Mathematics Survey. She wrote about encouraging students to find multiple solution strategies which could be shared. Her ideas were overall cognitively based. She wanted "...the students to give [her] examples of their findings in order to make connections between the different shapes." After one year of teaching, Mr. C.'s responses on the Mathematics Survey indicated a belief that you should "Get students to look at some solutions to the problem [so] they will begin to understand that the area and perimeter will sometimes give you different answers." Although his answers to the survey situations indicate a desire to use student thinking, he made few connections among solutions given and did not indicate using students' responses for assessment purposes.

During the SRI for the lesson on area and perimeter, Ms. W. referred to "listening to what [students] say, the questions they were having..." as the key to her action plan in implementing the lesson. She based her decisions "on what I know that they already know..." In addition, her belief on the importance of using student thinking appeared when she reflected on her lesson and possible changes she might make. "I felt that when I was teaching it, ...maybe I would show them a picture of what the book thought, but I thought , no, then they will do exactly what the book said." Mr. C.'s responses in his prelesson interview indicated that using students' thinking was not a priority goal in his decision making process but "making the numbers I guess, difficult enough so they wouldn't whip right through it" was a major focus. He further indicated in the interview that he wanted to focus on the concept of area. However, during the lesson he emphasized the multiplication process involved in finding area to the point that he did not see why students needed to use calculators. Comments from Mr. C. and Ms. W. about their lesson support findings from the Beliefs Survey. Ms. W. considered student thinking in developing a lesson image and the corresponding action plans while Mr. C. only expressed a desire to use student thinking.

The content knowledge of the two teachers was also different. Mr. C. had several additional mathematics classes, including a course on geometry specifically developed for 5–8 mathematics specialists. He felt confident about his background knowledge. In his responses to the situations on the Mathematics Survey involving geometry and measurement, he easily identified the problems the students had in the situations described. In his lesson he was able to draw upon his knowledge of the Pythagorean Theorem in response to a student's question. However, he tended to be lax in his use of terminology but did exhibit an understanding of the mathematics content he taught. Ms. W.'s mathematical background was weaker. She had only the two required content courses for K–8 preservice teachers. Her responses on the Mathematics Survey reflected a developing understanding of geometry and measurement but not necessarily a firm grasp of the content. Her lesson also reflected this. At times, she made mathematics errors without recognizing the problem. For example, she drew a right triangle using numbers for the lengths of the three sides that were impossible.

When asked during the SRI as to his goal for his lesson, Mr. C. responded "Mainly to use what we have been talking about with area. . . .I just wanted to take what we have looked at and make it into something they would see in real life. . . . So the goal was just to use what we have been studying in the past few days [perimeter and area]." These comments are consistent with what he wrote in his pre-lesson form. There was no mention of using students' thinking even when probed both before and after the lesson on his decisions for his action plans.

Early in the lesson in an effort to achieve his goal of "real life" he asked, "What do I have to do to find the perimeter of this room?" Students responded that it would be necessary to measure the length of the walls and then Mr. C. asked what would be done with those two numbers. John responded "Add them together and times it by two." Mr. C. responded "No, I am not going to times by 2 and add together. What did Jessica say to do?" Continued analysis of the discussion reveals that Mr. C. really meant to find the area of the room rather than the perimeter as he earlier had asked. Mr. C.'s failure to pursue the thinking behind John's response did not allow for clarification of the question being asked. Furthermore, pursuing John's thinking would allow for discussion of how to find perimeter by using both addition and multiplication rather than just using addition as done previously. In another situation in which Mr. C. asked students to find the areas of rooms in a house for which he had provided a blue print, he asked Tonya for the area of bedroom 3. When she answered incorrectly, he indicated that the answer was wrong and immediately asked another student for his answer. When probed in the SRI about this interaction, he said "She had the wrong answer, but I think she make a mistake in adding. So then I called on someone else. . . If she had been way off in her answer and said in the thousands or something like that then I would have, okay, let's look at what you did and try to figure out what you did wrong. She was only off by 20 so I figured it was just an addition problem." Again we see where Mr. C. used students' comments to evaluate correctness of their answers, following what Schoenfeld has called an IRE (initiates, responds, evaluates) approach.

The goal of Ms. W.'s lesson as written on her prelesson plan was "to review area, perimeter, and multiplication. . .I wanted to make sure they had a complete understanding of the difference between area and perimeter." During the lesson, the students were finding the areas of various figures using multiplication when Johnny drew an irregular figure on the board asking: "What if you can't multiply because you have a figure like this?" Ms. W. replied, "Okay, why couldn't we multiply that?" and Johnny answered. After his response, Ms. W. chose to come back to this idea later in the lesson. When asked about this decision, she thought that the irregular figure would generate a good discussion since Johnny didn't think it was possible. However, Ms. W. reported in her SRI that as part of her lesson image she had considered students doing this. She continued, "it ended up working really well later in the lesson. . .I probably should have dealt with them right there, but at the same time, I was thinking if they come upon this problem themselves, it's going to be more relevant when I come back to Johnny's." Ms. W. used Johnny's thinking to help students find the area of irregularly-shaped rooms in their house plans. She talked about how the example helped the discussion because "You take things for granted, you think that they would think about it that way and they had just never thought of it that way. I wasn't surprised."

In another incident, Ms. W. used student thinking to differentiate between area and perimeter. One of the students volunteered the size of her room so the students could find its area. After the students had time to work, Ms. W. solicited answers from about six students. Not everyone gave the same answer, so Ms. W. asked, "Why did some of you get 252 and some of you got 66?" She then had students explain how they determined their answer. One student explained that she added 12, 12, 21, and 21, while another student said he multiplied 21 by 12. Ms. W. then asked, "What is the difference, here?" Ms. W. then used the ideas solicited from the students to discuss area, perimeter, and multiplication in order to move toward her lesson goal.

Summary and Discussion

It is interesting to note that Ms. W.'s and Mr. C.'s overall belief scores were similar both at the beginning and immediately after the intervention. Yet, after one year of teaching they were significantly different. A question that arises is, Why?

During the intervention Mr. C.'s journal reflections indicated he would consider students' understanding during instruction, "Make sure that all students get a chance to explain their work to me and the rest of the class…this will help others to see the many different ways that a problem can be solved." He believed he would focus on understanding in his lessons; however this consideration was not evident in the lesson that was videotaped. During this lesson, he assigned students who finished solving their problems first to help others find the answer. It appears that Mr. C.'s overarching goal does not use students' understandings during instruction even though he reported they were important.

Ms. W. noted while teaching she often changed her mind about what she had planned to do based on students' comments. For example, having students work in pairs was a pedagogical change of plans made during one lesson. Ms. W.'s overarching goal appears to be teaching for sense making. Ms. W. considers students' thinking in her action plans and her lesson image.

Implications for teacher preparation programs include a need to address the issue of lesson image, goals, and action plan and how can these ideas be connected when considering students' thinking.

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PRESERVICE TEACHERS' IDEAS ABOUT TEACHING MATHEMATICS: CONTRIBUTIONS TO FRAMEWORKS FOR TEACHER EDUCATION COURSES

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Lesson plans reflect what preservice teachers think of mathematics, what they aim at in their lessons and what they believe they are able to do in the classroom. In this study, lesson plans on the area concept were used to investigate preservice teachers' content knowledge and pedagogical content knowledge. From it, some frameworks appeared that may be interesting for teacher education. Some of them have the structure "not only ... but also." With respect to the content knowledge of area, some preservice teachers are procedure-oriented, others are concept-centered also. With respect to the pedagogical content knowledge, some have a "teacher leading and learner following" orientation, others have a "learner constructing, teacher guiding" orientation also. In teacher education, the "also" components should be developed more in students.

This study aims to obtain insights into *ideas about teaching mathematics that preservice teachers bring to their teacher education courses*. As many earlier studies have shown, these ideas provide a basis on which the preservice teachers can construct their knowledge and understanding of mathematics education (Crawford et al., 1994; Cooney, 1994). They also help to determine the context in which the preservice teachers can develop personally — from viewing themselves chiefly as students, or "learners," to taking on the roles of "prospective teachers" aiming to promote learning by the students in their classes. The academic and personal development of the preservice teachers is a typical concern of teacher education courses. A better understanding of the preservice teachers' initial ideas can therefore contribute to addressing the goals of such courses and hence to designing more appropriate courses. For this paper, the emphasis is on helping the preservice teachers to construct and evaluate their lesson plans more consciously (Broekman & Weterings, 1987; Wubbels, 1992; Renkl, 1995; Desforges, 1995).

The authors of the paper belong to a group of mathematics teacher educators from various countries (the U. S. A., the Netherlands, Ireland and Sweden); they work in differently structured teacher education programs and at different levels (primary and secondary education). Part of their interest in working as a team comes from the challenge of identifying frameworks which can be used in differing cultural and educational settings, and of trying to gain insights which can illuminate mathematics preservice teacher development in the various contexts.

In the study, preservice teachers' ideas were inferred from their plans for a lesson (on the topic "area") and from their subsequent discussion of the plans with interviewers, and appropriate ways of presenting the ideas were identified. This paper outlines the theoretical frameworks on which the study was based; it describes the methodology used; and it then gives an account of the main results. Finally, possible applications to the design of teacher education courses are discussed.

Theoretical Frameworks

Preservice teachers' ideas about teaching mathematics are likely to stem from their knowledge of the subject, their experience of learning it and their beliefs about its nature. Much of the relevant research has been summarized by Thompson (1992), Fennema and Franke (1992), and Brown and Borko (1992). In the present context, a particularly useful theoretical framework is provided by the work of Freudenthal (1973, 1983). It suggests that as the preservice teachers prepare their lesson plans, they will be influenced in particular by their knowledge and views in three areas: the subject matter (the mathematical topic, with its specific structure and content); student learning (how students might approach the topic, their prior knowledge, approaches that will motivate them to learn, and so forth); and the reasons for which the topic is included in the curriculum (for instance, its historical role in the curriculum and its applications in daily life). Key ideas in Freudenthal's writings can also be recognized in the work of Shulman (1986) on the knowledge base of teaching. A second framework, therefore, considers the preservice teachers' content knowledge, their pedagogical knowledge, and their pedagogical content knowledge. As the paper is concerned specifically with mathematics education, the first and last of these three are of particular interest. For *content knowledge,* Hiebert's (1986) categories of conceptual and procedural knowledge provide further structure. For *pedagogical content knowledge*, the standard literature is less helpful; categories devised for the present study are outlined below.

Methodology

In order to investigate preservice teachers' initial ideas about teaching mathematics, the authors asked volunteering prospective teachers in their institutions to plan a lesson for introducing the concept of area to a class of students aged ten to thirteen, and to discuss it with an interviewer or interviewers, as indicated above. Where possible, the volunteers chosen were students who had little experience of "mathematics education" courses and who had not yet undertaken teaching practice, so their ideas were likely still to be close to those which they had brought into their teacher education programs. Each participant was given about an hour in which to prepare his or her lesson, the work being done in a room equipped with a suitable range of resources for teaching and learning about area. When the preservice teachers discussed their plans with interviewers, issues addressed included ones dealing with their educational and mathematical backgrounds, and hence with probable sources of their ideas about teaching mathematics as identified earlier.

The main dimensions for analysis — content knowledge and pedagogical content knowledge — were chosen partly because of their prominence in the literature, and partly because they matched the dimensions emerging naturally from the data. Members of the group tried out possible ways of categorizing the content knowledge and the pedagogical content knowledge on their own data; these categorizations were revised to take account of difficulties or inadequacies; and this process was repeated until agreement was reached among all group members.

As part of the process, a careful analysis of the different aspects of area of relevance to teaching was undertaken. It provided the group with an instrument by means of which the different teaching approaches could be coded, in particular as regards content knowledge. An account of the "aspects of area" instrument is given in van der Valk & Broekman (1997). When descriptions of the teaching approaches were agreed, the various preservice teachers' plans were examined once more, in order to place them appropriately in the resulting frameworks.

Results

Lesson plans of twenty prospective teachers from four countries (U. S. A., Ireland, Sweden and the Netherlands) were used in the study. The transcribed lesson descriptions and interviews provided a very rich set of data. The choice of area — a topic that typically appears in national curricula in the middle grades, and hence (for countries in which students move from primary to secondary education at around age twelve) spans different types of school — allowed for the participation of preservice teachers preparing to teach at different levels and entering their teacher education programs with different educational (and especially mathematical) backgrounds.

The first results of the study are concerned with identifying frameworks to categorize preservice teachers' ideas. One such framework was found, both in the mathematics education literature (Hiebert, 1986; Baturo & Nason, 1996) and by examining the data, to describe *content knowledge* in terms of *conceptual* and *procedural* approaches (van der Valk & Broekman, 1997), as indicated above.

For *pedagogical content knowledge,* the relevant literature (for example Shulman, 1986 and Lappan & Theule-Lubienski, 1994) proved less helpful. A framework was therefore devised using ideas of Berry and Sahlberg (1996). It has two dimensions, *teacher activities* and *orientations towards the teaching-learning process*. For teacher activities, three aspects were identified:

- "presenting subject matter"
- "presenting subject matter with interactive student activities"
- "promoting autonomous student activities."

Two orientations towards the teaching-learning process were distinguished:

- "the teacher leading and the learner following"
- "the learner constructing and the teacher guiding."

In the "teacher leading" orientation, the students have to follow the teacher's mathematical ways of reasoning; in the "learner constructing" orientation, the teacher encourages students to explain their own ways of reasoning, and guides that reasoning in a productive direction (the direction of doing mathematics). The three aspects of teacher activities, and the two orientations, together form a 3x2 matrix that can be used to describe prospective teachers' pedagogical content knowledge.

Further results stem from the use of these frameworks. For *content knowledge*, application of the "aspects-of-area" instrument developed in the course of the study (van der Valk & Broekman, 1997) allowed the preservice teachers' work to be classified as being *concept-centered* or *procedure-centered*. All preservice teachers had planned to use some procedure-centered approaches, but some also elaborated concept-centered approaches. A further group used concept-centered approaches, but did not elaborate them, seemingly because of a lack of content knowledge. From this, it appeared that some of the preservice teachers entered their teacher education program with a tendency to see mathematics—at least in a school context—as more procedural than conceptual (Berenson et al., 1997). This is not surprising; it reflects findings elsewhere (Lappan & Theule-Lubienski, 1994, and many others). However, the emergence of a rather similar range of approaches among the preservice teachers in all four participating countries—despite the different ages and mathematical backgrounds of the participants and the different national curricula and age groups they were preparing to teach—is perhaps worthy of note. The transcripts themselves provide interesting instances of conceptual and procedural approaches. The cross-cultural perspective helped to highlight some difficulties in the use of language regarding area; for example, several of the native speakers of English used the word "box" to describe a drawing of a square, and the possible confusion with a three-dimensional representation was not noted until it was pointed out by the native speakers of Dutch. Excerpts from the transcripts have provided useful material for developing preservice teachers' approaches. A model emphasizing "*not only procedures, but also concepts*" may help such preservice teachers to identify their current perspective and hence to develop a fuller appreciation of the mathematics they will teach.

With respect to *pedagogical content knowledge,* the two dimensions of *orientation* and *teacher activities* were considered. Analysis revealed that the group of preservice teachers contained two main subgroups. The larger subgroup showed a "teacher leading" orientation. Most members of this subgroup planned teacher activities that involved "presenting" the area concept and also "promoting student activities" (be they interactive or autonomous) on that topic. As for content knowledge, there were indications that some preservice teachers in the subgroup wanted to achieve a particular goal — in this case promoting interactivity — but were not doing so. The smaller subgroup showed a "student constructing" orientation. These prospective teachers all chose to present the topic with the aid of many student activities. A few preservice teachers used both orientations, alternating lecturing with autonomous (cooperative learning) activities. Again, therefore, with a view to aiding teacher development, models of the form *"not only ... but also"* can be formulated. Examples include "not only presenting, but also promoting student activities," "promoting not only autonomous, but also interactive student activities," and "not only the teacher leading and the learner following, but also the learner constructing and the teacher guiding."

Concluding Discussion

The methodology is of interest in itself (Broekman & van der Valk, 1998), because it can be used to address the challenge formulated by Haggerty: "to help each student to become aware of her or his own thinking and agenda so that she or he cannot only question ... existing beliefs but also so that each can provide an agenda for their own learning" (Haggerty, 1995, p. 108). Others (for example, de Jong (1997)) recognize the importance of examining preservice teachers' ideas *in action*, particularly with respect to the design (and, later, use) of lesson plans.

As pointed out earlier, the resulting models are intended to help preservice teachers as they move from viewing themselves chiefly as students, or "learners," to taking on the roles of "teachers" — eventually, of teachers who aim to promote learning by the students in their classes. Descriptions of the form "not only ... but also" are intended to recognize and reinforce their existing achievement, and hence to provide for development in (hopefully) a non-threatening manner. The models can be seen as aspects of a more general model: "not only *me teaching* but also *my students learning*." The work therefore refines, for the field of mathematics education, the stages of teacher concern identified by Fuller & Bown (1975).

In conclusion, the group feels that it has been successful in identifying frameworks and insights that are robust across cultures and age groups. In particular, it has identified the "not only ... but also" type of model as one which takes account of the personal and professional development of the preservice teachers.

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CLASSROOM TEACHERS BECOMING TEACHER EDUCATORS: "JUST" FACILITATORS OR ACTIVE AGENTS?[1]

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This paper considers two case studies of classroom teachers who took on specific professional development responsibilities in their own schools. Analysis of the two cases focuses on the teachers' definitions of their roles as leaders among peers and decisions to take action based on their developing understanding of their responsibilities.

Enacting the proposed reforms in mathematics education requires fundamental rethinking of basic assumptions about the mathematics students learn, how they learn it, and which classroom structures best support that learning (Cohen et al., 1993; Schifter & Fosnot, 1993; Sykes, 1996). In order to support change in substantial numbers of elementary classrooms, ongoing, long-term staff development is required at the local level. Such "scaling up" requires a large work force of teacher educators—a work force we do not have. The obvious source for expanding this work force is teachers themselves—teachers who are committed, prepared, and skilled at working with other teachers (Friel & Bright, 1997; Mumme & Acquarelli, 1996). Yet, the necessity of involving classroom teachers as primary agents in teacher education in mathematics raises many questions, from what the roles of these teachers should be to how the effort can be sustained financially to what kind of preparation teachers need as they move into the role of teacher educator.

The teachers considered here were part of a project, Teaching to the Big Ideas (TBI) (Schifter et al., in press).[2] In addition to the professional development work with 36 teachers, TBI developed a course about the ideas of arithmetic in the elementary grades for use outside of the project.[3] In this intellectually rigorous course, participants read and discuss teacher-written cases, view and discuss videotapes, engage in mathematics activities, and do a variety of assignments, including conducting interviews with students and writing their own episodes. It is this course that the TBI teachers, three years after they had begun the project and now as teacher leaders, decided to offer in their own school systems during the 1996-'97 school year. In addition to offering the seminars in their own schools, the teacher leaders would meet together monthly with TBI staff and keep a portfolio of reflections about their work.

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[2] Teaching to the Big Ideas was co-directed by Deborah Schifter, Education Development Center, Virginia Bastable, Summermath for Teachers at Mt. Holyoke College, and Susan Jo Russell, TERC.

[3] The course is called Developing Mathematical Ideas (Schifter, Bastable, & Russell, 1998). The first module, Building a System of Tens (eight 3-hour sessions), focuses on how students come to construct an understanding of the base ten numeration system. The second module, Making Meaning of Operations (eight 3-hour sessions), focuses on students' understanding of addition, subtraction, multiplication, and division of whole numbers and rational numbers.

In the summer of 1996, as they prepared to teach the course in the fall, teachers in the group expressed concerns about taking on the role of teacher educator. In TBI, they had established a learning community in which seriousness of purpose and a stance of inquiry were taken for granted. They were used to thinking hard about children's thinking and engaging deeply with mathematical ideas. However, they knew that many of their colleagues had different images of professional development—as an endeavor that provides clever classroom activities that could be implemented immediately. In this view, staff development did not have homework, did not require a multiple session commitment, and was certainly not expected to challenge strongly held ideas about learning, content, or pedagogy.

The teacher leaders left their summer institute with doubts about their own expertise in being able to bring about engagement and reflection in their seminars. Many of the teacher leaders described their roles as no more than facilitative: they weren't teaching, they argued; they didn't have any special expertise; they were simply providing the context and the material through which others could learn for themselves. However, while they hesitated to attribute potential learning to their own efficacy, they nevertheless had goals in mind for their groups:

> *I hope they will allow themselves the pleasure of listening to their students and 'hearing' what THEY have to say. I hope they will be able to appreciate their students' struggles and understand that understanding IS a STRUGGLE!!*
>
> *I expect teachers to develop the habit of posing questions for themselves about their practice: about what children understand, about what they themselves understand, about how to ask questions to stimulate rather than inhibit children's thinking . . . .*

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As the seminars began, the teacher leaders began to confront the tension between being "only a facilitator" and having strong beliefs about and goals for the outcomes of this staff development experience. One case, summarized below, illustrates this development. Data for this paper include the teacher leaders' portfolios, journals they kept during the summer 1996 two-week institute, and written notes as well as transcriptions of audio tapes of the monthly meetings.

Laurie's experience: "My job is only to facilitate. It is their job to learn."

Laurie and Beth co-taught a seminar for 15 teachers from their school in Riverside, a small urban center. Early in the course, Laurie already felt challenged by the task of running the seminar on top of her own full-time classroom teaching. She wrote after the first session:

> *As I wrapped up a day of teaching and realized that the session was about to begin I felt totally overwhelmed. . . . What helped me was to remind myself that this is not about* *my* *learning but about the teachers' learning, and that my job is only to facilitate. It is their job to learn.*

In the first semester, Laurie faced a variety of issues: a few seminar participants had very firm ideas about teaching mathematics and did not seem open to reflecting on them further, and a few were reluctant or unable to think hard about mathematical ideas. However, some good discussions occurred and Laurie noticed that many of the teachers *were* willing to think hard about new ideas. She noted, "Often they express their views as questions, as if they have heard this view and don't necessarily agree with it." Laurie also noticed that some teachers were consistently having a dampening effect on others in the group. She was concerned that these teachers did not seem to be open to learning from the study of children's thinking, but she was even more concerned that they were overshadowing the participation of some of the less confident participants:

> *I have noticed that the more rigid ideas come from people who have often seen themselves as more adept at teaching (or understanding) math. . . . What bothers me . . . is that I allowed this atmosphere of dismissiveness to continue. I believe that has put a stop to the expression of ideas by some of the most thoughtful and observant members of our seminar. . . . their ideas are not emerging and receiving full notice as I wish they would. Now that I have thought about this . . . I will try harder to make it happen.*

By the end of the first semester, Laurie was focusing on this issue with determination. In one of her pieces of writing, she described the approaches she was using to support the teachers—many of them teachers of the primary grades—who were thinking hard about children's thinking but were reluctant to express their thoughts. She talked with these teachers individually to let them know "how much I value their ideas and questions and how much I feel they contribute to the seminar." During the seminar, she brought comments they had made in small group discussion to the attention of the larger group and, once she had done this, she attempted to "slow down the discussion" so that those ideas were given careful consideration. She noticed the effects of these actions in the seminar; although some participants still seemed satisfied with the knowledge they had, others became engaged and eager to share their thinking. For example she recounted what happened as small groups worked to represent thousandths in some way. One small group was

> *secure in their own "knowledge" already, declared that "you can't use base 10 blocks to represent thousandths because it will just confuse the kids because they are so used to using them another way." They sat back, and even their body language conveyed they were not going to think harder about the issue. However, the other two groups were not deterred this time from sharing their excitement at their discoveries and from acknowledging to each other how their various interpretations enriched each other's understandings.*

Laurie ended the first semester feeling that she had made progress towards her goals and felt confident that "things will continue to improve, just as they do in my classroom." However, a few weeks later, after Laurie and Beth told the group they might offer the course again the following school year, one participant came to Laurie to express her dissatisfaction. If Laurie were to offer the course again, the participant asserted, "you will need to make big changes. I looked around the room and counted at least eight other people who are as dissatisfied as I am . . . it's so repetitive. At first it was interesting, but now we've got the idea about listening to the children's thinking. We need to move on."

At first this conversation shook Laurie's and her co-leader's confidence: "I told Beth about the conversation, and we were both quite taken aback although not entirely surprised. All indications to us in class discussions and, especially, writings, have been that the teachers are benefiting a great deal from the course. Although they gripe about the usual little things, on the whole we had thought things were going well." After considerable thought, they decided to devise a questionnaire to give to the participants that would help sort out reality from exaggeration, both for the participants and for themselves. The respondents rated most elements of the course as "somewhat beneficial" to "very beneficial." Eleven of the twelve respondents reported they were "changing the way I teach math," and seven agreed that "Working with these ideas over the whole year is helping me." The co-leaders had other data as well. In the teachers' most recent writing assignments, they saw clear indications of real interest and excitement as well as evidence of learning and growth. They also remembered encounters with teachers outside of the seminars, like the two participants in heated discussion in the teachers' room who told Laurie, "We used to be able to come in here and relax and talk about recipes. Now all we talk about is math!"

The questionnaire results provided data the leaders could use to understand the dynamics of the group and to talk directly with the participants. In light of the results, Laurie reflected on her own analysis of the group. She identified three sub-groups within the seminar:

> *One group of teachers is well launched in these ideas and working hard to implement them in their classrooms . . . A second group is somewhat intrigued by the ideas in the course. They are trying out a lot of things, but in the back of their minds they still believe that traditional procedures are the most "sophisticated" form of mathematics. . . . Talking about "mathematical understanding" generally leaves them cold. A third group is in between the other two groups. They are teachers who began the course with dissatisfaction with the way they were teaching math, but little vision of what they might be looking for. Some of them have really caught fire. Others are still wondering. They keep moving between the two other groups . . . .*

Considering these three groups as having different needs helped Laurie clarify her own role:

> *When I think about these three groups, my mission (!!) is pretty clear to me. I want to truly affirm and encourage the first group, especially since, ironically, many of them are teachers who do not feel strong mathematically . . . Next I hope to move more of the flip-flop group into that group. I'd like to see as many of them as possible end the course with a mission of their own . . . As to the second group, I'm trying to figure out how to shake them up without making them mad. When I respond to their writing I ask them lots of questions. Often I frame the questions as questions I have been asking myself . . . "I'm wondering just like you are why . . . " even though they weren't exactly wondering.*

At this point, Laurie is no longer describing her role as "only facilitating," but as a "mission." While her view that she can't shove learning down other teachers' throats, any more than she can force her students to learn, remains consistent, her description of the role of teacher educator in this setting has taken on a great deal of considered specificity.

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USING THOUGHT-REVEALING ACTIVITIES TO STIMULATE NEW INSTRUCTIONAL MODELS FOR TEACHERS

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The purpose of this research paper is to share the design and results of a multi-tiered teaching project involving students, teachers, and researchers in which thought-revealing tasks for students and teachers were used as the basis for professional development. The teacher-level activities were specifically designed so that it was possible to investigate the development of teachers' integrated mathematical, psychological, and pedagogical ways of thinking about their students' learning and problem-solving abilities. Results from the latter level of this teaching project suggest that the teachers involved have changed their ways of thinking about: the most important ideas in mathematics for their students; how to create classroom environments that are more conducive to the teaching and learning of thoughtful mathematics; and, how to observe, assess and document mathematical ideas in students.

Introduction

To learn mathematics for understanding, students need instruction that "encourages them to experience mathematics as a subject area that can in fact be understood" (Goldsmith and Shifter, 1993, p.1). To accomplish this, teachers must be provided with opportunities to refine, revise, and extend their own knowledge about the teaching and learning of mathematics. Generally, when considering teacher development, the same principles that apply to student learning should also be applied to teacher learning. For example, if we believe that students should be able to build models that help them to generate descriptions, explanations, and predictions in a variety of problem-solving settings and for a variety of purposes under differing conditions, then the same should hold true for their teachers (Lesh and Kelly, 1997; Lesh, Amit, & Schorr, 1997). "Telling" teachers about students' thinking or other new teaching strategies, is no more effective than "telling" students about a complex mathematical idea and expecting them to "understand" it. Indeed, teachers may change specific behaviors, teaching strategior curricular materials, while still missing key ideas and understandings about their students' ways of thinking. Effective teacher development must stimulate growth in teachers, including increased understanding of mathematical content, as well as a deeper understanding of how students learn mathematical concepts and ideas. It must also help them to analyze, interpret, refine, generalize, extend, and share their evolving models for teaching in a wide variety of settings and situations. With this in mind, we designed a multi-tiered teaching project in which teachers were provided with opportunities to improve their effectiveness in the classroom while becoming more familiar with their students' ways of thinking.

While there were many aspects to this muti-tiered teaching project, this research paper will focus on teachers' developing notions about the nature of mathematics, learning, and problem solving - as these understandings are reflected in their assessments and observations of their students' work, and their own classroom practices. More specifically, of the two distinct types of evolving conceptions investigated: the first having to do with the *shared* conceptions of the group of teachers involved in the project; and the second having to do with the conceptions of *particular* teachers within the group, this paper will restrict attention to the former type of conceptions.

Background

This research is a component of a teacher enhancement project in which researchers worked directly with groups of teachers from a number of sites located in urban and suburban school districts. With few exceptions, the same teachers remained involved over the entire course of the project. Each implementation involved teachers and researchers working together to review curricular and assessment activities, implement selected thought-revealing problem tasks in classrooms, and analyze the results. Planning, review, and analyses that focused on the teachers' knowledge of mathematics, how students learn particular mathematical ideas, and the pedagogical and psychological implications for instruction and assessment, occurred in workshops held within each district of school community.

Research Design

Our overall research design involved a sequence of activities in which teachers interacted with students, other teachers, and researchers. These interactions focused on thought-revealing activities which repeatedly challenged participants (both students and teachers) to reveal, test, refine, reject, revise, and extend important aspects of their ways of thinking. The thought-revealing activities were intended to promote learning, and simultaneously produce trails of documentation that show important aspects about the nature of the constructs that develop for both students and their teachers. For example, teacher level though-revealing activities might include: generating observation forms that colleagues could use to make significant observations about students as they are working in groups; and, identifying the strengths and weaknesses of students' results that could be used to assess the quality of the products that students produced.

In this research design, a teaching project for *students* was used as the context for a teaching project for *teachers*, which in turn was used as the context for a teaching project for *researchers*. Tier #1 of the project focused on the nature of students' developing interpretations of thought-revealing activities. The solutions that students produced involved the development of mathematically significant models (or constructs, or conceptual systems) for constructing, describing, explaining, manipulating, or predicting the structurally interesting aspects of systems that occur in real or imagined worlds. Tier #2 focused on teachers' evolving assumptions about the nature of students' developing knowledge and abilities, and about the teaching, learning, and problem activities in which they were engaged. Tier #3 focused on researchers' developing conceptions about the nature of teachers' and students' developing knowledge and abilities.

At each tier, the students, teachers, and researchers were continually challenged to reveal, test, reject, revise, refine, and extend their ways of thinking. Thus, they were all able to develop in directions that they themselves could judge to be continually "better" - even in the absence of a predefined conception of "best." Consequently, in our teaching project, it was possible to create conditions that optimized the chances that development would occur without dictating its direction.

A key feature of this project was that teachers were asked, while in a workshop setting with researchers present, to solve a series of thought-revealing activities that could later be used with their own students. Once they had solved the problem activity, they would share ideas with each other and the researchers about the main mathematical ideas, proposed implementation strategies, and anticipated student outcomes. Next, they were asked to implement these activities in their own classrooms (with researchers present for some activities). At follow-up workshops, teachers were asked to share their students' work on the activities, their thoughts about the mathematical ideas that were elicited in their students, implementation issues, and their assessments of student work. They were also asked to prepare observation forms that colleagues could use to record significant observations about students as they are work in groups.

Three characteristics of this research should be mentioned that relates to investigations involving the development of teachers and students. First, their development tends to be highly interdependent. Although for some purposes it is possible to deal with the development of students and teachers as though they were isolated and functioning independently from one another, in reality, their development tends to be interdependent and unable to be disassociated. Second, development for students and teachers is not simply the result of passive acceptance of information. Third, researchers tend to be integral parts of the systems they are hoping to understand and explain.

Teacher Level Activities and Results

Results indicated that when students' problem solving activities lead to the development of complex products, and when teachers attempted to make sense of these products, their interpretations became more refined and sophisticated over time. In particular, the amount and type of information that they noticed was strongly influenced by their assumptions about the nature of mathematical constructs that might be useful for addressing a given task, and factors that influence assessments of success in "real life" situations outside of schools. Consequently, their interpretations of student work changed dramatically over time. Interestingly enough, the preceding observation does NOT imply that it was impossible to get high agreement among teachers who were asked to assess student work. In fact, there was remarkable agreement amongst teachers at all sites.

To shed light on these results, we will consider data taken from teachers' own classroom implementations of thought-revealing activities for students that occurred at the beginning and at the end of one project. Limitations of space do not allow a complete description of the intermediate stages of development that occurred in teachers. A thorough discussion of that would far exceed the constraints of this forum.

In one particular activity used with fifth and sixth graders at the beginning of the project, students were given a newspaper article with a picture of a school banner. The banner had a picture of a lion whose face was made up of various geometric figures. The students were told that the actual banner had been lost, and it was their job to help fellow students construct a pattern for a new full size banner that would look exactly like the one in the picture.

Initially, the teachers were asked to solve the problem in a workshop setting with researchers present. When they solved the problem activity, the teachers noted that they had "used proportions to help scale-up the newspaper drawing to make the full banner". The teachers then implemented the activity in their own classrooms. Several weeks later, they again met with researchers in a workshop setting to share classroom results. The following are examples of student products that the teachers identified as exemplary:

Product I: We made a little pattern for the lioness out of construction paper. Then we traced a symbol (referring to a musical instrument) for the head. Then we traced a plastic lid for the ears. For the eyes we traced the bottom of a glue bottle. [F]or the nose we used a ruler. For the mouth we used a protractor.

Product II: Making the lioness isn't hard at all. All we did was make 5 circles, one for the head, two for the eyes and two for the ears, and just put half of the ears behind the head to make it look like ears. Then we cut out one triangle for the nose, and one trapezoid for the rest of the nose.

One teacher (who reflected the consensus of the group) noted that these were "very good student products because the students used many different objects to construct the lion head". She continued, "I really liked the way that the students were engaged in the activity, they used lots of different objects to construct the circles and other shapes". Throughout the entire time that the students' work was being discussed, none of the teachers noted that the lions drawn by the students for the banner were not at all proportional to the original lion in the newspaper clipping—even though the teachers had identified that as one of the most important mathematical ideas in the problem activity.

When the same teachers were asked to design an observation instrument to be used by teachers for the purpose of observing and assessing the students as they work in groups to solve problems, they responded with the following lists:

- [Make sure that students are] on task, understand the directions, [are] all involved, groups/pairs [are] getting along.
- [Did students] answer all questions thoroughly, remain on task, work cooperatively, help each other, follow directions.

The teaching project continued on for approximately two years. During that time, teachers met approximately once a month with researchers. At each meeting they would solve thought-revealing activities that could later be used with their own students. In addition, they would share ideas relating to other teacher level activities such as:

- Observing their own students as they were working to solve thought-revealing activities.
- Identifying the strengths and weaknesses of students' results.
- Assessing the quality of students' work.
- Conducting one-to-one interviews with selected students.
- Identifying follow-up instructional activities.
- Developing and refining reliable procedures for making insightful observations about students' work-in-progress

During the course of the project, teachers' assumptions about the nature of students' developing knowledge, and about the teaching, learning, and problem-solving activities in which they were engaged, changed dramatically. For example, approximately eighteen months later teachers implemented a new thought-revealing activity for students in their own classroom. Teachers were asked to identify exemplary student products and state the reasons for their choices. At this time, teachers' comments focused on: the strengths and weaknesses of different representational systems; the nature and types of justification used by students to defend particular responses; the use of student generated notation; the mathematical ideas elicited; the language and communication used by the students; and, the sense making abilities exhibited by students as they solved the problem.

To further highlight the differences in teachers between the beginning and the end of the project, consider the changes in the observation instrument that they generated for use by themselves and other teachers to observe and assess students as they work in groups to solve problems (taken directly from teacher notes and comments):

- Identification of the problem and understanding of the task.
- Choosing and using tools appropriately.
- Creativity in selecting and using strategies for solving the problem.
- Organization of problem data.
- Quality of the product (how it looks visually, organization, completeness).
- Persuasiveness of the justification for all parts of the results.
- Mathematical perseverance—staying with the task.
- Ability to monitor and assess the product, refining it appropriately, continuing through solution cycles as necessary.
- Working within a group to analyze the problem, share insights, questioning both the process and the results.
- Roles assumed within the group.
- Intellectual characteristics shown—reflection, confidence, curiosity, etc..
- Evidence of knowledge and skill with particular mathematical concepts.
- Differences in strategies chosen.
- Questioning that the solution "makes sense" - that it in fact appropriately answers the question posed and would be satisfactory to the client.

Conclusions

Data from this teaching project suggest that the teachers involved developed more sophisticated perspectives regarding what to look for when observing students engaged in problem-solving tasks and how to assess and document their students' thinking. While this report did not focus on the changes that took place in students, it is important to note that that the quality of their work changed dramatically over the course of the teaching project, as well. These results are encouraging as we continue to work help teachers create classroom environments where mathematics is taught in a more thoughtful manner, with a strong focus on students' thinking.

Notes:

[1] The data that is reported here comes from a low wealth surburban community in New Jersey. It should be noted however, that remarkably similar results were obtained at all other sites within the group, this paper will restrict attention to the former type of conceptions.

[2] The data that is reported here comes from a low wealth suburban community in New Jersey. It should be noted however, that remarkably similar results were obtained at all other sites.

[3] Space limitations do not allow us to provide the actual problem.

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TEACHER EDUCATION
SHORT ORALS

PRESERVICE TEACHERS' CONCEPTIONS ABOUT THE VALUE OF TEACHING FUNCTIONS

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What are the reasons given by preservice teachers for teaching mathematical functions? What are the common definitions of function given by preservice teachers? What are some relationships between preservice teachers' conceptions about the value of teaching function and their stated definitions of function?

Open-ended written surveys were completed by 72 preservice secondary mathematics teachers enrolled in a mathematics methods course in 1995, 1996, and 1997 and follow-up interviews with 10 teachers. In analyzing participants' conceptions of the value of teaching functions, we developed three categories: (a) *Intrinsic Value* (real-world connections), (b) *Pedagogical* (mathematical connections and curriculum prominence), and (c) *Excitement* (functions are beautiful mathematical phenomena) (Thorpe, 1989). In analyzing participants' definitions, we developed five categories: (a) *Correspondence*, (b) *Rule*, (c) *Dependence*, (d) *Operation*, and (e) *Expression.*

Few preservice teachers identified Intrinsic Value (33%) or Excitement (3%) as important reasons for studying functions and more than half (51%) indicated reasons for teaching about functions that we classified as Pedagogical. Nearly one-fourth (24%) did not express any reason for teaching functions. Definition responses were distributed as follows: Expression (33%), Operation (24%), Correspondence (19%), Rule (11%), and Dependence (11%). We saw interesting patterns in the survey data regarding relationships. Teachers who gave Intrinsic Value reasons for teaching about functions were more inclined to also provide Dependence definitions.

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REFLECTIVE ACTIVITY OF A PRESERVICE SECONDARY MATHEMATICS TEACHER

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The notion of reflection has become a significant theme in teacher education. This study was part of a larger project entitled Research and Development Initiatives Applied to Teacher Education (RADIATE)[1] in which reflection was encouraged and data were collected through an initial survey, weekly journals, class discussions, papers, and nine audio-taped interviews. Aims of this study were to describe and understand the reflective activity of Liz (pseudonym) as she participated in a four quarter secondary mathematics teacher education program.

Dewey's (1933) notion of uncertainty was helpful in identifying instances of reflection. Also guiding analysis was knowledge of one of Liz's beliefs about the role of a good mathematics teacher: The teacher should provide an encouraging classroom environment where material is clearly explained and students can comfortably ask questions (Chauvot & Turner, 1995).

In describing Liz's reflective activity, the most common themes were reflections on: her experiences as a learner of mathematics, her beliefs about mathematics, teaching or learning, the program experiences, and her teaching practices. The two strongest themes of Liz's decisions, or results of her reflections were 1) to gain knowledge of multiplistic ways of solving problems and of teaching and learning, and 2) that she needed to continue to learn. In understanding Liz's reflective activity, further analysis suggested that Liz's reflective activity was directed toward a search to improve her mathematical and pedagogical knowledge so that she would be able to clearly explain material to her students, an important aspect of her encouraging classroom environment.

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[1] RADIATE was directed by Dr. Thomas J. Cooney and Dr. Patricia S. Wilson and funded by the National Science Foundation (grant #DUE9254475) and the Georgia Research Alliance. Opinions and conclusions are those of the author and do not necessarily represent an official position of the funding agencies..

MATHWINGS™ INITIATIVE: TWO CASE STUDIES INVOLVING TEACHER CHANGE UNFETTERED TO FLY OR STIFLED BY STRATEGIES?

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The administrators in a rural region of a mid-southern state became disturbed by their students' substandard national mathematics test scores. To initiate current reform efforts, they chose to implement, in grades 3 through 5, the New American School (NAS) mathematics curriculum called MathWings™. As a component of Roots & Wings, this program integrates the visions of 1989 National Council of Teachers of Mathematics (NCTM*) The Curriculum and Evaluation Standards for School Mathematics* by incorporating Robert Slavin's cooperative learning strategies. This research highlights the recent qualitative research that investigates MathWings™ and its effects within the classrooms of two purposely chosen elementary mathematics teachers.

The two case studies that unfolded from the respondents' interview data and classroom observations reveal (1) a teacher's prior experiences heavily influence decision-making during change implementation, (2) classroom culture and activities provide a barometer indicating the status of math reform efforts, (3) formal/informal support structures influence reform implementation, (4) a teacher's stated desires and actual practices are incongruent during change implementation, and (5) the furtherance of math reform efforts were influenced by personal, professional, and emotional issues. For these teachers, change involved their commitment to learn different strategies, to risk implementing these tactics and to acknowledge change's uncertainty.

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CLASSROOM-BASED RESEARCH: THE EVALUATION OF PROFESSIONAL DEVELOPMENT PROJECTS

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The main purpose of this project was to investigate the effect of two professional development projects for high school teachers. One of these projects (funded by the Eisenhower Mathematics and Science Education State Grant Program) involved teachers in a single year of staff development, while the subsequent project (funded by the National Science Foundation) involved teachers in two to three years of intensive professional development. Both projects focused on content, pedagogy, attitudes, equity and leadership but for this presentation we will concentrate on content and pedagogy.

Although both investigators observed all 14 teachers in the study weekly, they each had different major foci for observations and interviews. One observer focused on issues of gender/ethnic equity and how these were manifested in teacher-student interactions, student-student interactions, and curriculum choice. This was of particular interest because both projects worked with schools which were eliminating tracking as a means of increasing the number of underrepresented students in academic courses. The other observer focused on how expanding views of algebra and geometry are integrated into the content and pedagogy. In particular, she was interested in how technology and modeling had become an integral part of the classroom instruction.

In this presentation we will discuss two categories of results: patterns in teachers' subject matter knowledge, and teachers' use of pedagogy. In the category of content, we found that topics such as probability, statistics, and discrete mathematics continue to cause conceptual difficulty even though these topics were addressed in the projects. Pedagogical results were complex, reflecting teacher awareness and some classroom changes. However, classroom inequities were still apparent in many classes and the power of technology has not been taped in most classrooms.

BELIEFS OF PRESERVICE MATHEMATICS TEACHERS: HOW ARE THEY AFFECTED BY STUDENT TEACHING ?

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Preservice teachers' beliefs are described before, during, and after participation in a semester-long student teaching experience. We focused on how the preservice teachers' encounters with pupils and their perceptions of pupils' attitudes toward mathematics and learning mathematics affected the preservice teachers' conception of their roles in the mathematics classroom.

Two volunteer preservice mathematics student teachers, Megan and Kate, participated in the study. Data was collected through a questionnaire, an initial interview, and an exit interview consisting of a card sort and concept map activity. The questionnaires and concept maps were based on previous work by Clarke (1997) and Raymond (1997). During the semester three pairs of classroom observations and follow-up interviews were conducted with each preservice teacher and two observations were conducted of lessons facilitated by each cooperating teacher.

Megan, despite working in a classroom in which mathematics was presented as drill, practice, and procedure, held on to her initial beliefs that mathematics requires creative thought, most problems can be solved in many ways, and learning mathematics requires explaining Kate's classroom interactions with pupils affected her earlier belief that learning mathematics requires mostly practice and led her to consider ways to better motivate students to learn.

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FINDING TIME FOR REFORM IN THE MATHEMATICS CLASSROOM

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When discussing mathematics education with teachers, I find agreement with many aspects of reform. However, there are serious constraints to change, most notably lack of time.

This report focuses on secondary mathematics teachers who attended an institute on mathematical modeling, where they studied successes in modeling, solved large problems, and wrote projects for classrooms use. Although teachers looked forward to teaching via projects, finding time to plan, do, and assess them was of paramount concern (see Blum & Niss, 1991). One teacher noted, "It seems we shorten the time the students are in our rooms and we continually add content — never removing any." Despite time concerns, several teachers found innovative ways to cope, including:

- Collaborate with other faculty to make projects a part of the regular curriculum.
- Reduce and redistribute the traditional review at the beginning of the school year.
- De-emphasize certain mathematical topics (e.g., computation), given available technology.
- Run projects in the background while teaching the mainstream curriculum.
- Use the so-called jigsaw method of group instruction, where students are teacher/experts.
- Locate well-developed projects that replace the traditional treatment of an important idea.

Innovative teachers who reflect on their processes are best equipped to effect classroom change (Clarke, 1994). At the session, I will describe two such teachers.

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TEACHER EDUCATION
POSTERS

WRITING TO DEVELOP PEDAGOGICAL CONTENT KNOWLEDGE

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In this study, writing was used successfully to engage preservice teachers in the construction of knowledge of probability and the communication of pedagogical content knowledge—alternative understandings of problems that teachers might encounter when working with middle school students. Two intact classes of statistics for preservice elementary teachers with a specialization in mathematics participated in the study. The control class was taught by a professor recently awarded tenure for his exemplary teaching and his research of the teaching and learning of probability. The experimental class was taught by an assistant professor whose teaching has been described as marginal by the department chairperson.

The pretest and the posttest were tasks developed and used by Jones, et. al, (1997) to assess students' understanding of probability. The pretest was administered during the first day of each class. The posttest was conducted during the last day of each class. The control group did significantly better than the experimental group on the pretest. A Chi-square test for homogeneity on pretest responses for the control group and the experimental group was significant (10.56 > 9.12 Chi-square df = 2 alpha = .01). Although the control group did significantly better than the experimental group on the pretest, the experimental group scored significantly better than the control group on the posttest. A Chi-square test for homogeneity on posttest responses for the control group and the experimental group was significant (7.5 > 6.63 Chi-square df = 1, alpha = .01). One explanation for the improvement exhibited by the experimental group is that having preservice teachers write alternative understandings of problems that teachers might encounter when working with students was a meaningful context for preservice teachers to study probability. Writing about probability problems in a meaningful context allowed for mediated thought and action in the context of probability.

USING REFORM CURRICULUM MATERIALS IN TEACHER EDUCATION: RELEARNING MATHEMATICS AND PEDAGOGY

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This project aims to understand how examination of reform-oriented curriculum materials might provide a context for preservice teachers to begin thinking about mathematics *for teaching* early in their development. The middle school materials of two projects (Mathematics in Context and Connected Mathematics Project) served as the primary texts for a course focused on mathematical preparation of prospective elementary teachers (prior to methods courses and field experiences). Course work included exploring the activities as students and using teachers' guides to plan and teach lessons to classmates.

Most preservice teachers felt challenged but also safe as they explored the middle school materials, expressing sentiments such as, *This has helped me relearn a lot of things and learn things I was never taught.* Almost every teacher embraced the novelty of the activities because, for instance, *I am learning about how to look for reasons and explanations as opposed to simply believing the rules.* Many teachers suggested having learned not only mathematical, but also pedagogical lessons from their work with the materials, namely the value of relevant problem contexts and discussions focused on students' ideas. Recognition and reflection on such pedagogical issues became even more prevalent when teachers planned and taught lessons based on the materials.

Preservice teachers clearly are *not* middle school students. They bring to their work with the materials a complex perspective based on past experiences as students, beliefs about mathematics and mathematical activity, and emerging projections of themselves as future classroom teachers. In many ways, the perspectives and needs of a preservice teacher seem to advance the middle school materials to their level so that substantial learning can occur. Further analysis of the nature of such experiences will bring us a richer understanding of what it means to learn in reform-oriented ways and how such learning will benefit teachers as they interact with children in elementary classrooms.

INTEGRATING THE TEACHING OF READING AND MATHEMATICS THROUGH RICH PROBLEM SOLVING CONTEXTS

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Mother: Do you like math in school?
Second grader: Math is too hard.
Mother: Oh, what sort of things are you doing?
Second Grader: Sums like 30 and 30 and 30.
Mother: Oh? So that is hard, is it?
Second Grader: Yes it is. I just can't do it. but if it was money it would be 90 cents.

Researchers are only beginning to understand the following problem: What role does context play in children's learning and how should this affect the way reading and mathematics are taught? This problem is at the heart of a two-year National Science Foundation grant recently received by a faculty team at Oklahoma State University. This interdisciplinary team brings together reading and mathematics faculty representing the colleges of Education as well as Arts and Sciences. Integrating the teaching and learning of reading and mathematics represents a change in the way these disciplines have been traditionally viewed. This may have a significant impact on how future teachers teach mathematics and reading in the schools. Reading and mathematics experts agree that reading and mathematics support each other. Therefore, they should be viewed together, learned together and taught together.

In this poster session, the presenters will report on the progress of the above NSF grant project, which is designed to develop curriculum materials aimed at integrating the teaching and learning of reading and mathematics for pre-service teachers. The curriculum materials are being developed for use in reading methods, math methods, and math content courses taken by pre-service teachers at Oklahoma State University. The main goal of the project is to increase future teachers' problem solving and mathematics learning by integrating the teaching and learning of math and reading through rich problem solving contexts.

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A DENSITY DISTRIBUTION OF CLASSROOM INTERACTIONS

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This poster presentation focuses on a rubric used to analyze classroom interactions as part of a study of the factors that influence the acquisition of pedagogical content knowledge for two groups of prospective secondary mathematics teachers - undergraduate mathematics majors and post-graduate scientists and engineers seeking mathematics certification - in the content domain of functions and graphs. In particular, the rubric focused on the following three dimensions: 1) teacher level of discourse/communication and cognition; 2) student level of discourse/communication and cognition; and 3) ways of "sense making"/the notion of mathematical authority.

Developing pedagogical content knowledge is a gradual and complex process. One way to better understand its development is to focus on those sites where its construction might occur. For prospective teachers, these sites are likely to include the development or construction of explanations which include not only explanations but also analogies, representations, examples and demonstrations, the planning of lessons, the teaching or simulation of teaching, and reflection on teaching. In particular, *instructional practices* were assessed through a vignettes task in which all of the subjects were asked to respond to student misconceptions and/or questions concerning major topics about functions and graphs.

Each of the subjects was assigned an ordered triplet that represented their responses to each of the five vignettes. For each of the vignettes, the distribution of responses, both within and between the two groups of prospective teachers, provide quantitative measure of the responses. Across all of the vignettes, these measures provide a means to determine whether the responses were mainly teacher-directed or student-centered and what notion of mathematical authority was used.

FROM STUDENTS OF MATHEMATICS TO TEACHERS OF MATHEMATICS: MAKING THE TRANSITION

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There is growing concern in the mathematics education community that the content knowledge teacher candidates acquire may be sufficient to produce correct answers but does not provide adequate subject matter preparation to teach mathematics for understanding (Ball, 1988; Cipra, 1992). The transition from student of mathematics to teacher of mathematics requires more than the ability to produce correct answers.

This study examines investigation notebooks prepared by elementary school teacher candidates enrolled in a content geometry course. The notebooks require preservice teachers to analyze their own experiences as mathematics learners and to reflect on the critical interaction between knowledge of content and pedagogy. Three central themes guided the focus of the analysis, the teacher candidates' ability (a) to communicate their understandings of mathematics content, (b) to make connections between and among mathematical ideas, and (b) to justify their conjectures. The students' reflections indicate that as the semester progresses their quest for "right answers" and "correct formulas" began to shift toward an interest in "making sense" of mathematics and exploring connections to their previous mathematical experiences. The students' reflections indicated they were beginning to think about their mathematics learning not only as students of mathematics but also as future teachers of mathematics.

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THE ROLE OF REFORM CURRICULUM IN LEARNING TO TEACH

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As preservice educators in an urban land-grant university committed to preparing teachers for our large urban public school district, we face a tremendous task in preparing elementary teacher candidates (TCs) to teach mathematics. Although this is partially because of their tenuous mathematical background, a more important issue is that TCs' experiences of mathematics are based in a different paradigm from those of the NCTM documents. We have found that the NSF-funded exemplary curricula provide an important resource for supporting TCs in making the transition to reform-based mathematics teaching.

For the last three years we have integrated NCTM-based curriculum into two courses in the Elementary Education program: Seminar in Teaching I (with a 60-hour field experience in city schools) and Teaching and Learning Mathematics. By using videotapes from elementary city teachers using the reform curriculum, we expose TCs to the broad diversity of elementary students' thinking. Then, we choose curriculum units in which the mathematics and ways of learning about it are quite novel to TCs. In doing this, we find that TCs become more receptive to their own thinking, elementary students' sense-making, and the pedagogical reasoning that must ground reform-based teaching. Finally, TCs use curriculum units to teach a lesson to peers and, when possible, to elementary students in classrooms.

Findings show that TCs learn early in their preparation to analyze the specifics of their reasoning and to connect that with students' sense-making and curricular decisions. In so doing, they learn how to integrate content and pedagogy intrinsic to the NCTM reforms.

TEACHER KNOWLEDGE
RESEARCH REPORTS

THE ROLE OF FORMAL AND INFORMAL SUPPORT NETWORKS AS TEACHERS ATTEMPT TO CHANGE THEIR PRACTICE FROM TRADITIONAL TO INNOVATIVE REFORM-ORIENTED TEACHING OF MATHEMATICS

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As teachers attempt to change their practice from more traditional forms (or what they know) to more reform-oriented practices aligned with the NCTM Standards (1989), they rely on both formal and informal support networks. In this study the formal networks were externally created to provide teachers with ongoing support and consisted primarily of the project staff. However, the teachers needed more and frequently relied on informal support networks which typically consisted of fellow teachers, spouses and others. This paper will examine the support networks of two teachers during their initial participation in a problem-centered second-grade mathematics project.

If we view teachers -like students- as active learners, then we must look at how teachers learn and for opportunities to support their learning. Crucial to teachers' learning and their ability to cope with daily dilemmas is their participation in peer support networks. These networks provide both practical and emotional support for teachers who are attempting to change their taken-for-granted teaching practices. Furthermore, peer networks provide an atmosphere conducive to learning and often reward teachers for learning.

Theoretical Framework

This study is based on a social constructivist perspective (Bauersfeld, 1995; Cobb, Wood, & Yackel, 1991). Teachers are viewed as active learners who construct knowledge in the course of their interactions and reflections. This case study specifically focused on the interactions of one teacher with members of her support networks and her reflections upon these interactions.

Method

This qualitative study used both the techniques of participant observation and interviewing to document the beliefs, practice, and subsequent learning of the teacher. The teacher's classroom teaching was observed and video-taped two days a week the first semester and eleven times the second semester. In this study the data sources were video and audio recordings of their mathematics lessons during their initial participation in this project. In addition, ethnographic field notes and pre and post teacher interviews were used in the analysis.

Case Study

Mary, one of the teachers in the case study, taught in a midwestern community in which the school system was predominantly Caucasian and had both a rural and urban population that encompassed a wide range of socioeconomic backgrounds. She had three years of first-grade teaching experience and was teaching second grade for the first time. Mary relied extensively on the teacher's manual of the textbook series used by the school system to teach mathematics and covered "every page in the book." She characterized her mathematics instructional approach as repetitive and uninteresting. Not surprising, she viewed mathematics as a "bunch of rules" which she did not enjoy learning or teaching.

Mary's Network

Mary was part of a network before her participation in the project and this prior network significantly influenced her attitude toward, and desire to participate in, the project. However, much of the prior network supported her former way of practice or methods of teaching and as Mary changed her practice she also changed her network. As she attempted to realize a new form of practice, her new network played a more vital role in her learning.

Part of Mary's prior network was another first grade teacher, who had attended a workshop with Mary on teaching problem solving in mathematics. The workshop was one of Mary's first experiences with the project. It was given by the project developers and advocated a problem-solving approach to teaching mathematics. Mary's reaction to this one day workshop significantly influenced her perception of the project. The following excerpt illustrated her initial reaction to the project and her reliance on a support network.

> *I went to the workshop last summer ... and of course in one day you can't learn it. It [the project] was really neat! Another teacher and I both went to it. We were excited, we wanted to do it, but of course we were in first grade and there wasn't a curriculum for first grade.*

Her descriptions of her reactions in terms of: "we", "we were excited", and "we wanted to do it" indicated that Mary and her fellow teacher provided each other with mutual support. Together they formed a network and attempted to realize changes in their teaching practices by implementing a problem-solving approach one day a week in their respective classrooms. It was important for Mary to know that she was not alone in her efforts. She also expressed her frustrations because other teachers questioned her attendance of this workshop on her own time.

Mary was influenced not only by her peers, but by her students as well.

> *We at least got the TOPS cards [problem solving materials] and started working with those. And the kids, they wanted to do that. They didn't want to do their paper and pencil [work]. They really like working in groups. They asked for it all the time.*

Her students could be considered part of her support network. Their positive reactions to a problem solving approach were an important influence in her participation in a markedly different approach to teaching mathematics.

While the prior network was important in her initial conceptualization of the project, her changing of schools, grade levels, and approach to teaching mathematics necessitated her participation in new support networks. In her new school there was an established network of experienced project teachers. Mary made these teachers a salient part of her support network. Initially, she used this network to help her organize her mathematics teaching and her sequencing of activities. In general, this network was invaluable during her first year.

> *Oh, I don't think I could have [organized the materials], I really don't because I didn't know what was supposed to come next or how many days you're supposed to spend on it. It was just too overwhelming!*

Not only were most of the activities new, but so was this approach to teaching mathematics. Taken together, it was too much for Mary to do without support.

In addition to the daily organization and the preparation of materials, she relied on this network for general support.

> *We eat together every day and usually we talk about math at some point during that time and it's really nice because if I don't understand something, then they can tell me how to teach it or maybe give me suggestions and I pretty much rely on that ... I can go to them, they have experience with it, and I always rely on them mainly for something new.*

> *While the experienced project teachers were invaluable to Mary, she relied on the most experienced project teacher for individual assistance.*

> *She is very, very helpful to me. I'm glad she's here because I can always go to her if I need something or have a problem.*

Mary initiated the development of this part of her support network.

Another important person in her network was her husband. She indicated that she talked about the project "all the time" with her husband. Mary believed that he, like many of her friends, did not see how the project worked, but he was supportive because she liked it.

Her support network was crucial as she encountered people uncertain about the project's approach to teaching mathematics.

> *They, [my friends, our school board, some parents, and other teachers in the building], just don't see how this could work. They don't understand how these kids could learn that way .... It's hard to explain to them.*

Mary believed in this program, but she had a difficult time convincing other people of its effectiveness. Her support network was vital as she encountered opposition to the project. Mary actively sought the establishment of a network for the support it offered.

Another key person in her network was the project staff member who visited her mathematics class every week. They would discuss specific children's mathematical thinking and events that had occurred in the classroom. The staff member offered suggestions and actively attempted to help Mary develop a new approach to teaching mathematics. Initially, these observations made Mary uneasy, but she relied on the staff member's suggestions as she developed a new form of practice. The staff member was an important part of her network and became an authority whom Mary looked to for guidance and support. The staff member provided the technical and practical support for Mary.

One unintentional member of her support network was myself, the researcher. As I was observing Mary twice a week, she had more contact with me than the staff member. Mary viewed me as an authority on mathematics and the project. In her view, I was there to help her realize this approach to teaching mathematics. On one occasion she had reversed the directions to an activity she was introducing to the class. As she was walking around the room she realized her error. After class she asked me why I had not corrected her when she was explaining the activity. Initially, she believed that her teaching practice was under scrutiny. For her, I was there to offer suggestions and correct her mistakes. As the year progressed, and as she realized that my role was not to criticize and offer suggestions, she began to view me as someone who could help her students. (I worked with her students during their group activity every time I visited her classroom.) Her feelings towards my presence changed from ambivalence to anticipation. I was, although unintended, a positive addition to her support network, providing more emotional support than technical.

After the mathematics lesson we would discuss specific events that had occurred in class. I offered encouragement, especially when she indicated that she felt she was stumbling. I tried to avoid offering suggestions but, instead, tried to indicate that she was progressing and doing a good job. My presence and our discussions became a form of support as she attempted to change her practice.

Mary relied on support networks from her initiation to the project to her newly realized established practice. Her support networks provided her with the technical and emotional support she needed, and were crucial to her change in beliefs and practice. Her network helped her establish an atmosphere conducive to learning which also rewarded her for learning. Without this support Mary could not have made the changes, or learned as much as she did.

Implications

The development of teachers' support networks can significantly influence teachers' learning and their subsequent change in practice. Teachers do not learn in isolation but, like students, learn as a member of a community. Teachers' need both practical and emotional support as they attempt to change their practices. Those seeking to reform or change educational practice may readily set up the appropriate practical support but they might also consider promoting peer support networks which provide the needed emotional support. Networking may be as simple as providing opportunities for participating teachers to eat lunch together, as was the case with Mary. Consequently, it is important that we be aware of these informal support networks of teachers and that we promote their development whenever possible. Finally, we must remember that teachers actively construct their own knowledge and as educators of teachers , we can only present opportunities for teachers to learn.

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ENHANCING THE PEDAGOGY OF UNIVERSITY FACULTY THROUGH MENTORING AND REFLECTION: PRELIMINARY OBSERVATIONS

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While much research exists on the process of mathematics teacher change at the K-12 level, there is little, if any, such research on post-secondary faculty. This paper describes a study of university faculty who participated in a project to encourage systematic monitoring and regulation of their teaching through reflection. Using a model originally developed to encourage K-12 teacher-change and mentored by their mathematics education colleagues, two-computer science faculty engaged in a plan/teach/debrief sequence. Results suggest that non-education university faculty have little language to talk about teaching, have difficulty thinking about their own teaching actions, and have difficulty letting go of their own agenda when they facilitate other faculty in the reflection process.

Research on the process of K-12 teacher change in response to current reform efforts as suggested by the National Council of Teachers of Mathematics (1989, 1991, & 1995) is abundant (Clarke, 1997; Eisenhart, Borko, Underhill, Brown, Jones & Agard, 1993; Hart, 1996, 1997). Models to support, encourage, and implement change have been documented (Grouws & Schultz, 1996), and while results and perceptions of the relative success of the process vary, the work is prolific. This is not the case, however, for post-secondary mathematics faculty.

Clearly the university community has begun to respond to the call for reform in mathematics teaching. There isstrong evidence that at least some members of the academic community believe it is important to improve undergraduate mathematics instruction. In documents such as the American Association of Higher Education's publications on teaching portfolios and on using cases to improve college teaching, as well as practical guides such as Case (1994), Krantz (1993), and many editions of the MAA Focus and UME Trends, we find numerous recommendations on how to do this. However, there is little research on the subject of faculty change. Those research publications available (e.g., Dubinsky, Schoenfeld, & Kaput, 1994; Kaput & Dubinsky, 1994; Kaput, Schoenfeld, & Dubinsky, 1996) have no studies with an analysis of the teaching process, reflection, or change. The predominate focus of post-secondary mathematics education research has been on curriculum issues (i.e., the Calculus reform; studies of how students learn or have certain misconceptions in specific domains of mathematics; comparisons of various instructional innovations; and affective, gender and ethnic issues in student learning).

The paucity of research on pedagogy at the university level—on why, how, or if college faculty change their pedagogy in light of current recommendations for reform (e.g., standards for post-secondary mathematics teaching [Cohen, 1995])—gave rise to the research described in this paper.

Reaching Out

The project grew out of a collaboration between three mathematics educators and a computer science faculty member from the Department of Mathematics and Computer Science. We (the mathematics educators) had solicited the expertise of the computer scientist to work with us on a K-12 teacher change project. The project was implementing The Reflective Teaching Model, which engaged teachers in a set of peer-facilitated activities over an extended period of time. Through the shared experiences of working on the project together and through some additional informal conversations about teaching, the computer scientist suggested that she might be interested in trying to engage in some reflective practices. She and a colleague were planning some together. Would we be interested in working with them? Yes, of course, we would.

Gaining access to another faculty person's classroom, where that faculty person is willing to expose his or her teaching to a colleague and is willing to analyze, study, and reflect on that teaching, is not easy. We had often spoken of trying to work with university faculty in much the same way we had worked with K-12 faculty, but the pitfalls were many and the opportunities—until this point—had not been there. We were delighted to have the occasion to challenge our model and ourselves in this new arena.

The Model

The Reflective Teaching Model (RTM) is a teacher-change model that was conceptualized in earlier research on the role of metacognition in teachers' pedagogical thinking. The RTM provides a framework for educators to systematically observe, experience, and reflect upon teaching and learning and thus learn to challenge and explore their own teaching practice and to construct and use new knowledge about teaching and learning. The experiences and practices are clarified through peer-mentored opportunities to systematically reflect upon them. The RTM is grounded in two theories, constructivism and metacognition, and is based on a fundamental belief about sharing authority. Each of these is embedded in and motivates the experiences and activities we provide teachers as they examine their practice and attempt to change.

Constructivism and Learning. Constructivism is a theory of how people learn (von Glasersfeld, 1983). Central to the theory is the idea that learning is an active process of trying to "make sense" of new experiences. As we integrate new ideas and information into our existing knowledge structures, the new knowledge becomes unique to our own thinking.

If we apply this perspective on learning to teaching, we must reject the assumption that we can simply transmit information and expect that understanding will result. Faculty, viewed as learners of new thinking and pedagogy associated with their discipline, need opportunities to share their thinking, to evaluate, argue, and justify their reasoning, and time to construct new knowledge about teaching and learning associated with their disciplines. They need a dynamic and interactive learning environment in which to develop strategies beyond traditional methods.

Metacognition and Thinking. Metacognition is a theory of how people think. The theory refers to our ability to think about what we are doing and thinking while we are experiencing it (Schoenfeld, 1987). It is our ability to reflect on experiences and to learn from them. For a student, it is the ability to monitor how he or she is doing on a mathematical problem. For faculty, it is the ability to determine how a lesson is going, where the pitfalls are and how to regulate teaching behavior while teaching. Good problem solvers and teachers do these things almost automatically, but it is possible to increase metacognitive activity through carefully planned reflective experiences.

The RTM provides faculty with experiences to help them learn how to reflect on their teaching in a systematic, productive, and constructive way through peer interaction and facilitation. By thinking about teaching before the experience (planning) and by carefully reflecting about the experience after it has occurred (debriefing), the teacher becomes aware of the choices and decisions he or she can or could have made during a lesson. As the instructor becomes more metacognitive and raises to consciousness the awareness of newly constructed knowledge and the related choices this offers during the instructional process, the learning environment can become more flexible and open.

Sharing Authority. Finally, the RTM is based on the fundamental belief that sharing authority is a critical factor in creating environments that are most conducive to constructing new knowledge about teaching and learning. The ability of a faculty member or teacher to relinquish control, allowing others to share in the generation of intellectual developments, is a subtle but significant shift in roles. It builds trust, ownership, and cohesion among those involved. In the classroom it involves valuing student ideas. For teacher educators, it involves valuing the contributions and perspectives of teachers or, in our case, of non-education faculty with whom you are working. In general, sharing authority is the ability to be receptive to, value, and encourage various perspectives, to honor knowledge gained through experience and/or to respect the reasoning and thinking of others (Hutchings, 1993). This perspective frames all of the experiences in the RTM and sets the tone for reflective practice through collegial interaction and peer mentoring.

The Pilot Project

The objective of the pilot project described here, then, was to begin research on university faculty pedagogy and the process of change at this level. The research was participatory action research, that is, we were initiating the intervention and studying the process simultaneously. The model for intervention was an adaptation of the Reflective Teaching Model (RTM) in which colleagues/peers interacted to facilitate and support each other in the reflective process in a systematic way.

In this pilot, mathematics education faculty served as mentors to their computer science colleagues, modeling the mentoring process and encouraging deep and significant monitoring of their teaching behaviors. The team participated in a plan/teach/debrief cycle from the Reflective Teaching Model. It occurred as follows.

Plan. The planning session focused on lessons to be taught by the two computer science faculty members in separate sections. The planning was facilitated by the mathematics educators using a semi-structured set of questions to initiate thinking about the lesson. In a non-judgmental atmosphere, the member of the team who was to be teaching the lesson was encouraged to give serious consideration to her ideas and plans.

Teach. Following the planning, the mathematics educator and one of the computer science faculty members observed the teaching computer science faculty member, taking notes and making observations. The teaching faculty member attempted to implement the plan which inevitably varied from what was anticipated and planned.

Debrief. After each lesson the team met and debriefed, allowing the team member who taught the lesson to think back on the experience of teaching and examine the choices that were made and the outcome of those choices. The tone was again non-evaluative, using another set of semi-structured questions that encouraged careful examination of the experience. The teaching faculty member was asked how the lesson went, what worked, what didn't work, what she would do differently, etc. The debriefing portions of the cycle were videotaped, allowing team members to collectively view the experience. Those videotapes are the primary source of data for this report.

Results

Using qualitative methods, we have examined field notes and videotapes from the experience, looking at faculty behaviors as they participated in activities that encouraged the monitoring and regulation of their pedagogy. Preliminary observations are made here for the purpose of raising questions and issues that may lead to further understanding of the process and to guide further research. It is not suggested that this initial work is comprehensive—just a starting point.

We looked at the data from two perspectives: first from the point of view of the faculty member who actually taught the lesson and then from the point of view of the faculty member facilitating the reflection. Three main themes surfaced in the analysis: (a) Faculty have limited language to talk about teaching, (b) faculty are unable to see themselves in the act of teaching and talk about their choices, acts and decisions, and (c) it is extremely difficult for faculty to let go of their own agenda when facilitating another person's reflection.

The faculty members we worked with from computer science did not have much language to talk about their teaching. Education is a new domain for them with unfamiliar jargon and new modes of communication. Traditional concepts such as wait time, student centered activities, modes of questioning, redirecting questions, and changing modalities are just as alien to these faculty as are newer ideas such as students constructing their own knowledge or reflecting on their thinking.

Faculty members also found it difficult to analyze themselves—their actions—with any degree of detail in these initial cycles. Leading questions that were intended to force a faculty person to think about a choice he or she made and how useful that choice was usually elicited responses about what the students did or said. Faculty were unable to turn the camera on themselves, think about their own behavior, assess that behavior and make suggestions about alternative possibilities. When asked why they asked a particular question, they were more likely to focus on what the student did than on what they did.

The final theme came from analyzing the faculty facilitator. In each case when a new faculty member facilitated a reflection, this faculty member was driven by her own agenda, rather than reading the issues raised by the teaching faculty member and pursuing those.

Final Comments

The limited results described here only allow us to raise tentative hypotheses. When we contrast the work with university faculty to that done with K-12 teachers we notice two things. First, with K-12 teachers we held an initial inservice to help the community of teachers and educators build a common language to talk about their teaching. Phrases like 'constructing your own knowledge' and 'reflecting on practice' become part of the community jargon. No such orientation was done with the university faculty. Second, everything experienced in the RTM is modeled. In our university experiment, we did not do this. Particularly lacking was modeling of the process of reflection and facilitation of reflection. In retrospect, we may have had a more successful experience if the computer scientists had first observed us teach, reflect on our teaching, and talk about the experience.

The process and caveats of faculty change in the postsecondary setting are not well documented. This is a first attempt to explore that arena. The two computer science faculty who participated in this study were receptive to the notion of reflecting on their teaching and were/are willing to attempt more sessions. This is an important first step in creating an environment for change. They suggested themselves the need for more modeling of the process by experienced teams so that they (and others) might observe and learn. We are encouraged by the results.

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DEVELOPING TEACHERS' ABILITY TO IDENTIFY STUDENT CONCEPTIONS DURING INSTRUCTION

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This paper explores how teachers develop the ability to interpret student ideas that arise during instruction. Four teachers participated in a monthly video club in which they watched and discussed excerpts of video from their classrooms. Though the teachers initially focused on pedagogical strategies that might have been used by the teacher in the video, they soon learned to focus instead on trying to make sense of what happened in the video excerpt. Furthermore, they were able to develop a number of techniques for interpreting student actions that appeared on video. These techniques included paying close attention to individual student utterances, connecting a student's comment with the student's prior experiences, and examining the relationships among different ideas expressed by students. The teachers reported that as a result of their participation in the video club they paid more attention to student ideas that surfaced in their classrooms.

Reform in mathematics education calls for an adaptive style of teaching in which teachers make decisions, during the very act of instruction, about what mathematics to pursue with students. Critical to these decisions is the teacher's ability to identify and interpret student conceptions that arise in class and to use these ideas as the basis for decision-making. The focus of this paper is to explore how teachers develop such abilities.

Through a series of *video club* meetings, middle school mathematics teachers watched and discussed videotapes of their practice and became increasingly comfortable interpreting student comments and actions that appeared on video. I claim that the teachers not only accumulated new knowledge in the form of new understandings of student methods, but that they also developed new understandings of how to interpret student ideas as they arise during instruction. Such knowledge then informed the teachers' instructional practices as they listened for student comments and modified their instruction with these comments in mind.

Teachers' Content Knowledge and the Implementation of Reform

Mathematics education reform places extensive demands on teachers' content knowledge. First, reform requires that teachers rethink and extend their subject matter knowledge — their understanding of the mathematical skills and concepts that students are to learn. Teachers are also asked to use new pedagogical strategies, thus requiring the development of pedagogical content knowledge — an understanding of how to represent the subject matter to facilitate student learning (Shulman, 1986). Therefore, reform requires a combination of subject matter knowledge and pedagogical content knowledge, a combination that I refer to as *content knowledge*.

Numerous studies have found that implementing mathematics education reform calls for changes in teachers' content knowledge. In some cases, it appears that teachers do not have enough content knowledge in a particular area (e.g., Putnam, 1992). Thus, change involves acquiring new subject matter knowledge or pedagogical content knowledge. In contrast, other studies document cases in which teachers' existing content knowledge is in conflict with the goals of reform. Here teachers need to unlearn a set of familiar teaching strategies in order to implement reform successfully (Cohen, 1990).

Recent research has begun to examine the process through which teachers adapt their content knowledge during the implementation of reform. Central to this process appears to be the development of new understandings of students' ideas and methods. For instance, Heaton (1994) examined her own attempt to align her teaching practices with reform recommendations. To do this, Heaton found that she needed to learn how to hear students' ideas about mathematics as they surfaced in the classroom. Similarly, Sherin (1996) argues that recognizing novel student behavior during instruction triggered teachers to develop new understandings of mathematics. In light of such findings, it is critical to explore methods of providing teachers with opportunities to reflect on their teaching and to develop new ways of thinking about students' ideas.

Structured forums in which teachers reflect on their practice have the potential to act in this capacity. For example, case discussions (Barnett & Friedman, 1997) promote teacher learning and enhance teachers' ability to implement mathematics reform. I believe that video clubs can also serve as such a forum. Previous research documented that video clubs offer teachers opportunities to re-interpret their practice and had a catalytic effect on subsequent classroom instruction (Gamoran, 1994). The video clubs discussions reported in this study focused on identifying and interpreting student ideas. Moreover, rather than introducing teachers to a specific set of student methods or typical student misconceptions in a particular mathematical domain, the video clubs were designed to help the teachers develop their own interpretive techniques.

Research Design

Four sources of data were collected for this study. First, the researcher organized a video club at a middle school in the San Francisco Bay Area. Four mathematics teachers participated in the video club which met monthly from September, 1996 through June, 1997. Each video club meeting involved watching and discussing an excerpt of video from one of the teacher's classrooms. The video club meetings, which lasted approximately 45 minutes, were videotaped. Throughout the year the four teachers were either videotaped or observed on a bi-monthly basis; this formed a second source of data. Third, in June, each of the four teachers was interviewed about their experiences in the video club. The video club meetings and interviews were transcribed.

The fourth data source comes from a more intensive collaboration with one of the four teachers. In addition to the monthly video club meetings, this teacher was videotaped twice a week, met weekly with the researcher, and kept a journal of reflections on his teaching throughout the school year. Analysis of the data uses a number of analytic methods; primary among them is fine-grained analyses of videotapes (Schoenfeld, Smith, & Arcavi, 1991). In addition, techniques designed by the Video Portfolio Project (Frederiksen, Sipusic, Gamoran, & Wolfe, 1992) are used to analyze both the classroom and the video club data.

Results

In summary, an interesting and broad transformation occurred. During the course of the video clubs, the teachers moved away from a focus on "what could have happened" and towards an emphasis on interpreting and making sense of "what did happen" in a specific episode of classroom instruction. This shift seemed to facilitate the development of analytic techniques on the part of the teachers that they then used for interpreting classroom video. Subsequently, the teachers claimed that they found ways to apply similar techniques in their teaching practices. This transformation is described in more detail in the remainder of this section.

The teachers did not initially focus on interpreting the student ideas that appeared on the video excerpt. The teachers' initial reaction to watching video was to comment on alternate pedagogical strategies that the teacher shown in the video might have used. Thus, they responded to the video by suggesting pedagogical strategies that could have been used in the situation that appeared on the video. In addition, they saw the video as a potential source of new pedagogical routines. One teacher explained that this focus on pedagogy was similar to what he looked for when observing in other teachers' classrooms. "If I go into a classroom, I'm critiquing. I'm looking at what the teacher's doing and saying 'Oh, they could do this. Oh, this is a really good idea.' Or, 'They could have done this here.'" It seemed natural for him to use the same lens in the video club. As a result of this interest in pedagogy, the discussions often emphasized what *might have* happened in the lesson portrayed on the video rather than interpretations of what *did* happen.

As the video club progressed, however, this initial focus on pedagogy seemed to diminish. Two features of the video club contributed to this shift. First, because the video excerpts that the group watched came from the classrooms of teachers who participated in the video club, the group needed to find a way to discuss the video clips without being overly critical or evaluative of the teacher involved. This issue came up explicitly in the following discussion taken from the fourth video club meeting.

Teacher 1: I want to make sure we're up front about that comments are not personal or intended to be critical.

Teacher 2: No, I think it's a real issue. I would agree.

Teacher 1: I noticed one time when I criticized Dave, he looked a little hurt.

Teacher 3: I was. I was having a bad day... It's weird to have some one come visit your classroom, but then to have it on videotape where they can rewind and watch what they want to watch.

The group's concern about their vulnerability during these discussions helped to open up the possibility of looking at classroom video in other ways.

A second feature of the video club capitalized on this opportunity. The facilitator, Sherin, and the teacher with whom Sherin worked most closely, David Louis, modeled an alternate way of commenting on the video. First, they tended to emphasize events within the video itself. Second, they focused the group's attention on the explanations that students offered in the video.

For example, during the second video club, the group watched an excerpt in which students discussed containers of various shapes. The students were trying to determine the graphs that best represented the height of the water level in the containers as water was added at a constant rate. One of the teachers began to describe generally how the same lesson had played out in his own classroom. Sherin attempted to draw the group back to the video and asked, "So let's go back to Daniel's comment. He says, 'That one's kind of curved. But this one changes suddenly instead of going up gradually.'" Sherin continued by stating that she was unsure of the precise meaning of Daniel's statement and invited the video club members to comment.

Another example occurred later in the same meeting. The group was discussing how the teacher in the excerpt could have determined whether those students who were not speaking were still engaged in the whole-class discussion. In this case, Louis turned the group's attention towards one of the students in the class, Jared, who had remained silent during most of the discussion. "Jared is completely out of the discussion until the very end, when, at an appropriate point he says, 'Well I agree with Tina because...' And I would have never expected him to jump in like that. And if he never jumped in, I might have thought that he was not listening." Here Louis moved the conversation away from a discussion of pedagogical strategies that could have been used. Instead, he focused the group on making sense of the data that presented itself on the video.

The teachers were able to develop techniques for interpreting student actions that appeared on the video. Once the teachers began to focus on trying to understand the student ideas that appeared on the video, they developed a number of effective techniques for doing so. One such technique was to pay close attention to individual utterances by students. In other words, the teachers analyzed particular statements for clues about the students' understandings. For example, the group explored students' conceptions of the terms linear and exponential after watching an excerpt of a whole-class discussion on this topic. The teachers focused on specific comments by students and used the transcript of the videotape as an additional resource.

A second technique that the teachers developed was to connect a student's statement with students' prior experiences with the concept in question. In this way, the teachers tried to uncover the source of a particular idea. For instance, when a student stated that "linear is like it's adding," the group discussed how recent experiences comparing the differences between consecutive y-values in a data set might have led the student to make this claim.

Third, the teachers compared the methods and comments of a number of students as a way of exploring the relationships among the ideas that students raise. For example, the teachers contrasted three types of responses from students who examined a set of values representing an exponential function. Some students claimed simply that "there is no pattern," while other students inferred from this lack of pattern that "it's not going to be a line." Finally, there were students who explained that the numbers "get larger and larger," "but not at even intervals," and that this is a result of "multiply[ing] by the same number." In sum, in the video club, the teachers engaged with student ideas in a number of new ways. Moreover, the data suggest that the teachers adapted these techniques for use during classroom instruction.

The teachers developed related techniques to interpret student actions during classroom instruction. In individual interviews near the end of the school year, the teachers reported that their participation the video club had influenced their classroom teaching. In particular, each of the teachers claimed that they were more aware of student ideas that surfaced in their classroom. As one teacher explained "I think I'm better at listening to students." He went on to describe how in the video club, the group would spend a great deal of time trying to sort out a particular idea that a student had raised. "We'd argue about it for an hour." As a result, in his own teaching, he began to pay closer attention to the meaning behind students' comments. "I stop and try to figure out what students are saying…There's been instances where we just stop and slow down and figure this out [because] this is going to be valuable. 'Wait, are you trying to say this…?' 'Can you clear that up for me?' It's made my Q & A with the students less directed."

Preliminary analysis of the classroom observations support these claims by the teachers. For example, Louis often uses a "revoicing" strategy in his teaching (O'Connor & Michaels, 1993), in which he rephrases a student's idea for the class. In doing so, Louis makes the task of listening for and interpreting student ideas a formal part of his role during instruction. In this way and in others, the teachers learned to pay attention and react to student comments in the moment of instruction.

Conclusion

In this paper I have argued that in order to implement mathematics education reform, teachers need to learn new ways of interpreting student ideas that come up during instruction. This work suggests that video clubs have the potential to support such efforts. In particular, video club meetings provide a context in which teachers can examine classroom practice with a new lens. Rather than focusing solely on pedagogical strategies, video serves as a useful medium for in-depth investigations of student ideas. Further research is needed to delineate the ways in which participation in a video club subsequently affects teachers' attention to these ideas in the classroom.

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CHARACTERIZING A PERSPECTIVE ON MATHEMATICS LEARNING OF TEACHERS IN TRANSITION

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Teachers who are participating in the current mathematics education reform are changing their perspectives about mathematics, learning, and teaching from perspectives commonly associated with traditional teaching. The research reported here identifies a particular perspective on mathematics learning, a "perception-based perspective" that may be useful in understanding part of the developmental process of many of these teachers in transition. The articulation of this perspective is based on empirical evidence from a study of teachers and prospective teachers in a professional development program. A perception-based perspective represents a shift from traditional perspectives in its emphasis on first-hand experience with mathematical relationships. From a perception-based perspective, these relationships exist in an independent reality that can be perceived by each learner. Thus, the perspective also contrasts with a conception-based perspective which emphasizes the experiential reality of the learner and the key role of current conceptions in what is perceived, understood and learned.

Understanding teachers' perspectives as they struggle to participate effectively in reform of mathematics teaching can contribute to mathematics educators' efforts to work more effectively with teachers in transition. One area of teachers' perspectives that we take to be important in determining teachers' practices is their perspectives on mathematics learning, whether that perspective is explicit or implicit. In this paper, we derive from research data a perspective on learning that may be useful in characterizing the perspectives on mathematics learning of teachers in transition.

Background and Methodology

This report is based on research conducted during the first 2 years of the 4.5-year Mathematics Teacher Development (MTD) Project[1]. The Project is focused on understanding how teachers' practices, including perspectives, develop from ones based on traditional conceptions of mathematics, learning, and teaching towards ones based on conceptions that are more consistent with those underlying current reform efforts. Toward this end a teacher development experiment methodology (Simon, in press) is used which combines whole-group teaching experiments in teacher education courses with case studies of individual participants. The participants–henceforth referred to as "the teachers"–are prospective and practicing elementary teachers (grades 1-6) who work collaboratively as students in 5 mathematics teacher education courses in consecutive semesters. In addition, the prospective teachers participate in a 3-year series of internships in the classrooms of the practicing teachers.

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Data for this report were taken from the second of the MTD courses, which focused on children's mathematical thinking. The analysis presented centers on discussion of development of a concept of number. The courses, conducted as teaching experiments, were videotaped and transcribed. Ongoing analysis took place between class sessions and resulted in decisions for the subsequent session. Following the completion of each course, the research team conducted a retrospective analysis (Cobb, in press) based on either explicit a priori research questions or on themes that had emerged in the ongoing analysis. The retrospective analysis involved line-by line analysis of the videotapes (supported by transcripts) in which relevant data were identified and interpretations of the data were negotiated. Analysis of additional data caused revisiting of earlier data and renegotiation of interpretations in order to arrive at explications of the data compatible with the full set.

Conceptual Framework

We specify briefly here aspects of our conceptual framework that figured prominently in our analyses of data. These include the psychological component of our perspective on learning mathematics and a brief indication of the concepts that we use to think about children's development of number. We work from an emergent perspective (Cobb & Yackel, 1996) that includes both a social and a psychological component. Underlying the social component is a view of learners as participants in particular social groups that have norms, practices and knowledge that is taken-as-shared. This view was not very related to the perspectives of the teachers involved. Our focus in this report is a psychological interpretation of mathematics learning because of the important contrast we have identified between our psychological perspective and what we have come to understand as the psychological perspective of the teachers in this study.

Psychologically, we view knowledge development from a constructivist perspective. This is a *conception-based* perspective, i.e., knowledge develops through transformation and reorganization of current conceptions. A learner's current state of knowing influences learning in two ways. First, it defines the learner's experiential reality; that which a learner perceives (attends to) and interacts with is structured by her current knowing. This is our way of using Piaget's core notion of assimilation. Second, it is this current knowing that must be transformed (accommodated). Thus, the possibilities for learning are afforded and constrained by the current state of knowing. From this perspective on knowing (complemented by a social perspective), mathematics is created by humans as a way of organizing their experiential worlds.

We now turn to the development of a concept of number. Here we articulate only those aspects of our conceptual framework that are implicated in the data analysis for this report. Children initially do not view their worlds in terms of quantities of objects, that is, number knowledge is not innate. Thus, their experience of the world does not initially include a perception of number. Through coordinating two actions, rote counting (recitation of number words in order) with the touching of individual objects (using one-to-one correspondence), and reflecting on records of experience of that activity, the child abstracts the idea that the last number word stands for the manyness of the collection of objects (Steffe, von Glasersfeld, Richards, & Cobb, 1983). At this point we would consider the child to have a primitive conception of number.

Inquiring into Children's Development of Number

Early in the second semester of the MTD Project, the researcher/teacher educator, Simon, engaged the teachers in discussions of children's development of number. What resulted could be characterized as a problematic and mostly unsuccessful attempt by Simon and the participating teachers to communicate despite significant, but, to that point, unidentified differences in perspectives. The analysis of these discussions allows for an exploration of the teachers' perspectives on mathematics learning focusing on the group as an interacting whole. This focus does not afford conclusions about differences in individual perspectives.

The exploration of children's development of number began with assigned readings that described stages of children learning to count. Simon facilitated a discussion clarifying the meaning of the various stages described in the readings (e.g., rote counting, one-to-one correspondence, counting-on) and identifying the typical sequence in which those stages unfold. He then tried to engage the teachers in thinking about how children progress from one stage to the next. In particular, he invited them to consider how a child who can only recite a sequence of number words while touching one object for each word progresses to using those number words to quantify manyness. The teachers explained this transition as the result of repeated experience relating number words with perceived quantities.[2]

L: They experience it and after enough experiences, then they start making the connection, "Oh that's what she means when she says two."

M: I think that if they had different groups of cups . . ., they say, "Okay

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[2] Repeated and extraneous words have been omitted for ease in reading the transcripts.

> a group that looks like this [inaudible] is a five." Maybe they don't know what that means but they know that the word five goes with that. And then they have their group of three. So they say, "Well a group that looks like this is a three."

Simon pointed out that if children do not see the objects in terms of quantity, they cannot identify that attribute of the collection to connect with the number word. However, several subsequent comments by the teachers suggested that they had difficulty conceiving of children who have no perception of quantity.

> H: They're not born with number but they immediately need number . . . .
>
> N: They start wanting to know like how many is in that group.
>
> Simon: If they don't know what number is, how do they know the question of how many?
>
> N: . . . They're seeing a big group of things. They can figure out that there's cups, and there's different numbers of cups.

Our analysis of these data identified two related characteristics of the teachers' responses: they had difficulty constructing a mental image of a child who did not see the world in terms of number and they had an unproblematic view of the learning of number that involved the association of number words with examples of the quantities. E's comments contributed significantly to our interpretation of these data:

> E: What I'm wondering, and I don't know is, [do they see] individuality? And can they look at those cups and see that they're all separate–maybe not knowing how many there are or whatever? When you give your son two cookies, he sees that they're separate things that he's getting? . . . So that when he looks there, he'll see a number of them, like numerous ones, and see them all separately? . . . When I look at that, I see five separate things, or without using the word five, I see separate things.

E could not understand how children would not see 5 objects unless their vision was insufficiently developed. For her as for many of her colleagues, it was a matter of perception of (direct access to) an external reality that there are 5 objects, regardless of whether one does not yet know a number word to associate with that number of objects.

Discussion

Based on an analysis of these and a considerable quantity of additional data, we postulate a perspective on learning that we refer to as *perception-based* and which we believe may usefully characterize the perspectives of many teachers in transition. According to this perspective mathematical relationships exist in an independent external reality. This implies that these relationships are perceivable by all learners and that what is perceived is the same for each person. Thus, number can be seen in a group of objects, place value can be seen in Dienes blocks, and multiplication can be seen in an array. In our example, the teachers considered that number can be learned by associating number words with the universally available perception of the quantity to which it is associated. This perspective is consistent with what Putnam (1988) called a "representational view of the mind," in which learning is seen as developing internal representations to mirror relationships perceived from the external world (Cobb, Yackel, & Wood, 1992).

This perception-based perspective on mathematics learning can be contrasted with both a traditional school mathematics perspective and with the conception-based perspective that we articulated briefly in the description of our conceptual framework. In the traditional view, students *passively receive* mathematical knowledge by listening to and watching others, usually mathematics teachers and by reading about mathematics (in textbooks). The perception-based perspective takes as fundamental for learning *direct personal perception* of mathematical entities. That is, to ensure the desired quality of learning, it is necessary for learners to "see" for themselves the mathematical relationships that exist among mathematical objects. Such direct perception is considered to derive from representations using manipulatives and pictures, from seeing patterns, and from experience associating related phenomena (e.g., a quantity and a number name). This perspective challenges the notion that teachers can transmit their perceptions/knowledge to the students and promotes the creation of experiences in which the learners can perceive relationships first hand.

In contrast to a perception-based perspective, a conception-based perspective eschews the notion of direct interaction with a reality different from the one structured by the individual's current conceptions. Perception is defined as an interaction in one's personal reality, which is structured by one's conceptions. The implications for teaching of the two perspectives are dramatically different. From a perception-based perspective, teaching involves creating opportunities for students to apprehend (perceive) the mathematical relationships that exist around them. From a conception-based perspective, the focus is on understanding the students' conceptions (assimilatory schemes) and determining ways to promote transformations in certain of those conceptions. Teachers use popular teaching tools and strategies (collaborative groups, manipulatives, class discussions) differently and for different purposes, depending on the perspectives that they hold.

Why might a perception-based perspective be a common construction of teachers in transition? It is natural for people to assume that what they experience is how the world really is. It requires a particular understanding and concerted effort to step back from this perspective and view oneself and others as living in their experiential realities. Teachers who participate as students in reform-based mathematics experiences often have the experience of "perceiving mathematics as it is." Cobb, Yackel, & Wood (1992) explained that this experience of "mathematical truth and mathematical certainty" (p. 3) is an essential part of doing mathematics often missing from traditional school mathematics. As teachers' communicate about their "perceptions" of the mathematics, they come to believe that others see what they see, which supports the assumption that these relationships are preexisting and universally perceivable (Cobb, Yackel, & Wood, 1992.).

The postulation of a perception-based perspective as a result of this study can serve in two ways. First it may afford a better understanding of how a number of teachers in transition are viewing learning. It may account for teachers' emphases on opportunities for the students to "see" mathematical relationships and help to explain how teachers interpret professional development opportunities. What does learning to listen to students mean from this perspective? What sense do teachers make of their opportunities to learn mathematics with understanding? Second, by comparing a perception-based perspective with a conception-based perspective, we can posit directions for ongoing efforts to foster mathematics teacher development. Teacher educators face the challenge of helping teachers to understand that learners who differ in their mathematical knowing see the world differently (their experiential realities), to understand particular ways that students might perceive of and think about their worlds, and to inquire into how new or modified conceptions might be developed by these learners.

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TEACHER KNOWLEDGE
SHORT ORALS

UNDERSTANDING THE EVOLUTION OF ONE TEACHER'S PHILOSOPHY OF MATHEMATICS THROUGH THE NARRATIVE OF HIS BIOGRAPHICAL HISTORY: PART I, HIS INITIAL CONCEPTIONS

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The goal of our research is to understand the evolution of one teacher's philosophy of mathematics during his teacher education program and the impact of his philosophy on his classroom practice. In this paper we will report on the teacher's conceptions about mathematics as he entered the teacher education program (thereafter called his initial conceptions) and the critical experiences that, from his own perspective, influenced those conceptions. As a main research method, we are using the biographical approach that involves the use of heuristic tools such as critical event and formative experiences. Damian Torres, the participant of the study, is writing a journal of reflections in which he reflects and comments on his evolving philosophy of mathematics and provides answers to questions or issues related to the nature of mathematics.

Damian Torres's initial conceptions about the nature of mathematics included a utilitarian view, a platonic view, a logical view, and a problem-solving view. Damian Torres saw mathematics as a service subject because it is used to solve daily-life problems faced by ordinary people, and advanced practical problems faced by some people having a professional degree such as engineers, architects, etc. He also held a platonic view about the nature of mathematics. He thought that mathematics was a finished, immutable and absolute body of knowledge. He said that mathematical formulas and theorems were true yesterday, are true today and will be true tomorrow. Damian Torres also views mathematics as a logical, objective, and rational discipline. He stated that the solutions to mathematical problems can always be checked because their solution processes are based on logical reasoning and not on personal opinion. Finally, he also conceived mathematics as problem solving. He mentioned that mathematics provides problems to practice and apply the formulas and theorems to as well as to develop our thinking skills. Damian Torres said that some factors that shaped his initial conceptions include his experiences in solving daily-life problems and the problems and theories posed and described in mathematics and science textbooks.

SECONDARY TEACHERS' KNOWLEDGE OF FUNCTIONS: THE ROLE OF KNOWLEDGE AND CONCEPTIONS IN THE CLASSROOM

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Previous research has recognized content knowledge and pedagogical knowledge as important cognitive aspects of teacher knowledge. This study 1) examined the extent and organization of experienced secondary mathematics teachers' subject matter and pedagogical content knowledge of functions, 2) examined their conceptions of the nature of mathematics, and 3) characterized how this knowledge and their conceptions are applied in the classroom.

In phase one of the study data concerning 20 teachers' knowledge of functions were collected. This provided the background for phase two, case studies of two participants including extended classroom observations and interviews. This design allowed for the careful selection of cases to provide insight into how teachers' knowledge bases are used in the classroom. To gather and triangulate information on teachers' knowledge of functions and their conceptions of mathematics, a variety of instruments were used: a survey of knowledge; a survey of conceptions of mathematics; two concept mapping tasks; an interview based on the concept mapping tasks, including two card-sort tasks; and an interview based on the survey and conceptions instruments.

The results showed practicing teachers had a better understanding of functions than did preservice teachers, but 20% of the practicing teachers did not demonstrate a uniformly deep understanding of the function concept despite their considerable experience teaching the concept. The second phase of the study resulted in three grounded hypotheses which describe the impact of teacher content knowledge and conceptions on classroom teaching. First, teachers' concept image impacts teaching through the learning opportunities the teacher affords students. Second, the use of applications in teaching depends upon the availability of real world applications in the knowledge base of the teacher. The existence of this knowledge does not ensure its application. Finally, conceptions of mathematics influences the use of content and pedagogical content knowledge in the classroom in both subtle and obtrusive ways. One particularly salient conception dimension was the perceived usefulness of mathematics.

TEACHER KNOWLEDGE
POSTERS

POSING MATHEMATICAL PROBLEMS BY GENERALIZING: A STUDY OF PROSPECTIVE SECONDARY MATHEMATICS TEACHERS

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Mathematics is "alive" because it has problems whose solution has not been found; but mathematics has problems to be solved because somebody has posed those problems. Thus, problem posing is a central activity of mathematics. However, little research has been conducted regarding people's abilities to pose mathematical problems. Many mathematics theorems involve general formulas or patterns. For example, the sum of the measures of the interior angles of a *n*-gon is $(n\text{-}2)180^\circ$ and the number of diagonals in a *n*-gon is $\frac{n(n-3)}{2}$ etc. Then, generalizing is a fundamental aspect of mathematics. In this study we examine the problems posed by 15 prospective secondary mathematics teachers. The students were given the following situation with corresponding diagrams: "We notice that we can draw two diagonals in a quadrilateral, five diagonals on a pentagon, and nine diagonals on a hexagon. Look at the examples and think about similar questions or problems that occur to you. Write down as many different problems or questions as you can." We were particularly interested in whether they would pose the general problem related to how many diagonals there are in a *n*-gon. The teachers had not received any formal instruction on problem posing. However, at the time the task was given to the students, they had experience working on finding general formulas for geometric patterns.

Students posed a diversity of problems that varied on mathematical complexity. Thirteen of the fifteen students posed the general problem of finding the number of diagonals in a *n*-gon. All the students posed numerical problems about the number of diagonals on polygons with a number of sides less than 12. Other mathematical problems posed by students were related to the number of triangles or regions formed in some of the polygons. Students also posed questions of a pedagogical or methodological nature. The most common question was related to whether a pattern can be noticed between the diagrams and the number of diagonals in each diagram. Then, we can conclude that those students had an ability to pose interesting mathematical problems.

STUDENT CONTRIBUTIONS CORRECTING AND STRENGTHENING TEACHERS' CONCEPTIONS OF MATH

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This study concerns what teachers learn from students. Recent claims suggest that one of the reasons teachers fail to enact pedagogical and curricular reform in their teaching of math is because of constraints on their own understanding of the mathematical concepts. As a result, teachers retain intellectual control of their classrooms by using traditional teaching methods with a unilateral flow of knowledge. In contrast, I claim that a bilateral flow of knowledge between teacher and students has the power to increase teacher knowledge of math concepts and operations.

An intensive year-long case study of one teacher was undertaken at a small urban elementary school. Analysis of transcripts of classroom video and teacher interviews has generated three levels at which teacher mathematical leaning was stimulated by student discourse. These three levels move from learning about specific facts or terms to more general reconceptualizations of math:

1. *Teacher refines the meaning of a mathematical term.* Students defined terms in their own words and with their own examples; this enhanced teacher understanding.

2. *Teacher learns a new strategy for solving a mathematical situation.* By allowing students to explore solution strategies and then discuss them, the teacher learned new solutions.

3. *Teacher understands a new frame for viewing and analyzing mathematical situations.* As students found extensive patterns in numbers and connections between operations, the teacher became aware of these patterns and connections.

As expectations concerning the "transfer of information" from teacher and students change, an environment that supports "discourse for understanding" on the part of students *and* the teacher results. The teacher gains math and pedagogical content knowledge that powerfully affects learning in the classroom and has the potential to affect future teaching.

THE IMPACT OF TEACHERS' KNOWLEDGE ON STUDENTS' LEARNING OF FRACTIONS

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This study was developed to inform a theory on the interrelationship of teachers' conceptual knowledge of fractions and students' development of conceptual knowledge of fractions from a social constructivist perspective. Two teachers from among four possible candidates were selected on the basis of the differences in their written responses on a test of fraction knowledge and their location in the same school system. The cases of these two teachers generated from data collected through videotapes of the unit on fractions the teachers conducted, interviews of students and teachers, collections of student work, and students responses to a test of fraction have been examined to determine the patterns of relationship between teachers' knowledge of fractions and students' developing knowledge of fractions.

Previous studies which attempted to correlate teachers' knowledge of subject matter with students' acquisition of subject matter knowledge using quantitative methods did not show highly significant results leading the researchers to believe that teachers' subject matter knowledge was not a powerful factor in students' development of subject matter knowledge. More recent studies using qualitative methods have found that teachers' subject matter knowledge does influence their instructional practice. However, no studies using current learning theory and research methods have examined the interrelationships between teachers' knowledge of subject matter and students' development of subject matter knowledge. This study bridges that gap.

From this study it is evident that teachers help students to interpret a problem/conceptual space such as fraction number sense and negotiate with them meanings and schemas relating words, symbols, images (visual and concrete), and contexts in which fractions are appropriately used. These negotiations are primarily influenced by the student's sense-making, but are constrained by the variety and appropriateness of the teacher's own schema of the problem/conceptual space.

PERCEPTIONS OF PRESERVICE ELEMENTARY TEACHERS TOWARD USE OF MATH MANIPULATIVES

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In order to engage students in the process of doing mathematics, active manipulation of concrete models, is necessary for construction of mathematical concepts to take place. There is little research on preservice teachers' perceptions about the use of manipulatives as well as the relationship between manipulatives and attitudes toward mathematics.

The subjects for this study were 32 preservice elementary teachers, enrolled in a one-semester math methods class, in a small regional university in the Midwest . This study used several methods of inquiry: self-report survey on attitudes toward mathematics, questionnaire about attitudes toward and use of manipulatives in the classroom.

The perceptions about the appropriateness of manipulatives, as opposed to more traditional items such as worksheets, increased during the semester. The attitudes of the preservice teachers became more positive by the end of the semester. Although the subjects continued to believe in the use of games for the classroom, they began to realize the need for true manipulatives, such as base ten blocks and Cuisenaire rods.

In order to bring about the necessary changes in teacher preparation, preservice teachers need to experience mathematics as an active process themselves. Teacher educators must provide initial training in preservice courses as well as continuing training as part of inservice experiences. If favorable attitudes toward mathematics by teachers can help students' attitudes and beliefs about mathematics, a major objective in the preparation of preservice teachers should be experiences that help develop a philosophy of education that would incorporate theses feelings. These positive attitudes can lead to greater participation and achievement by all involved.

MATHEMATICAL TASKS CHOSEN BY PROSPECTIVE TEACHERS IN THEIR PROFESSIONAL SEMESTER

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The Standards (NCTM, 1991) maintain that teachers must help students develop conceptual and procedural understanding of mathematics. Conceptual knowledge of mathematics is characterized by Eisenhart (1993) as the knowledge of the underlying structure of mathematics—the relationships and interconnections of ideas that explain and give meaning to mathematical procedures. Procedural mathematical knowledge is defined by Hiebert and Lefevre (1986) to be made up of two parts: the formal language of mathematics and rules, algorithms, or procedures used to perform mathematical tasks. Research findings show that procedural knowledge is emphasized in most teachers' lessons. This preliminary study examines tasks selected by three prospective teachers and the extent to which these tasks reflect the intent to teach procedural and conceptual knowledge. Sources of data were audio and videotapes of teaching episodes, the prospective teachers' lessons plans, and the researcher's field notes. Data were analyzed using multiple sorts to define which tasks emphasized procedural knowledge and those that emphasized conceptual knowledge. Preliminary results indicated that the preservice teachers depended heaviiy on procedural tasks, especially in the first few weeks of their student teaching. Two of the three continued to emphasize procedural tasks while one was able to incorporate conceptual tasks as the semester progressed.

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TECHNOLOGY
RESEARCH REPORTS

INVESTIGATING TEACHERS' INSIGHTS INTO THE MATHEMATICS OF CHANGE

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The mathematics of change and variation is one of the "big ideas" highlighted in the NCTM Standards and reflected in many new curricular reform efforts (cf, TERC, 1997). As with most big ideas, forming robust images of how to mathematize these situations is very challenging for students and teachers as well. One of the goals of the SimCalc Project (Kaput & Roschelle, 1997) is to develop software-based activities that can support students' thinking about ideas of relative motion by enabling them to mathematize the movements of characters in a simulation environment. For such project efforts to be successful, it is critical to understand how students can coordinate and revise their current views of mathematical formalisms (which are often devoid of real-world relevance) with their present efforts to describe change. In this paper, we discuss the ways in which preservice teachers' actions with the software constrained and enabled the emergence of a variety of mathematical insights.

Theoretical Framework

The theoretical framework guiding our analyses is based on a phenomenological perspective that focuses on the ways in which learners report revising their mathematical ways of knowing as they interact with symbols on a computer screen and with each other. This perspective draws on the work of researchers such as Nemirovsky and Monk (in press) and Miera (1998) who describe students' experiences and interpretations with computer-based symbols from the students' perspective. We also build on the work of Carpenter and Fennema (1992) in that we seek to analyze not only teachers' interactions with the software but also their interpretations, understandings, and explanations as they, in turn, interact with their own students.

It has been said that computers both constrain and enable specific understandings based on their affordances (cf. Norman, 1994; Pea, 1992). In our view, this interpretation tends to assign agency to the computer instead of the student. That is, it is not the computer software itself that constrains or enables students' interpretations, but instead it is the quality of their reasoning and the social nature of their interpretations that supports or discourages theirs efforts to revise their current ways of knowing. For research purposes, the implication of this shift is that we try to focus not on the affordances of the software itself, but instead on the quality of the interpretations and reorganizations the students create.

Data Corpus

The subjects for this study were pre- and in-service secondary teachers enrolled in one of two technology-based courses taught by each of the authors of this paper. Therefore, we will refer to the students in our classes as "teachers" to distinguish them from the students they tutored. The goal of these courses was to expand teachers' ideas of functions in conceptual ways by engaging them in a variety of computer-based activities in which the goal was not simply to compute a correct answer (as is the case with most calculus courses), but instead to explain *why* these answers appeared as they did. Prerequisite courses included at least one semester of elementary calculus, although many students had completed a full undergraduate major in mathematics.

The data corpus consists of 1) the instructors' journals, and 2) copies of all teachers' written work (a concept map, the "Cheetah and Gazelle" assignment, the Bouncing Ball assignment, journals, and a project report). The written work was analyzed to capture the teachers' reflections on their software-based activities. The project report was designed to reveal teachers' reflections on their tutoring work with one or more students.

The two activity sequences described above were designed to be explored with the Mathworlds software, a simulation world developed by a team of researchers at the University of Massachusetts at Dartmouth. This program is a dynamic microworld for exploring one-dimensional motion in which any combination of three graphs (position *v.* time, velocity *v.* time, and acceleration *v.* time) can all be linked to an animated simulation and to each other. The goal of these various linked representations is to enable users to explore motion from different perspectives (Kaput & Roschelle, 1997). For example, students were asked to model the following problem from the Cheetah and Gazelle activity:

> Cheetahs can run very fast, but only for short distances. Horses cannot run as fast, but can run much farther at their top speed. Imagine the following situation. A horse gallops by a dozing, well-camouflaged cheetah, who wakes up and decides it's dinner time. He accelerates with constant acceleration to his top speed of 30 meters/sec. in 10 seconds, while the horse, who has different dinner plans (he's a vegetarian), continues running at his top speed of 20 meters/sec. The cheetah can run only 200 more meters after he reaches his top speed. Does he get to have a vegetarian for dinner?

One example of how this question could be explored in the Mathworlds software is shown in Figure 1.

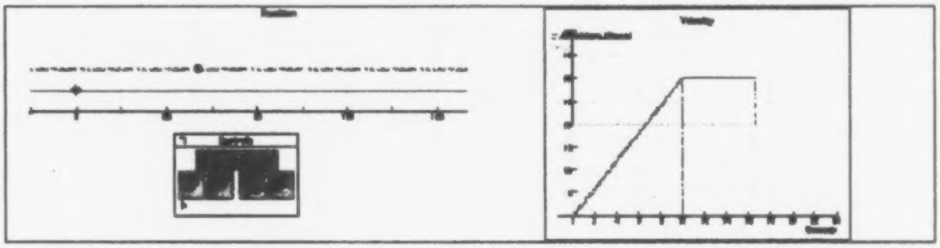

Figure 1. *Dots simulation and Velocity graph used with Cheetah and Gazelle activity.*

Methodology

In order to investigate the nature of teachers' experiences and the sources of their insights while acting with this environment, the instructors asked the teachers to do the following:

1) Complete the Cheetah and Gazelle activities beginning with graphs of velocity. The reason for limiting the teachers to using velocity graphs was to push them to reason about relative position using relative velocity.

2)Explore the "Bouncing Ball" activity, which involves using a motion detector to graph the position and velocity of a bouncing ball and to then to simulate this experience using Mathworlds.

3)Create a three-lesson activity sequence (using MathWorlds) directed toward middle or high school students, and then implement the lessons with at least one student. After each session, the lesson plans were required to be revised based on student learning. The teachers were also required to write a report on the goals of the lesson, what the student learned, and what they learned about the student's thinking.

The methodology for analyzing these data was based on the goal of documenting the quality of student reflections and consequent reorganizations of their formally-held views. Thus, we were not, for example, looking for ways in which MathWorlds supported students' reasoning, but instead were focusing on ways in which the teachers acted with the software to revise their own understandings and those of their tutees. In the following section, we describe some of the insights and constraints that arose as students engaged in the activities and tutoring sessions.

Results and Discussion

Our analyses indicated that the teachers came to view several mathematical concepts in new ways. From our view, these insights were enabled and constrained by their efforts to reflect on and make sense of their activities. In the following paragraphs, we briefly discuss three insights that emerged as the students acted with the software and two cases in which the students' interpretations were constrained by their use of the software.

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Interpretations that Enabled Insights to Emerge

One of the most profound and widely reported mathematical insights was that any given velocity graph determines a family of position graphs. This insight occurred when teachers noticed an unexpected phenomena: When they moved the velocity curve vertically, the linked position curve moved along with it. On the other hand, when they moved the position curve vertically, the linked velocity curve did not move along with it. In an effort to reconcile this apparent contradiction, many teachers referred to their prior knowledge of calculus. In so doing, they reinterpreted their actions by noting that as they were moving the position graph up and down, they were changing the initial starting point but were not changing the speed at which the animals moved. At this point, these teachers reported an "Aha!" insight regarding the meaning of the ubiquitous "+C" they had been trained to add when computing indefinite integral functions. In the full paper, we present more detailed examples illustrating how the teachers' actions with the linked representations of the position and velocity graphs supported their efforts to reorganize their views of both the velocity graph and the Fundamental Theorem of Calculus.

A second insight that emerged from the activities was a phenomenological interpretation of Reimann Sums. When we asked the teachers to figure out where the animals would end up, we assumed that most teachers would count the squares on the velocity graph (representing one meter traveled in one second) or simply calculate the triangular area under the linearly increasing velocity curve. Instead, many used Reimann Sums to approximate distance traveled at constant speeds. Although these teachers knew that forming Reimann Sums was an appropriate approach to finding area based on their prior calculus knowledge, the activity of actually *drawing the bars* supported a reinterpretation of their views of what a Reimann sum was, and also the relation between area under a velocity curve and its relation to a graph of position.

A third insight that emerged as students engaged in the Bouncing Ball activity was a stronger intuitive feel for and appreciation of linearly decreasing speed and its relation to position. In particular, when given a linearly decreasing velocity graph and asked to make a position graph that would show the position of a ball at each point in time, 40% of the students calculated position by multiplying the velocity at a given time and the given time measure and then plotting this product and the corresponding time to create a position *v* time graph. After checking their work with the linked representations in the Mathworlds, the students were surprised to see that their position graphs were incorrect. This conflict between what they expected and what he saw lead to fruitful explorations in which the instructor encouraged the class to compare the graphs of a ball moving at a constant rate with one moving at a linearly decreasing rate. For some students, this discussion lead to an "Aha!" experience wherein they began to revise their view of the Mean Value Theorem in terms of the linked relationship between the velocity and position graphs rather than simply in terms of a static drawing of a function and its derivative at a point on a graph. During this discussion several students asked why their former teachers had not made this connection, and, more generally, whether formalisms should precede or follow phenomenological experiences. This issue is further addressed below.

Interpretations that Constrained Insights

One issue that often arose in the teachers' reflections was whether students should be taught formal definitions for mathematical conventions such as slope and y-intercept before or after they are asked to explore these ideas in the microworld. When planning their tutoring sessions, approximately 50% of the teachers organized their lessons to begin, as one teacher wrote, with "qualitative experiences of speed rather than formulas which they forget anyway." This dilemma parallels Doerr's (1997) distinction between exploratory and expressive computer software. Although the class did not reach a consensus, the teachers did agree that the answer to this paradox may not be to choose one approach over the other, but to ensure that both phenomenological exploration and the discussion of formalisms be included in their lesson plans. Clearly for most of these teachers, revisiting the exploration of the relationship between the velocity and the position functions in MathWorlds provided new insights into previously held views of calculus.

A second interpretation, which emerged as the teachers worked with their own students, was that there can only be one answer, and it must match that shown on the computer. For example, during one tutoring session, a senior mathematics major was tutoring a talented high school sophomore. The teacher asked the student to draw a position graph in which a clown was traveling at a constant rate of 6 m/s for 3 seconds. The student reasoned that the clown would travel 18 meters away from his start after three seconds. He then plotted the points (3,18) and (0,0) and then drew a line connecting these two points as shown in Figure 2(a). The tutor then asked him to draw a velocity v. time graph showing his velocity at each second. In response, the student drew the graph shown in Figure 2(b). Because this graph did not match the tutor's expectations of a constant velocity graph as would be included in the MathWorlds, she dismissed it as incorrect. However, on later reflection with her instructor, she realized that in the student's explanation, he had described a graph of rate by *seeing* velocity in each of the staircases as shown in Figure 2(c). For him, these were two entirely different graphs, depending on how they were interpreted.

This example illustrates one incident in which a student's creativity could have lead to productive explorations in the MathWorlds. Instead, the tutor was constrained by her interpretation of what a velocity graph should be. In fact, it appears that for her, working with the MathWorlds consisted

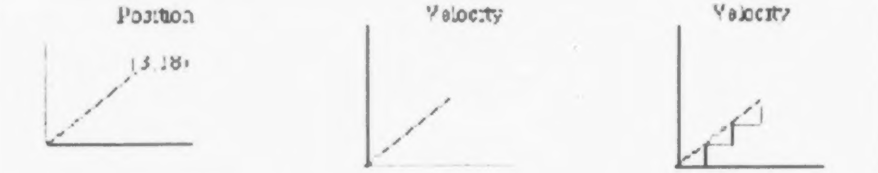

Figure 2(a). Position v. time graph

Figure 2(b). Student's drawing of velocity v. time graph

Figure 2(c). Preservice teacher's later drawing of [illegible] *explanation* of velocity v. time graph.

Figure 2

of playing a game of "guess and check" where she attempted to manipulate a graph to obtain a desired output.

Summary and Conclusions

In summary, we have found that the teachers' attempts to resolve conflicts between their expectations and what they saw on the screen supported and constrained their interpretations of the mathematics of change. Their insights often took the form of "Aha!" experiences in which formal theoretical relationships such as the Mean Value Theorem and the Fundamental Theorem of Calculus, which had been studied in their prior calculus studies, suddenly came to life. In our larger paper we explain in more detail our methodology of focusing not on the affordances of the software (e.g., the linked representations), but on how the teachers came to make sense of their activities as they acted with the graphs and simulations. In this way, we follow Cobb (in press) and Miera (1998) in suggesting that it is not the tool per se, but *acting with* the tool to reinterpret current ways of knowing that supports learning about both the content at hand, and more globally about teaching in general.

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TECHNOLOGY, STUDENTS' UNDERSTANDING OF GRAPHS, AND CLASSROOM INTERACTIONS

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Data collection devices used with graphing calculators support students' development of their understanding of graphs. The use of these tools provide new challenges in teaching and learning of mathematics and the changing interactions between teachers and students. The study involved a high school mathematics teacher teaching a week-long unit to her 70 geometry and algebra students. The research design included quantitative and qualitative data. Data collection included student pretest and posttest data, fieldnotes, videotapes, and student self-report data. A t test for non-independent samples revealed a significant difference ($p>.0001$) in pretest and posttest scores of students' understanding of graphs. Analysis of the fieldnotes, videotapes, and student self-report data reveal changing roles for teachers and students.

Students have difficulty interpreting graphs. They often misinterpret what a graph represents and ignore the global features of graphs such as rates of change, intervals of increase or decrease, and extrema. According to Herscovics (1989), "a picture is supposed to be worth a thousand words, but this does not prove to be quite true for secondary students looking at graphs" (p. 74). Bell and Janvier (1981) and Kerslake (1977) observed that students often confuse the graph with the actual event and mistakenly use the visual configuration of the graph to describe the actual event. Janvier (1981) found that students essentially read graphs in a "point-by-point" manner, like they read tables of values. Janvier observed that students were typically asked to plot a graph from a table of points but were not asked questions that focused their attention on the global features. Thus they tended to use a graph as a table of ordered pairs rather than for more general information extrapolated from the global features.

Data collection devices that augment the capabilities of graphing calculators have the potential to focus instruction on the global features of the graphs rather than the point plotting. These devices seem to be especially valuable for enhancing students' ability to interpret graphs and create graphical representations of actual events. With these devices, students can collect real time data of actual events, making and testing conjectures about the graphs of these events. The purpose of this study was to investigate the use of data collection devices, specifically motion detectors, on students' ability to interpret and predict graphs (i.e., distance vs. time and velocity vs. time graphs) and on classroom interactions.

Graphing calculators and data collection devices are increasingly accessible to the mathematics community. Recent reform initiatives support the use of these technologies in the implementation of recommendations proposing "greater emphasis on conceptual understanding, on algebra as a means of representation, and on algebraic methods as a problem solving tool" (National Council of Teachers of Mathematics (NCTM, 1989, p. 150). The small amount of existing research on graphing calculators and data collection devices suggests a significant impact of these technologies on students' development of concepts and students' classroom interactions. Ruthven (1990) found evidence that graphing calculators promote the recognition of classes of functions both graphically and symbolically. Quesada and Maxwell (1994) found that college students who were taught precalculus with graphing calculators had significantly higher scores on a common final exam than those taught by traditional methods. Slavit (1996) observed that the use of graphing calculators in an algebra classroom was associated with higher levels of discourse including higher-level questioning by the instructor and more active learning on the part of the students. Borba and Confrey (1996) stressed the importance that teaching mathematics with technology is more than placing mathematics on a machine; it necessitates the evolution of curricula along with the technology and an understanding that student approaches will continue to change as a result of the resources made available to them.

Theoretical Framework

For this study, a unit that integrated the use of graphing calculators and motion detectors to help students develop their ability to interpret and predict graphs was developed. Vygotsky's (1978) "zone of proximal development" framework guided the construction of the unit. According to Vygotsky, the development of knowledge is a social process involving exploration, language, and communication. When an individual is within the zone of proximal development in the process of developing new knowledge, they can be assisted and guided to expand their understanding to a higher level. The instructional unit in this study engaged the students in explorations with and communication about graphs created with motion detectors.

Another framework used in the development of the instructional unit is based on Bruner's (1988) notion of enactive, iconic, and symbolic representations leading toward cognitive growth. This notion has been recast as a framework consisting of phases—manipulating, getting-a-sense-of, and articulating—within a developing spiral (Open University, 1982). The mathematics tasks in the unit engaged the students repeatedly in these three phases. Through the use of motion data collection devices, students manipulated graphs by exploring changes in the motion creating the graphs. As students discussed patterns and generalities in the graphs and the motion creating the graphs, they were engaged in the getting-a-sense of phase.

Through continued discussion and exploration, the students' mathematical ideas about time, distance and rate became increasingly manipulable within the articulation phase.

Methodology

This study was conducted in a high school in the southwest and involved a high school mathematics teacher and her 70 students (21 geometry and 49 algebra II students) in five classes (2 geometry and 3 algebra II). The week-long mathematics unit used in the study was developed by the researcher in collaboration with the teacher. All classes were taught by the teacher; the teacher learned to use the equipment through the support of the researcher. This investigation was conducted toward the end of the academic school year; thus the students had completed a majority of the mathematics taught in their respective courses. Also, all of the students had completed a course in algebra I.

The research design for this study involved qualitative and quantitative data collection. A pretest and posttest was used to measure students' growth in interpreting and predicting graphs. The test was composed of multiple choice and short answer questions involving descriptions or graphical representations of actual events. For example, on one item the students were given the following scenario:

> Kendra is speeding along the highway and is stopped by a police officer. The officer gives her a ticket and then she continues on her way. Make a graph of the speed as a function of time.

Videotapes and fieldnotes of each lesson were used to investigate classroom interactions and self-report data was used to gather student feedback on the unit.

Data analysis involved several phases. Initially, the student pretest and posttest scores were compared using a t-test for non-independent samples. Results across test items were analyzed further to look for growth patterns across types of questions. The videotapes and fieldnotes were analyzed for classroom interactions. The videotaped lessons were divided into segments that were coded according to types of interactions (i.e., role of teacher and students). The student self-report data were analyzed and coded into various categories. This data provided student feedback on the lessons and some insight about students' attitudes toward the classroom interactions promoted within the lessons.

Findings

Comparison of the pretest and posttest data indicated that the unit involving the use of data collection devices increased students' ability to interpret and predict graphs and their understanding of the global features of the graphs, such as rate of change. A t-test for non-independent samples revealed that the average mean difference of matched pretest and posttest scores of all the students was satistically significant at p>.0001 level (t = 14.24, df = 69). The percent of increase in the mean scores was 28.90%. Analyzing each item more closely revealed that all students increased their understanding of items involving distance vs. time; however, on items involving velocity vs. time, student understanding did not increase equally. Several students treated these items as distance vs. time graphs. Students' understanding of the motion detector and distance vs. time graphs did not easily translate into an understanding of velocity vs. time graphs.

Based on the analysis of the videotapes, fieldnotes, and informal conversations with the teacher, the classroom interactions between the students and teacher found in these lessons were distinct from the typical interactions in these classes (i.e., traditional teacher-directed class). The teacher became a resource person/facilitator, a fellow investigator, a consultant, and a technology assistant. In the role of resource person/facilitator, the teacher provided graphs and scenarios for the students to explore with the motion detectors and facilitated students' discussion of their explorations and findings in whole group and small group discussions. As a fellow investigator, the teacher was responsible for encouraging students explorations and problem posing/solving. For example, when the students tried to model a circle with the motion detector, a device that collects one data point per instant in time, teacher encouragement lead to some interesting "what if" solutions on the part of the students. In the role of consultant, the teacher was available for students to ask questions about their explorations and their thinking in relation to the graphs they were making. As a technology trouble-shooter, the teacher provided assistance and troubleshooting with the technology being used. For example, when the students did not quit a program correctly and their calculator was caught in a loop, the teacher was called on to remedy the problem.

During these lessons, the students became investigators, explainers, collaborators, and problem solvers/posers. During these lessons, the students were engaged in much exploration and investigation of graphs, actual motion used to create the graphs, and the technological tools used to collect the data for the graphs. In small group and whole class discussions, the students were often engaged in explaining their ideas. For example, students explained to one another why a vertical line could not be made with the motion detector and the programs we were using to run it. As collaborators, the students helped one another with calculations and checked their own work against that of others in their small group. The students also posed and solved problems throughout the lessons. For example, they sketched a graph they thought they could make with a motion detector and then they attempted to make it. Also they posed "what if" type problems that helped them develop creative solutions to the question of creating a circle with the motion detector.

From an analysis of the self report data, students overwhelmingly reported that what they liked most about the unit was working in small groups with the motion detectors. The students felt that working in small groups gave them an opportunity to experiment and learn from analyzing their own motion graphs and provided an opportunity for them to help each other. In response to what students liked least, many students indicated that they enjoyed the entire unit. Of the responses that reported something liked least, finding the algebraic representations for piecewise-defined functions, seemed to be liked least. Looking back over the implementation of the unit provides insight for this attitude. Determining piecewise-defined functions was the last lesson taught and because of time constraints the students were not provided with the time needed to effectively explore this concept.

Conclusions

The results of this investigation support the use of data collection devices and motion detectors in developing students' understanding of graphs and their global features. This technology provides students with tools to experiment in creating different graphical representations that involve motion and time, two important variables in the analysis of real data. Through this experimentation in a social context, the students expanded their understanding of graphs and their roles in classroom interactions. Additionally, this study suggests the potential of this technology in promoting positive attitudes about learning mathematics.

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TECHNOLOGY
SHORT ORALS

WHAT KIND OF HEURISTICS DO CHILDREN USE TO EXPRESS CONVINCING ARGUMENTS IN A COMPUTER ENVIRONMENT?

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Given rich experiences and appropriately designed tasks that challenge their curiosity, children can build powerful representations of mathematical ideas. They often call upon previously built ideas, re-examine and extend them, when given opportunities to do so (Pirie & Kieren, 1994). Young children often build convincing arguments that lead to proof making when asked to explain their thinking (Maher & Martino, 1997; Maher & Martino, 1996). This research seeks to investigate children's reasoning and proof making within a two-session teaching experiment in a computer environment. Twenty-four fourth-grade students were individually presented with a combinatorial problem that was designed to elicit the use of heuristics and proof making. The children were provided with a set of tools from a computer environment that would enable them to manipulate objects on the screen. The guiding questions include: (1) What heuristics and strategies do children demonstrate? and (2) What justifications do children offer to support their solutions? The task-based interviews were given one month apart, and were conducted by the same researcher. Protocols guided the conduct of the interviews. Data collected include: (1) videotapes of the interview; (2) printouts of solution screens; and (3) notes made by the interviewer and the observer. Coded transcripts and solution printouts were examined by additional researchers to verify accuracy. The findings, consistent with previous research in non-computer environments, include a new technique that we call "skipping". The presentation will provide general results and a detailed case study (to include videotape excerpts) of one student's path toward proof building.

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TACKLING "HARD" OPERATIONS: A COMPARISON OF STUDENTS IN REFORM AND CONVENTIONAL MATHEMATICS CLASSES[1]

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Individual differences and similarities in mathematical knowledge and understanding are being closely analyzed as part of a longitudinal/cross-sectional study of the impact of a reform-based middle school mathematics curriculum. A student questionnaire was administered to approximately 1500 students in grades 5-7 who studied either a particular reform or conventional mathematics curriculum. This paper explores results of two open-ended questions from this questionnaire.

Responses with regard to age, gender and ethnicity, and inclinations of math class were entered into a database. Favorite operations such as, "fractions/add/subtract," and manipulatives such as, "using blocks" were among the groups' most common answers. Interestingly, after a query about least liked items divided the data into reform and conventional groups, both groups treated what they liked least about math, "having a lot of homework", differently. Further analysis showed a distinction between groups. The reform group was motivated to practice and master the "hard" homework in class listing class activities like "standing," "writing in a journal," and "working with a partner". Many in the conventional group did not list strategies to learn the "hard" concepts in or out of class. A query to further distinguish students' individual differences such as gender and ethnicity, and a closer examination of instructional formats found in teacher logs and classroom observations will bring to focus ways in which each groups' curricula complements student differences.

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A GEOMETRICAL ENVIRONMENT TO DEVELOP NUMBER AND OPERATION SENSES: A STUDY WITH 7-8 YEAR OLDS USING GRAPHIC CALCULATORS

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A number of studies have proved the potential of using the calculator's numerical environment in elementary school mathematics to help children develop number sense. The present report discusses some preliminary results from a study aimed at exploring the calculator's graphic capabilities to help young pupils develop number and operation senses for decimal numbers. The study was carried out with fifteen children (six seven year olds and nine eight year olds) who took part in a Mathematics Workshop (two hours per week during ten weeks). The theoretical referent adopted in this study relies on a didactical approach developed by the author through recasting Bruner's research on the acquisition of the mother tongue (1982, 1983).

Results

Meaning for decimal numbers. The tasks of the type "fill in the gap between linear graphs until it gets black" helped pupils realize the existence of fractional decimal numbers. *Relative magnitude of decimal numbers*. The work done by the children in the classroom and their answers during a post-interview suggest that the geometric relationship 'to be between two straight lines' helped children develop significant insights about the order for decimal numbers. *Operations sense*. Being supported by the calculator all the children but one were able to correctly answer questions of the type 'can you find two numbers that added give 0.321?'. Some theoretical and methodological issues as well as the results of the study will be wider discussed in the presentation.

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TECHNOLOGY POSTERS

USING SIMULATION TECHNOLOGY TO STIMULATE POWERFUL IDEAS ABOUT THE MATHEMATICS OF CHANGE

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The mission of the SimCalc Project is to enable ALL middle and high school students, beginning with those who are least advantaged, to develop full understanding and practical skills relating to mathematical ideas involving the Mathematics of Change and Variation, including fundamental concepts of Calculus. These powerful ideas have been shown to contextualize, enliven and organize other important mathematics that students are expected to learn and that many find difficult (e.g. signed numbers, areas of geometric figures, coordinate systems, decimal multiplication, slope, rate, ratio, and proportion). A combination of innovative curriculum and intelligent use of engaging technologies such as simulations and visualization tools make "advanced" topics (e.g. connections between varying quantities and their accumulation, velocity, acceleration, limits and other fundamentals of Calculus) accessible to students.

This research project is a collaboration of the SimCalc Project, Rutgers University-Newark and the Newark Public School System. Disadvantaged, at-risk students from Central High School in Newark have been working with SimCalc to learn concepts of signed numbers, prime and composite numbers, in the context of the mathematics of motion and velocity. These students who never have had the opportunity to work with technology or advanced mathematics have been given an opportunity to invent powerful ideas and reinforce ideas that previously had been problematic for them. The research involves documenting the language that the children used to express their mathematical thinking, how it was revised and refined over time, and how the motion simulation experiences helped to conceptualize the mathematics. This language has been captured via audio and videotape sessions and written responses to questions.

STUDENTS' USE OF REPRESENTATIONS IN MATHEMATICS EDUCATION

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The use of multiple representations with or without technology is one of the important topics in mathematics education that has been discussed for decades. The use of representations is advocated by many educators and also supported by the NCTM Standards (NCTM, 1989). Here, multiple representations are defined as external mathematical embodiments of ideas and concepts to provide the same information in more than one form. It is suggested that multiple representations provide an environment for students to abstract and understand major concepts (McArthur et. al., 1988; Yerushalmy, 1991). Thus, first we need to understand how students see and use these representations. In short, my questions are; what affects students' choice of representation to solve a mathematics problem? and how does the computer setting affect students' choice of representations? After observing a remedial college freshman mathematics class and having one computer lab hour with this class, I developed a survey based upon observation data and relevant literature. I asked their strategies and preferences related to the use of representations. I tried to find what affected the choice of representation, the rationales for using a particular representation and influences of technology. Previous knowledge and experience, personal preferences, and other external effects were the main themes.

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USING SATELLITE IMAGES TO CONNECT SCHOOL MATHEMATICS TO THE REAL WORLD

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The objective of this study was to develop the MSTISE (Mathematics, Science, and Technology for Individual, Society, and Environment) model with satellite images to enhance connecting school mathematics to the real world, as an integrated approach from elementary grades through high school grades.

The National Council of Teachers of Mathematics is fully supporting the integration of technologies and mathematics and recommending connecting mathematics to real-world situations (NCTM, 1989). However, the current classroom practice for mathematics education in schools is still far from this line of recommendation. As a plausible approach for solving that problem, the MSTISE model was introduced, and examplary lesson plans using the MSTISE model with satellite images was suggested. In addition, using the MSTISE model could help students develop their higher order thinking skills, problem solving skills, and facilitating use of communication and technology.

The study mainly used the satellite images that are produced by **t-ris**" (Telonics Remoting Imaging System). In the study, students schedule paths and process images in a real time, user-friendly environment. As students process their own data and examine their own images in real time, they are more curious, eager to learn, and willing to cooperate with other students (Busch, 1994). The technology with integration of other subjects could be an excellent vehicle for enhancing students' attitudes towards mathematics, science, and technology.

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WORLD WIDE WEB USE IN MATHEMATICS EDUCATION

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I create and maintain databases and web pages available for World Wide Web (WWW) use at the Office for Mathematics, Science, and Technology Education (MSTE) at the University of Illinois <http://www.mste.uiuc.edu>. This site and many other similar web sites available are mathematics education resources for parents, siblings, students, and teachers. The poster presentation features how these resources are being used and whether or not mathematics education benefits from the WWW.

The poster illustration includes defined and recounted history of the Internet and the WWW. Then I discuss the current methods of WWW use in mathematics education such as; stand alone lessons, helping students with mathematics questions, researching data sets, connecting to other classes, connecting to other teachers for lesson ideas, and connecting to businesses. The available uses show the possibilities in ideal circumstances, but only tell part of the story. Among disadvantages, many schools have discovered that educating teachers on computer and Internet use can be difficult and quite costly. However, Means and Olson stated that if teachers carefully integrate the Internet into the classroom then, "students discover concepts and apply new skills rather than to tell students how tasks should be done" (1994).

The poster presentation addresses implications for mathematics education and the need for further research on the effectiveness of WWW use in the mathematics classroom.

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UNDERSTANDING KINEMATIC GRAPHS: CONCEPTUAL CHANGE VIA MEDIATING TECHNOLOGY

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As the mathematics education community continues to embrace teaching with technology, important questions arise regarding the creation of beneficial paths of instruction. For example, when studying the phenomenon of motion, currently two "high-tech" methods of data analysis are available and developmentally appropriate for secondary students: (a) real-time and (b) virtual data collection systems. Real-time data collection refers to the ability to instantaneously see a motion graph "in the moment of the action" (for example, using a sonic motion detector and a graphing calculator). Virtual data collection involves examining concepts of position, rate, and acceleration after an event has occurred, usually through a digitized video recording of the incident. As with any representational system, each of these tools has merits and drawbacks. Thus, the challenge is to negotiate a "best" route for using the systems to mediate and promote meaningful learning.

To determine how each system induces conceptual change, eighth grade students were assigned to two treatment groups: the real-time data collection group and the virtual data collection group. Over a period of three weeks, students in both cohorts performed parallel tasks that challenged their intuitive notions about kinematic graphs. Transcripts of social interactions, students' writings, technological artifacts, and field notes were collected to identify themes between and within these groups. A qualitative analysis of the data suggest three trends: (1) students using the real-time data collection system benefit from their control over the physical phenomena, (2) students using the virtual data collection system benefit from the replay-ability of the virtual phenomena, and (3) students using the virtual data collection system benefit from the "hot-linked" representations (graph-to-movie, table-to-movie, and table-to-graph).

THE DISTANCE GAME—DEVELOPING MATHEMATICAL IDEAS

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The Distance Game is software which promotes understanding of numerous mathematical ideas. While the focus of the game is to find a randomly chosen point in a coordinate plane, students have been observed developing ideas other than that of graphing in the coordinate plane. While graphing in the coordinate plane, students are given the distance from their last chosen point to a randomly chosen point. When given this distance in decimal notation, students must make sense of what this number means to play more efficiently. Observations of two middle grade students playing this game revealed that they at first thought of the number as two separate numbers, i.e. 2.6 was seen as (2,6). As they continued to play and recognize that this was not efficient, they questioned what the distance in decimal notation really meant and began to make more sense of decimals, i.e. 2.6 was between 2 and 3. In addition to plotting points and revising ideas about decimals, students have opportunities to make sense of other mathematical ideas such as strategic reasoning, averages, units, and Pythagorean triplets.

Based on observations, students have been observed learning significant mathematics and making meaning of mathematics as the computer provides feedback. This poster session will share students' learning experiences with Distance Game as well as provide an opportunity for participants to experience the game.

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Whole Numbers Research Reports

LEARNING THE STANDARD ADDITION ALGORITHM IN A CHILD-CENTERED CLASSROOM

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Despite the NCTM's recommendation to create elementary mathematics instruction that builds on children's natural computation methods, there continues to be societal pressure to teach children the conventions of adult mathematics, including a specific procedure for addition ("carrying" or "trading"). Some studies have suggested that instruction in such algorithms interferes with children's development of numerical reasoning. This report examines children's learning of the conventional addition procedure in third grade, after several years of instruction that encouraged them to develop their own computational procedures. It was found that, in this context, the children did not "unlearn" place value and most maintained their self-generated computation methods, however, while some children benefited from this instruction, others did not.

The National Council of Teachers of Mathematics (1989) recommends creating a curriculum that builds on children's mathematical intuitions towards a strong conceptual foundation for their further mathematical learning. As teachers attempt to reform their instruction, they find there is tension between the goals of building instruction on children's thinking and of introducing children to the conventions of adult mathematics including symbols, language, and procedures. For example, much of the traditional K-3 curriculum is devoted to developing place value understandings, culminating in the standard written arithmetic procedure for addition ("carrying"). Research shows, however, that children naturally develop an understanding of the relative values of ones, tens, hundreds, etc. by constructing increasingly sophisticated counting strategies to solve addition problems (e.g., Cobb & Wheatley, 1988; Kamii, 1986). While the procedure usually taught in school works from right to left (ones to tens, then on to hundreds, etc.), children's self-generated addition methods generally involve working from left to right (e.g., Fuson et al., 1997). It is therefore not surprising that children often resort to learning these standardized algorithms through rote memorization and without understanding the underlying mathematical concepts (e.g., Labinowicz, 1985). McNeal (1995) shows how asking children to replace their own computational methods with the teacher's less-understood procedures can have the effect of reducing autonomous problem-solving behavior. Kamii (1994) makes a similar argument.

The teachers in The Miquon School, a private elementary school located just outside of Philadelphia, drew on the school's Deweyian philosophy and on their own knowledge of children's mathematics to reorganize instruction to support children's natural development of computational methods. The K-3 teachers agreed to delay teaching the conventional addition algorithm until third grade in order to foster children's development of a stronger conceptual foundation from which to learn the algorithm with understanding. Despite school administrators' consent to and parents' acceptance of this delay, the third grade teachers feel pressure to present the algorithm in order to keep students from falling behind those in more traditional schools.

Our study developed from one teacher's perception of difficulties in the teaching and learning of the standard addition algorithm. This teacher, Betty Tilley, felt tensions between her usual mode of instruction, which encouraged children's mathematical discoveries, and her desire to introduce them to the conventional method. She wanted to explore what her third graders were learning (or un-learning) when she introduced the algorithm, the cause of their difficulties, and whether the teaching of this procedure should be further delayed. In a study funded by the Spencer Foundation, we combined the perspectives of teacher researcher (Tilley) and university researcher (McNeal) in two interrelated strands of inquiry: 1) Tilley's management of the tension between the conventional procedure she feels she must teach and their personal methods of computation, and 2) the students' learning of this procedure in a particular classroom context. The strand of our inquiry focusing on the teacher's thinking is reported elsewhere (Tilley & McNeal, 1997). This report addresses the children's learning of the standard addition procedure.

Methods and Data Sources

To provide a description of Tilley's usual instruction for later comparisons, we videotaped mathematics lessons in her classroom once or twice a week from September to December, starting with the first week of school. Field notes taken by McNeal focused on children's computational strategies. Students' written work was collected for all videotaped lessons. Instruction in the addition procedure took place in February so we videotaped those classes and subsequent addition activities that we thought might be relevant.

Our report here is based primarily on clinical interviews similar to those of Kamii (1994) and Labinowicz (1985) with 18 of the 20 children supplemented by our other data. Prior to the study, Tilley had developed the strategy of postponing introduction of the algorithm as late in the year as her curricular plans would permit. Her aim in the autumn was to provide the children with multiple opportunities to develop their own strategies for solving addition problems involving regrouping.

The first set of interviews with the students was therefore scheduled in January to precede the targeted instruction in the standard algorithm. Interview tasks were selected to elicit students' computational strategies and explanations of those strategies using concrete materials. The chosen tasks match some of those used by Kamii (1994) and were posed using language with which the children were familiar. Students were asked to solve 16 + 17 written vertically; 98 + 67 and 7 + 52 + 186 (written horizontally) without pencils; and 4 + 35 + 24 written vertically but misaligned (the "4" was written in the tens' column). Students' understandings of place value were documented through analysis of their strategies and explanations. Vertical and horizontal problems were both posed in order to probe children's changes of strategy and to encourage them to use both the conventional and their own strategies, particularly in the final interviews. In May, about two months following instruction in the conventional procedure, the same tasks were posed again. If children used the conventional procedure, they were asked to explain it using concrete materials. If they used an invented strategy, the interviewer demonstrated the algorithm saying that another student had used this method and asked the student to explain this method using concrete materials.

Tilley worked hard to build instruction that would connect the children's invented strategies with the algorithm, and to find ways to make it problematic for her students. Having little success in building connections that were meaningful to the students, Tilley eventually decided that she had to simply tell the students the steps of the procedure. This mode of instruction conflicted with her usual style, and raised concerns for her that the students would lose their mathematical autonomy and interest. (For more details of Tilley's instruction, see Tilley & McNeal, 1997.) Analysis therefore focused on changes in children's mathematical thinking, in their reliance on the teacher, and in their attitudes towards mathematics (e.g., confidence in their answers).

Data and Analysis

In January, before instruction in the conventional algorithm, 12 and 13 of the 18 children solved 16 + 17 and 98 + 67, respectively, by starting with the tens (see Table 1). Of the remaining children, those who did not start with the ones solved these two problems by decomposing the addends in other ways, such as 98 + 67 = (98 + 2) + 65. Most students used unconventional ("other") ways to solve 7 + 52 + 186: eight began with 50 + 180, three began by adding 7 + 52, and one student first collected 100 from 186, then added 50 + 80, etc. The methods used for the last problem were split with half of the students beginning by adding the tens' digits.

Comparing the two interviews (Tables 1 and 2), we see that students used a variety of methods; no method completely disappeared after instruction had emphasized the conventional algorithm. While instruction focused on the addition algorithm, problems had primarily been written in vertical format, so the appearance of both horizontal and vertical format, three addends, and the requirement to do some "without pencils" probably influenced the children's choice of method. There is, however, a shift in the

Table 1
January Interview

	16 + 17	**98 + 67**	**7 + 52 + 186***	**4 + 35 + 24***
started with tens' digits	12 (67%)	13 (72%)	3 (18%)	9 (53%)
started with ones' digits	4 (22%)	2 (11%)	2 (12%)	6 (35%)
other method	2 (11%)	3 (17%)	12 (71%)	2 (12%)
single-digit language	0	1 (6%)	0	1 (6%)
place value language	18 (100%)	13 (72%)	14 (82%)	16 (94%)
single digit language, then switched	0	4 (22%)	3 (18%)	0
1st answer correct	15 (83%)	13 (72%)	9 (53%)	16 (94%)
self-corrected	17 (94%)	17 (94%)	15 (88%)	—

Total number of students = 18
* One student did not answer this question.

methods for the problems written in vertical format, 16 + 17 and 4 + 35 + 24. In May, the method of choice for these became starting with the ones' digits. This fits with the instruction. There was little change in the distribution of methods used to solve 7 + 52 + 186 which may be explained by the complexity of this problem and the requirement to do it "without pencils." Most interesting is that, for 98 + 67, 4 students (22% of 18) switched to beginning with the ones' digit to solve this in May, despite the requirement to solve it mentally. Possible explanations include a greater level of comfort with this newer method or the development of new routines due to instruction emphasizing this procedure.

In January, there were only 2 occurrences of students using "single-digit language" to describe their solutions. That is, no one explained 16 + 17 by saying that 6 + 7 = 13 and 1 + 1 + 1 = 3. Most students used "place-value language," spontaneously describing, for example, the "9" in "98" as "90." Some students' explanations of 98 + 67 and 7 + 52 + 186 included switching from single-digit back to place-value language by saying for example, "9 + 6 = 15, so 90 + 60 = 150." We took this as a primary indicator of place value understanding.

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Table 2
May Interview

	16 + 17	98 + 67	7 + 52 + 186*	4 + 35 + 24*
started with tens' digits	3 (17%)	8 (44%)	3 (17%)	3 (17%)
started with ones' digits	11 (61%)	6 (33%)	3 (17%)	11 (61%)
other method	4 (22%)	4 (22%)	12 (67%)	3? (?)
single-digit language	1 (6%)	2 (11%)	1 (6%)	5 (28%)
place value language	13 (72%)	12 (67%)	17 (94%)	9 (50%)
single digit language, then switched	4 (22%)	4 (22%)	0	3? (?)
1st answer correct	17 (94%)	13 (83%)	11 (61%)	11 (61%)
self-corrected	–	17 (94%)	18 (100%)	17 (94%)

In May, most students continued to use place-value language. Although the total number of occurrences of single-digit language increased from 2 to 9, two students account for 5 of these 9 with 4 students using single-digit language just once in this interview.

Finally, no student asked about the "4" in 4 + 35 + 24 prior to instruction in the conventional procedure (data not visible in either table). One student did answer 99 and did not correct this when asked to reread his work. In May, nine students responded differently than they had before: five asked about the meaning of the "4" before producing a correct interpretation and answer, two responded with "99" which they self-corrected, and one student corrected his answer when prompted to reread the problem and his solution. We interpreted the questioning of the meaning of "4" to indicate their awareness of a contradiction in the position of the numeral and its meaning. No student left their answer as 99.

Conclusions

Looking across students, we found their experiences fell into 3 categories: some benefited from the new procedure (2 students produced more correct answers including reasonable explanations); some adopted it with ease (7 students used and explained the algorithm as if it were their own); and some adopted it with difficulty (9 students used the procedure correctly but faltered in their explanations). Of this last group, 5 students showed shaky confidence and expressed undesirable emotions toward the algorithm during the final interview (e.g., "I don't like that math," "we just practiced it...not really getting the idea of what's happening", "I'm supposed to do it this way").

Looking across students' activities in class and their methods and understandings prior to instruction in the algorithm, we were struck by the complexity of their different responses to instruction in this conventional procedure. The students whose strategies had differed considerably from the standard procedure were those who had difficulty and even negative emotional responses to using the new procedure, despite their prior confidence and accuracy. The 7 students who were not only comfortable and accurate using the algorithm, but could flexibly select the written procedure in some contexts and use self-generated mental strategies in others, were those children who had begun to develop their own versions of the standard procedure, such as aligning addends vertically for their own convenience, prior to instruction. The two students who benefited from introduction to this procedure, Jon and Tom, benefited for very different reasons. Jon was the weakest in the class in his understanding of place value and his flexibility of thinking, yet he seemed to blossom under the step-by-step instruction. The conventional procedure seemed to provide an opportunity to engage in the same work as his classmates and to derive confidence from the structure and predictability of the procedure. Tom was a sophisticated and logical mathematical thinker who had difficulty keeping track of his mental calculations. The conventional procedure seemed to free him to consider the meaning of the steps of the procedure and the patterns evident in applying it repeatedly in the case of multi-digit addends.

We cannot confirm Kamii's claim that instruction in the algorithm is "harmful to children's development of numerical reasoning" (1997, p. 58). In Tilley's classroom, the students did not "unlearn" place value; they all showed an understanding of place value before and after instruction in the algorithm, and overall did at least as well at producing correct answers on the first try in May. This leaves us questioning which contexts encourage "unlearning" and which do not.

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MEGAN: "SEVENTEEN TAKE AWAY SIXTEEN? THAT'S HARD!"

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In this paper the specific concern for improved numeracy skills is addressed through a discussion of an innovative mathematics early intervention program which the author and classroom teachers have developed and trialed. This research has highlighted the vast difference in children's mathematical knowledge and the type of whole number strategies they used to solve tasks set in different contexts. Children who were successful with the tasks from the initial interviews appeared to exhibit proceptual thought (Gray & Tall, 1994). However children experiencing difficulties with mathematics used procedural strategies such as "count-all" or tried to remember rules and procedures. For example, Megan (Year 3) was heard to say "Seventeen take away sixteen. That's hard!" For Megan it was hard. Like many children who are struggling with mathematics she was reliant on remembering rules and procedures even when these were inefficient and unreliable.

Introduction

There is considerable concern currently being expressed by teachers, parents and politicians about the standard of literacy and numeracy skills of young Australians. In March 1997 Australian state and territory education ministers agreed to a national goal that "every child leaving primary school should be numerate, and be able to read, write and spell at an appropriate level" (Masters and Forster, 1997, p.1). A national plan to support this goal requires education authorities to provide support for teachers in their task of identifying children who are not achieving adequate literacy and numeracy skills and in providing early intervention strategies for these students.

This paper focuses on *Mathematics Intervention*, a collaborative project involving the principal and staff of Boroondara Park Primary and mathematics educators from La Trobe University (Pearn, Merrifield & Mihalic, 1994; Pearn & Merrifield, 1996). *Mathematics Intervention* aims to identify, then assist, children in Year 1 at risk of not coping with the mathematics curriculum as documented in the National Statement on Mathematics for Australian Schools (Australian Education Council, 1991). The program features elements of both Reading Recovery (Clay, 1987) and Mathematics Recovery (Wright, 1991) and offers children the chance to experience success in mathematics by developing the basic concepts of number upon which they build their understanding of mathematics. Children are withdrawn from their classes and work in small groups to assist with the development of their mathematical skills and strategies.

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In 1997 testing was extended to include students from Years 3 and 4. In one of these clinical interviews, Megan (Year 3) was heard to say "Seventeen take away sixteen. That's hard!" For Megan it was hard. She had been given a card on which was written 17 - 16 and had been asked to give an answer as quickly as she could. She saw this purely as an algorithm which could only be solved as seventeen things take away sixteen of those things. As she used her fingers to solve most mathematical tasks this was in fact very difficult.

Theoretical Framework

Mathematics Intervention incorporates mathematical activities and strategies based on recent research about children's early arithmetical learning (Steffe, von Glasersfeld, Richards & Cobb, 1983; Wright, 1991) and about the types of strategies used by children to demonstrate their mathematical knowledge (Gray & Tall, 1994).

The five counting stages were developed in theoretical work by Steffe, Cobb, von Glasersfeld and Richards (1983) and in summary are:

1. *Perceptual* . Students are limited to counting items they can perceive.
2. *Figurative*. Students count from one when solving addition problems with screened collections. They appear to visualize the items and all movements are important. (Often typified by the hand waving over hidden objects.) If required to add two collections of six and three items, students must first count the six items to understand the meaning of "six", then count the three, then count the whole collection of "six and three" items.
3. *Initial number sequence.* Students "count on" to solve addition and missing addend problems with screened collections. Students no longer count from one but begin with the appropriate number. If adding two collections of six and three, students commence counting with six and then count on: That is, six, seven, eight, nine.
4. *Implicitly nested number sequence.* Students focus on the collection of unit items as one thing, as well as the abstract unit items. They can "count-on" and "count-down", choosing the most appropriate to solve problems. They generally count down to solve subtraction problems.
5. *Explicitly nested number sequence.* Students are simultaneously aware of two number sequences and can disembed smaller composite units from the composite unit that contains it, and then compare them. They understand that addition and subtraction are inverse operations.

Research studies by Gray and Tall (1994) have shown that young children who are successful with mathematics use different types of strategies to those who are struggling with mathematics. For Megan, when solving

17-16 the focus was on the procedure of counting back by ones from 17 and not on the relationship between seventeen and sixteen. Students like Megan, struggling with mathematics, are usually procedural thinkers dependent on the procedure of counting and limited to the "count-all" and "count-back" procedures. Gray and Tall (1994) defined procedural thinking as being demonstrated when: "... numbers are used only as concrete entities to be manipulated through a counting process. The emphasis on the procedure reduces the focus on the relationship between input and output, often leading to idiosyncratic extensions of the counting procedure that may not generalize" (p. 132).

Successful Year 3 and 4 students chose the appropriate strategy for each task and were more flexible in their mathematical thinking. In response to the task: 17 -16, successful students usually gave instantaneous answers and, when asked to explain, responded "I know that 17 is one more than 16." For these students the task was almost trivial. According to Gray and Tall (1994) use of known facts and procedures to solve problems, along with the demonstration of a combination of conceptual thinking and procedural thinking, indicated that these children were proceptual thinkers. Gray and Tall (1994) defined proceptual thinking as: "... the flexible facility to ... enable(s) a symbol to be maintained in short-term memory in a compact form for mental manipulation or to trigger a sequence of actions in time to carry out a mental process. It includes both concepts to know and processes to do" (pp. 124-125).

Methods or Modes of Inquiry

The initial assessment for *Mathematics Intervention* requires teachers to assess the extent of the child's mathematical knowledge by observing and interpreting the child's actions as he/she works on a set task. Children were clinically interviewed using tasks developed by teachers to evaluate children's understandings and mathematical skills. A clinical interview is described in Hunting and Doig (1992) as: " ...a dialogue or conversation ... held between an adult interviewer and a subject. The dialogue is centered around a problem or a task which has been chosen to give the subject every opportunity to display behavior from which mental mechanisms used in thinking about that task or solving that problem can be inferred" (p. 203). Each interviewer observed children carefully to determine the strategies being used to solve the tasks as it was essential to understand whether they were counting physical objects, counting on, counting back or "just knew it".

Data sources or evidence

Over the last five years 275 children from Years 1 and 2 at Boroondara Park Primary School have been individually interviewed by a clinically trained teacher using an instrument designed, administered and consequently

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modified by three teachers. Interviews were conducted at the beginning of each new school year. A shortened interview was developed using "critical tasks" for Year 1 students which were felt to accurately predict those children needing to be included in the program (Pearn, Merrifield, Mihalic, & Hunting, 1994). This modified interview takes ten minutes and has been used since 1994 (Pearn & Merrifield, 1996). The interview included verbal counting tasks and two tasks based on the counting stages.

In 1997 testing was extended into Years 3 and 4 when 57 students were interviewed by two teachers using tasks being developed and trialed by two of the teachers involved in the original Year 1 testing. The Years 3 and 4 interview (Pearn & Merrifield, 1997) included tasks testing knowledge of verbal counting sequences, whole numbers, oral responses to addition and subtraction facts and word problems. These were designed so children were given every opportunity to demonstrate the various strategies they used in solving mathematical tasks.

Results

This research has highlighted the vast difference in children's mathematical knowledge and the type of whole number strategies they use when solving tasks set in different contexts. Children from Year 1 and Year 2 successful with the tasks from the initial interviews were able to count by ones forwards and backwards, counted fluently by twos, fives and tens from a given number, and demonstrated their ability to choose and use the appropriate strategy of count on and count back. These children appeared to exhibit proceptual thought (Gray & Tall, 1994) and were at Stage 4 or 5 of the Counting Stages.

Clinical interview results indicated that Year 1 and Year 2 children requiring *Mathematics Intervention* were experiencing difficulties with the verbal counting sequence and were at either Stage 1 or Stage 2 of the Counting Stages. That is, they used the procedural strategy of "count-all". When unsure of an answer these children would guess with no attempt to confirm their answer. One of the most significant findings from five years of testing of Year 1 students has confirmed that there appears to be a link between children needing both *Mathematics Intervention* and Reading Recovery. There appears to be a need for further research and the necessity for a more integrated approach in teaching mathematics.

Results from Year 3 and Year 4 interviews have shown that children successful with mathematics are flexible in their mathematical thinking and use a variety of strategies and usually at Counting Stage 4 or 5. Children struggling with mathematics relied on rules and procedures even when these were inefficient and unreliable. These students were at Counting Stage 3, that is, they could "count on" and could not "count back" or "down to" as evidenced by the number who were unsuccessful with subtraction .

Table 1
Results of 1997 Grade 3 and 4 whole number tasks (in percentages).

Year Level	63 = 39 + Δ	52 - 17 =	subtraction facts	subtraction problem	addition facts	addition problem	multiplication problem	division problem
Year 3 n = 24	46	42	54	42	71	88	63	63
Year 4 n = 33	73	67	61	52	70	91	55	79
Total n = 57	61	56	58	47	70	89	58	72

Most Year 3 and Year 4 students were successful with verbal counting tasks and in identifying the number 'one more than', 'ten more than', and '100 less than' 342. But, as shown in Table 1, students found the missing addend and subtraction tasks much more difficult due to a reliance on rules and procedures. Hiebert and Lefevre (1986) commented on the reliance of Grade 3 and 4 students on rules and procedures and noted that: "By the time students are in third and fourth grade, they have acquired a large array of symbol manipulation rules. In general, the rules are more sensitive to syntactic constraints than to conceptual underpinnings" (pp. 20-21). For example, six students from the same class all responded that 52 - 17 was 45 because "you take the small number from the large one". This difficulty with subtraction is also reflected in Table 2 where typical responses to the subtraction word problem are given. Although most students were able to identify the word problem as subtraction, 53% of the students were unable to complete the computation successfully.

Table 2
Typical responses to the subtraction word task.

Task	Lynda	Mike	Barry
Richard is 131 cms tall. Mary is 17 cms shorter than Richard. How tall is Mary?	Successfully counted back by ones from 131 keeping track on her fingers. Gave correct answer	Took 10 away from 31 then 7 away from 21. Gave correct answer.	Written algorithm. 131 - 17 126 Incorrect answer.

Conclusion

This paper highlights the differences between successful students' strategies and strategies used by students "at risk". Like many children who are struggling with mathematics Megan was reliant on remembering rules and procedures even when these were inefficient and unreliable. Students "at risk" of not succeeding at mathematics, do not possess either "rich mathematical ideas" or have the powerful strategies that will enable them to use their mathematical knowledge to improve and enhance their mathematical thinking.

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THINKING IN UNITS: AN ALTERNATIVE TO COUNTING AS A BASIS FOR CONSTRUCTING NUMBER

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This paper suggests that over-emphasis on counting in young children's mathematical activity can have negative effects while a collections approach that encourages the construction of units develop powerful thinking strategies for adding and subtracting. While counting plays an important role in doing mathematics, many children who rely on counting often do not move to more meaningful ways of adding and subtracting, even in middle grades. An approach to number that emphasizes thinking in collections using dot patterns is described. Evidence is provided for the effectiveness of activities that encourage the construction of units while discouraging counting. In the collections approach, mental imagery plays an important role in number constructions.

Many factors contribute to a child's construction of number in an abstract sense. Steffe et. al. (1983) present a theory of number development that is based on counting. In this paper we are suggesting that activities that encourage counting may be limiting for some students. An approach to early number that emphasizes units rather than counting is described.

For a number of years we have been engaged in research into children's mathematical thinking, particularly their construction of number and the imagery involved in their sense making. Our research has focused on individual students as well as whole class settings at several age levels. In each case the research is in the tradition of a teaching experiment approach, as elaborated by Steffe (1991), where we have interacted with students over a year or longer as we posed tasks that had the potential to engage them in learning and thus probed their emerging mathematical constructions. In developing our thesis in this paper we draw on this research base.

Preliminary investigations suggest that many students, even in middle school, still rely on counting as a primary method of adding. For example, to determine 8 + 7, many grade seven students will count-on saying, 9, 10, 11, 12, 13, 14, 15. These students seem to have stagnated in their construction of number with an inefficient, procedural method. That is, they have developed neither number relationships nor efficient procedures and obviously have not "memorized" 8 + 7 = 15. Counting dominates their thinking and unfortunately remains their method of finding sums and differences. It also seems that efforts to get students to memorize are not being successful. On the other hand, students who think in collections and use thinking strategies are able to approach both routine and nonroutine mathematics tasks flexibly and effectively (Nicholls, et. al., 1991). When they use thinking strategies, students construct a network of schemes that leads to rapid responses to addition and subtraction situations as well as using relationships they have constructed.

As students enter school, learning to count is one of their central mathematical activities. When students begin determining the total number in two sets they will usually count-all and later move to counting-on. Much is learned through counting. However, an over- reliance on counting rather than number patterns leads many students to continue using counting while others have shifted to more powerful reasoning. In the primary grades there is often a push for students to memorize addition facts. No grade three teacher wants to see students counting to find sums. Teachers work very hard to help students memorize their addition facts. So why are so many students still using counting in grade 7?

While it is recognized that addition and subtraction algorithms taught in school are procedural and may not be meaningful to young students, rarely has counting been viewed in the same way. When students operate procedurally, attempting to remember the steps in the procedure, they may not give meaning to their activity. But in the same way, counting may have little meaning. In an interview setting, Alex (grade one) counted a set of six objects and said there were six. When then asked to solve a problem using the set, he recounted them (in fact, more than once) as he tried to use a counting strategy. He had to make the six each time - he had not constructed six as a mathematical object. His counting was a procedure that did not result in taking the set as a collection with numerosity. Wirtz (1980) noted that counting seemed to interrupt the flow of thinking about what was going on. He wrote, "... during the time involved in counting one-by-one, the connection between the intention (i. e., comparing dominoes) and the outcome broke down" (p. 3). This is a stinging indictment of counting as the basis for early mathematical activity.

By a collections approach to early number development we mean focusing on students construction of units (Wheatley & Reynolds, 1996). A child has developed an abstract concept of number when she has mental imagery associated with a numeral, number word or set of objects and can flexibly use the unit in adding or subtracting and can decompose the unit into singleton units. The development of thinking strategies such as compensation (7 + 9 is 16 because 8 + 8 =16; taking one from nine is compensated for by putting one with seven) is enhanced by thinking in units. In a collections approach, students may at first only form pattern images, what Piaget (1952) calls a figural pattern as opposed to operational knowledge. As children experience fourness in many settings they can abstract four from the particular objects or picture seen and their knowing becomes operational.

A student has constructed four as a mathematical object when the unit (four) simultaneously is four separate units and one unit. Counting does not encourage this construction since it is a sequential activity and the last word said is the name of the abstract number; there is a tendency for students to think only of the single objects and not "make" the collection. Tammy, a third grade student, was so routinized to counting that she could not give meaning to number abstractly. She would look at a domino and count the dots rather than knowing the number based on the pattern. It was as if the collection did not exist, only the single dots. Megan, a fifth grader who had been taught using Touch Math in which points on numerals are counted in adding and subtracting, had made little sense of numbers and could not function in a grade five mathematics class. Her counting-on using points on numerals was very fast and efficient. In fact, it was so efficient that she did not feel the need to develop another method of adding in school mathematics settings. Counting seemed to block her sense making. On the other hand, Adam, a first grader, could deal meaningfully with arrangements of six or seven dots even though his response to 3 + 1 = was to haltingly count-all. Adam had developed a collections scheme of number. By this we mean he could take six as a unit composed of units.

In a year-long study, second grade students experienced a collections approach to number with thinking strategies as a goal. In one activity used frequently, students were briefly shown an arrangement of transparent plastic chips in a pattern on an overhead but not long enough to count and asked to determine how many. As the year progressed, the students moved easily into using thinking strategies to add and did not rely on counting. Through this and other activities, students formed images of dot patterns from which they could abstract number. The students became mathematically powerful as demonstrated on paper and pencil tests and in clinical interviews. Their journals also provided evidence of their abstract sense of number.

Activities with ten frames (Wirtz, 1980) in which some dots in a 5 by 2 grid are shown briefly on the overhead and students are asked to say how many they saw, encourage thinking in collections rather than counting. Through use of this activity students come to think in units rather than just sequentially saying number names. In Norman, Oklahoma a grade two class used ten frames extensively along with other activities (Wheatley, 1996) that encourage the use of imagery and showed evidence of thinking in collections. For example, two ten frames were shown briefly on an overhead projector as seen in Figure 1. The following transcript indicates how students transformed an image of dot patterns in determining the total number of dots.

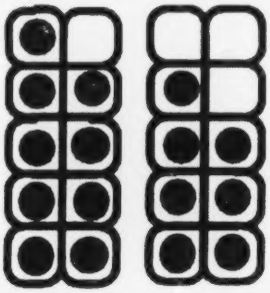

Figure 1. *A Double Ten Frame Display*

Meliah: "I took those over to those" (indicating the 3 dots at the top of columns in the left ten frame, that she mentally 'moved' to fill the right ten frame).

Teacher: "Why?"

Meliah: "Because if you do that you got six left and ten on that side."

Kelly: "I saw four empty boxes; I knew that 20 minus 4 would be 16."

Cale: "I saw 4 and 4 and 4 and 3 and 1 (meaning 4s arranged as 2 rows of two dots, starting with the left ten frame seeing four at the bottom and another four above that, then the right ten frame with four at the bottom, then the remaining 3 on the right and 1 on the left)."

The examples above show the different ways students determined the total number of dots. Meliah transformed her image of the ten frames by moving dots from the left ten frame to the right with the intention of making a ten. She was then able to use her knowledge of our numeration system to combine six and ten to make 16. Cale decomposed the collection into manageable units that he could combine. Kelly used an even more sophisticated strategy involving complements. Knowing there were 20 spaces in the two ten frames, she took the number of empty spaces as a mathematical object (4) and was able to formulate the subtraction task, 20 - 4, and determine the difference to be 16. She just knew 20 - 4 = 16. In each case, there was evidence of students thinking in collections that they could transform rather than relying on rote counting.

These students were reasoning with numbers without relying on any procedure such as counting. This activity of mentally transforming dots in ten frames led to the use of thinking strategies in addition. For example, on another day in determining 9 + 7, Meliah said she made the 9 into 10 by moving 1 from the 7 to the 9 so that it was 10 and 6. There was no need of her to count on from 9 by ones as so many middle grades students still do. We also contend this way of thinking about addition is superior to just memorizing facts - they are giving meaning to their activity. As the students continue to do addition problems in this manner, they come to know their addition combinations without drill and timed tests. Wirtz (1980) observed that the children enjoyed 'pushing the dots around in their heads.' Wheatley and Reynolds (1996) provide evidence for the power of mental transformations of images as children construct number concepts.

Imaging that is initially based on figural patterns of objects can be useful to children in constructing number in a meaningful way - a way that leads to using powerful thinking strategies for adding and subtracting. Grade one and even younger students can construct number from patterns of dots and subsequently use this figural material in a process of reflective abstraction to construct number (von Glasersfeld, 1987, p. 297) . Hatano (1979) developed a successful system of early arithmetic instruction based initially on figural patterns from which students developed number relationships. In commenting on a collections approach, Wirtz (1980) wrote, "When they become more involved with written symbols, those symbols will have a deeply ingrained referent in the experience of the learners." (p. 5)

Through activities with dot patterns, ten frames and other activities that encourage thinking in collections, young students construct an abstract concept of number without resorting to a procedure to "make" the number each time by counting. Of course, the collections approach in which a child can mentally transform a set to a familiar pattern for which a number name is known, depends on conservation of number (Piaget, 1952). But to encourage counting can actually interfere with the construction of number in that it is a rote process that may be devoid of any mathematical meaning. This is not to say that counting cannot be meaningful but that for many students it becomes a substitute for sense making.

Summary

This paper outlines an alternative to the construction of an abstract concept of number based on counting. Students will always count and it is a useful procedure. However, students who rely heavily on counting in computing and problem solving may not be constructing essential mathematical relationships necessary for success in more demanding mathematics tasks. While counting is an important component of acting mathematically and students may make important mathematical constructions by curtailing their counting activity, there is the real danger they will come to rely on this procedure as a method for getting answers without reflecting on their activity. Tasks that encourage students to think in collections provide rich opportunities to construct abstract mathematical relationships and become powerful problem solvers. Rather than viewing counting as the basis for number constructions, we present evidence that a collections approach that encourages the construction of units may be more effective. Children build mental images of dot patterns that become the material for abstracting number. Thinking in collections and using thinking strategies paves the way for quantitative reasoning as described by Smith (1997).

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WHOLE NUMBERS
POSTERS

PLACE VALUE UNDERSTANDING OF MONTESSORI STUDENTS

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Research has shown that elementary school students have poor place value understanding, a concept vitally important for understanding most of arithmetic. Several factors are seen to be responsible for this: lack of uniformity in the English system of naming numbers, textbook instruction that focuses on memorization, and over-reliance on manipulatives that represent adult ways of "seeing" place value concepts. Although the English system will not be readily changed, reformers have sought instructional methods and materials to foster deeper understanding.

Maria Montessori (1870-1952) developed a philosophy of instruction that melds methods and materials—one that creates a supportive environment for a child's natural physical, intellectual, and emotional growth. The role of the teacher is to "follow the child," learning about the child's thinking by observing his activity. The teacher can then provide follow-up materials, create new materials, or modify existing materials to help the child reach a deeper level of understanding. Thus, by learning from many children about their growth in understanding, Montessori developed materials seriated to promote conceptual progression of many topics, including place value (1973).

To determine the effects of a Montessori environment on students' mathematical knowledge and concepts, this study examines the place value understanding of first-, second-, and third-grade Montessori students. Videotaped clinical interviews using tasks found in Cobb and Wheatley (1988) will be analyzed to determine trends in place value understanding across grade level and type of Montessori school. Qualitative analysis of pilot study data suggests that students from Montessori preschools hold a deeper understanding of place value concepts.

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CHILDREN'S MULTIPLICATIVE REASONING

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Research shows that students have intuitions about mathematics long before they experience formal mathematics instruction (Resnick, 1986). Unfortunately, they often have difficulty connecting their intuitions with formal mathematics. In an attempt to examine students' formal and intuitive representations of multiplication, we interviewed 28 sixth-graders and asked them to complete several tasks related to multiplication.

Multiplication is an operation that is "taught" to students beginning in the third grade, and is "retaught" every year thereafter. One would assume that by the time a student is in the sixth grade, he or she would have a thorough understanding of multiplication. However, when students were asked to generate a multiplicative word problem in context, less than half of them were successful in their attempts. The responses varied from addition to subtraction, division, and even comparison of fractions. For the students who wrote non-multiplicative word problems, the interviewers attempted to put the student in disequilibrium by having the student model the problem using manipulatives. Even when the students saw that their model yielded different results, they invented reasons to justify the original answer. This poster session will address the rationalizations that these students made, as well as possible implications for teaching multiplication in context.

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